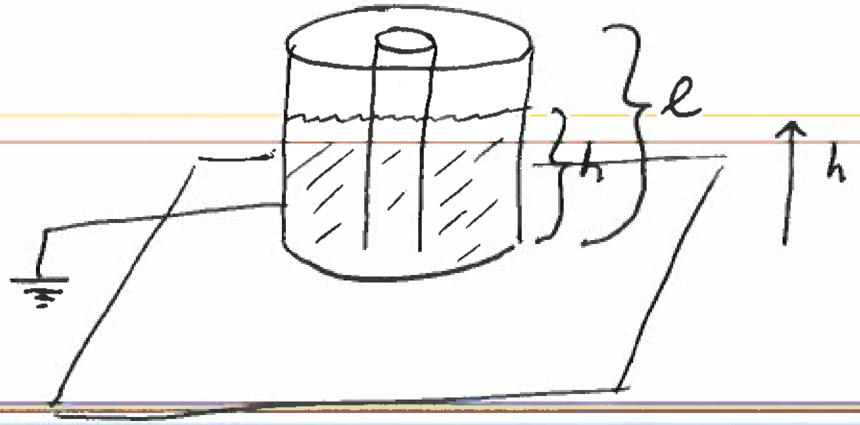

inner cylinder radius a, charge $-Q$

outer cylinder radius b, charge $+Q$

inner cylinder is at electric potential V, outer at 0

dielectric oil has χ_e and density ρ

σ is the surface charge per area in air, σ' in oil

First, find the capacitance of the system:

In air:

$$\int \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E\, 2\pi s \ell' = \frac{-\sigma 2\pi a \ell'}{\epsilon_0}$$

$$\vec{E} = -\frac{a\sigma}{\epsilon_0 s}\hat{s}$$

$$-\int \vec{E}\cdot d\vec{\ell} = V$$

$$V = -\int_a^b -\frac{a\sigma\, ds}{\epsilon_0 s} = \frac{a\sigma}{\epsilon_0}\ln s\Big]_a^b = \frac{a\sigma}{\epsilon_0}\ln\frac{b}{a}$$

In oil:

$$\int \vec{D}\cdot d\vec{a} = Q_{enc,\,free}$$

$$\vec{D} = -\frac{a\sigma'}{s}\hat{s}$$

$$\vec{D} = \epsilon\vec{E} \Rightarrow \vec{E} = -\frac{a\sigma'}{\epsilon s}\hat{s} \Longrightarrow V = \frac{a\sigma'}{\epsilon}\ln\frac{b}{a}$$

Since both the air & oil parts are at $\Delta V = V - 0 = V$

set $$\frac{a\sigma}{\epsilon_0} \ln \frac{b}{a} = \frac{a\sigma'}{\epsilon} \ln \frac{b}{a}$$

so $$\frac{\sigma'}{\epsilon} = \frac{\sigma}{\epsilon_0} \implies \sigma' = \frac{\epsilon}{\epsilon_0}\sigma = \epsilon_r \sigma$$

Find the total charge Q:

$$Q = \sigma' h 2\pi a + \sigma(\ell - h) 2\pi a$$

$$= \epsilon_r \sigma h 2\pi a + \sigma \ell 2\pi a - \sigma h 2\pi a$$

$$= \sigma 2\pi a h(\epsilon_r - 1) + 2\pi a \ell \sigma$$

$$= \sigma 2\pi a h \chi_e + \sigma 2\pi a \ell$$

$$= \sigma 2\pi a(\chi_e h + \ell)$$

$$C = \frac{Q}{V} = \frac{\sigma 2\pi a(\chi_e h + \ell)}{\frac{a\sigma}{\epsilon_0} \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{2\pi\epsilon_0(\chi_e h + \ell)}{\ln\left(\frac{b}{a}\right)}$$

$$F_{upward} = \frac{dW}{dh} \qquad W = \frac{1}{2} V^2 C$$

(holding V constant)

$$F_{up} = \frac{1}{2} V^2 \frac{dC}{dh}$$

$$\frac{dC}{dh} = \frac{2\pi\epsilon_0 \chi_e}{\ln\left(\frac{b}{a}\right)}$$

$$F_{up} = \frac{1}{2} V^2 \frac{2\pi\epsilon_0 \chi_e}{\ln \frac{b}{a}} = \frac{\pi\epsilon_0 V^2 \chi_e}{\ln\left(\frac{b}{a}\right)}$$

$$F_{down} = mg = \rho V g = \rho\pi\left(b^2 - a^2\right) h g$$

$F_{up} = F_{down}$ for equilib.

$$\rho\pi\left(b^2 - a^2\right) h g = \left(\frac{V^2 \pi\epsilon_0 \chi_e}{\ln\left(\frac{b}{a}\right)}\right)$$

$$h = \frac{V^2 \epsilon_0 \chi_e}{\rho\left(b^2 - a^2\right) g \ln\left(\frac{b}{a}\right)}$$