The University of Georgia
Department of Physics and Astronomy

Graduate Qualifying Exam – Part II

August 17, 2019

Instructions:

- Start each problem on a new sheet of paper. Write the problem number on the top left of each page and your pre-arranged prelim ID number (but **not** your name) on the top right of each page. Leave margins for stapling and photocopying.
- Put your answers only on one side of the paper. Please **do not** write on the back side.
- If not advised otherwise, derive the mathematical solution for a problem from basic principles or general laws (Newton's laws, Maxwell equations, Schrödinger equation, laws of optics, laws of thermodynamics, etc.). Use common mathematical symbols and avoid writing lengthy texts.
- You can use the formula sheet you prepared as advised by the Graduate Coordinator as a guide for your derivations.
- You may use a calculator for basic operations only (i.e., not for referring to notes stored in memory, symbolic algebra, symbolic and numerical integration, etc.). The use of cell phones, tablets, and laptops is not permitted.

Part II contains problems 6 - 10.

Problem 6 (1 part):

A free nonrelativistic particle of mass m is prepared in a wave packet

$$\psi(x) = C\, e^{-\alpha x^2},$$

where C is a normalization constant. Find the *normalized* state wave function at later times, $\psi(x, t)$.

Problem 7 (2 parts):

1) Using first-order perturbation theory, derive the leading correction, E_1, to the hydrogen-like ground state energy of a negatively charged particle of charge $(-q)$ and mass m subjected to the *screened* Coulomb potential,

   $$V(r) = \frac{q}{4\pi\varepsilon_0 r}\, e^{-r/\lambda}, \text{ where } \lambda \gg a_0,$$

   with a_0 being the corresponding Bohr radius.

2) Calculate E_1 **in electron-volts**, assuming $m = 3 \times 10^{-31}$ kg, $q = 2 \times 10^{-19}$ C, $\varepsilon_0 = 8.8542 \times 10^{-12}$ F/m, $\hbar = 1.0546 \times 10^{-34}$ J·s, $\lambda = 20$ Å. If needed, the value of elementary charge is $e = 1.6022 \times 10^{-19}$ C.

Problem 8 (4 parts):

1) What is the momentum of a particle whose total energy is four times its rest energy?

2) What is the speed of the particle?

3) What is the kinetic energy of the particle?

4) Derive an expression for the relativistic kinetic energy of a particle in terms of p, γ, and m.

Problem 9 (4 parts):

An atom is ordinarily found in its ground state, or state of lowest energy. However, by supplying some energy, one can lift it to an excited state. An excited state of an atom X is sometimes denoted X* and, because of the additional energy ΔE, has a mass m* slightly greater than that of the ground state (m):

$$m^* = m + \Delta m$$

where $\Delta m = \frac{\Delta E}{c^2}$. If left in isolation, the excited state X* usually drops back to the ground state emitting a single photon.

$$X^* \to X + \gamma$$

If the atom were immovable, the photon would carry off all the additional energy.

$$E_\gamma = \Delta m c^2$$

In reality, conservation of momentum requires the atom to recoil, so that a little of the energy goes to kinetic energy of the recoiling atom.

1) Using conservation of momentum and energy and assuming that the excited atom X* was at rest, show that

$$E_\gamma = \Delta m c^2 \left(1 - \frac{\Delta m}{2m^*}\right)$$

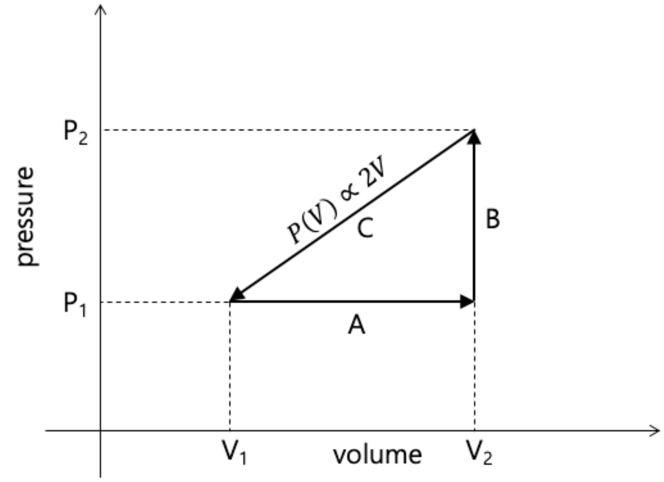
2) The energy needed to lift a hydrogen atom from its ground state to the lowest excited state is 10.2 eV. Evaluate the fraction ${}^{\Delta m}/_{m^*}$ for this state.

3) Does it make a significant difference whether you use m or m^* in the denominator?

4) What percentage of the available energy $\Delta m c^2$ goes to the photon in the decay of this excited state?

Problem 10 (4 parts):

An ideal gas with three degrees of freedom undergoes a cyclic process, as shown in the following figure. Assume that during step C, pressure is proportional to the volume in accordance with $P \propto 2V$.



Find, for each of the steps, A, B, and C,

1) The work, W (expressed in terms of V_1 and V_2), done on the gas.

2) The change in the internal energy, ΔU (expressed in terms of V_1 and V_2), of the gas.

3) The heat, Q (expressed in terms of V_1 and V_2), transferred to the gas.

4) Using your results found in Parts 1), 2), and 3), show that for the whole cycle, $\Delta U = 0$ and $Q_{total} = -W_{total}$.