Graduate Qualifying Exam – Part I

August 10, 2019

Instructions:

- Start each problem on a new sheet of paper. Write the problem number on the top left of each page and your pre-arranged prelim ID number (but **not** your name) on the top right of each page. Leave margins for stapling and photocopying.

- Put your answers only on one side of the paper. Please **do not** write on the back side.

- If not advised otherwise, derive the mathematical solution for a problem from basic principles or general laws (Newton's laws, Maxwell equations, Schrödinger equation, laws of optics, laws of thermodynamics, etc.). Use common mathematical symbols and avoid writing lengthy texts.

- You can use the formula sheet you prepared as advised by the Graduate Coordinator as a guide for your derivations.

- You may use a calculator for basic operations only (i.e., not for referring to notes stored in memory, symbolic algebra, symbolic and numerical integration, etc.). The use of cell phones, tablets, and laptops is not permitted.

Part I contains problems 1 - 5.

Problem 1 (1 part):

A small ball of mass m rolls in a horizontal circle around the inside of the inverted cone shown in the Figure. The walls of the cone make an angle θ with a horizontal plane. The coefficient of static friction between the ball and the cone is μ_s; rolling friction and air resistance are negligible.

What minimum speed v_{min} must the ball maintain to remain at a constant height h?
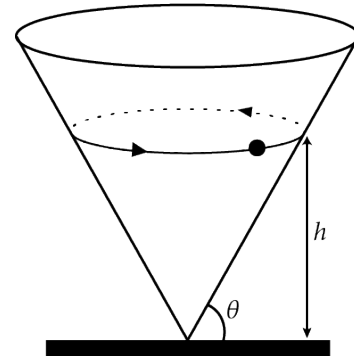
Problem 2 (3 parts):

For the step ladder shown in the figure, sides AC and CE are each 2.44 m long and hinged at C. Bar BD is a tie-rod 0.762m long, halfway up. A man weighing 854 N climbs 1.80 m along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find
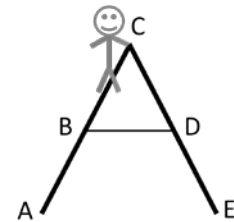
1) the tension in the tie-rod,
2) the magnitude of the force on the ladder from the floor at A,
3) the magnitude of the force on the ladder from the floor at E.

Problem 3 (2 parts):

A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown in the following figure.
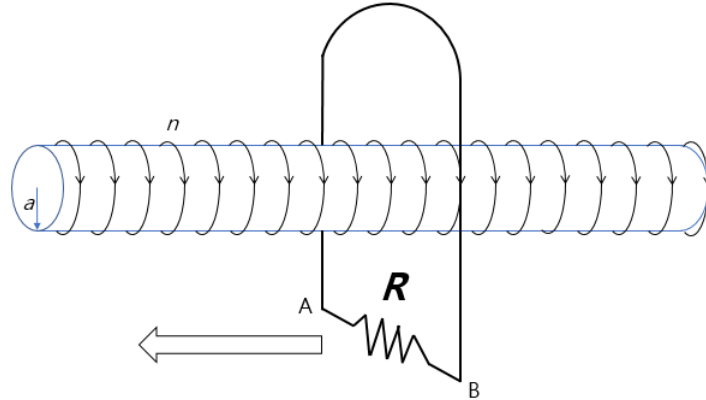

(1) If the current in the solenoid is increasing at a constant rate $(dI/dt = k)$, what current flows in the loop, and which way (A to B or B to A) does it pass through the resistor?
(2) If the current I in the solenoid is constant but the solenoid is pulled out of the loop (toward the left to a place far from the loop), what total charge passes through the resistor?
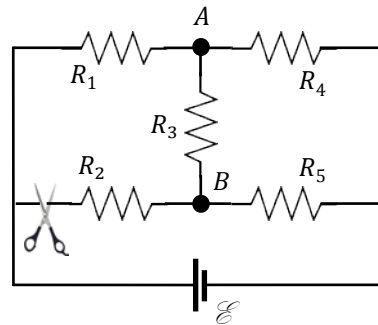
Problem 4 (3 parts):

Part 1:

Refer to the figure on the right. Assuming ideal circuit elements with $R_1 = R_2 = R_3 = R_4 = R_5 = 1 \ \Omega$, and $\mathscr{E} = 6$ V, calculate the current I_1 flowing through resistor R_1.
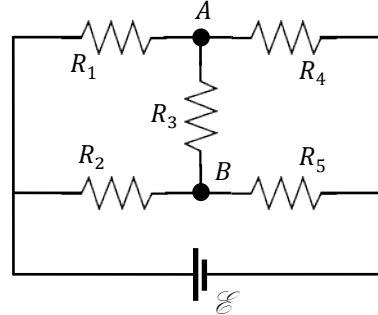
Part 2:

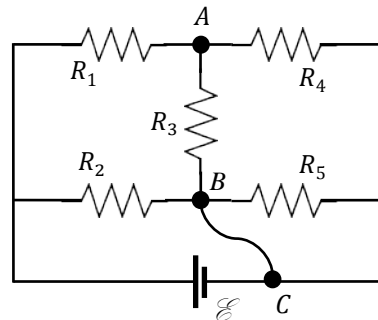Assume now that the wire on R_2 has been cut.

Calculate the current I_1 flowing through resistor R_1. (Use same values for all the circuit elements as given in Part 1.)

Part 3:

Use same values as in Part 1. Assume that an ideal wire was inserted in the circuit to connect points B and C, as shown in the figure on the right.

Calculate the current, I_1, flowing through resistor R_1.

Problem 5 (1 part):

A normal eye uses a compound microscope to look at amoeba. The microscope has an objective of focal length f_{obj} = 2 cm and an eyepiece of focal length f_{ep} = 2.5 cm. If ameba is placed 2.4 cm from the objective, what is the magnification of this microscope? Assume that the near point distance of the normal eye is d_{NPN} = 25 cm.

Reminder: The magnification, M, of a microscope is given by

$M \equiv$ (Linear magnification of objective) $\cdot$ (Angular magnification of eyepiece)