The University of Georgia

Department of Physics and Astronomy

Graduate Qualifying Exam ─ Part II

3 January 2018

Instructions:

- Attempt all problems. You *must* show your work and/or clearly explain your answers in order to be able to earn a passing grade for a problem.
- Start each problem on a new sheet of paper. Write the problem number on the top left of each page, and your pre-arranged prelim **ID number** (but *not* your name) on the top right of each page. Leave margins for stapling and photocopying.
- Put your answers only on one-side of the paper. Please **DO NOT** write on the back side.
- This is a closed-book exam. You are permitted to bring one page of notes (equations, definitions, physical constants, etc.) per exam day. You must hand in this page of notes with the exam each day.
- You may use a calculator, but *only* for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.).

Part I has five problems, numbered 6 — 10.

Problem 6: (two parts)

A harmonic oscillator is in a state such that a measurement of the energy would yield either $\frac{1}{2}\hbar\omega$ or $\frac{3}{2}\hbar\omega$, with equal probability. What is the largest possible value of $\langle p \rangle$ in such a state? If it assumes this maximal value at time $t = 0$, what is the wavefunction $\Psi(x,t)$?

Problem 7: (two parts)

Consider two systems S and $\boldsymbol{S'}$ that are both in motion in the x-direction. $\boldsymbol{S'}$ is moving with a very high velocity $+V$ with respect to system S. Let a particle move in the x-direction at a speed $\boldsymbol{u'_x}$ with respect to the system $\boldsymbol{S'}$, so that the particle is moving faster than $\boldsymbol{u'_x}$ with respect to S.

a.) How fast will the particle appear to move when viewed from system S?
b.) How does your answer compare with the non-relativistic result?

Problem 8: (four parts)

A cylinder contains one liter of air at room temperature (300 K) and atmospheric pressure (10^5 N/m^2). At one end of the cylinder is a massless piston, whose surface area is 0.01 m^2. Suppose that you push the piston in very suddenly, exerting a force of 2000 N. The piston moves only one millimeter, before it is stopped by an immovable barrier of some sort.

(a) How much work have you done on this system?

(b) How much heat has been added to the gas?

(c) Assuming that all the energy added goes into the gas (not the piston or cylinder walls), by how much does the internal energy of the gas increase?

(d) Use the thermodynamic identity to calculate the change in the entropy of the gas (once it has again reached equilibrium)

Problem 9: (three parts)

The Hamiltonian for a hydrogen atom is given by

$$H = H_0 + H_r + H_{SO}$$

where

$$H_0 = -\frac{\hbar^2}{2m}\vec{\nabla}^2 - \frac{e^2}{4\pi\epsilon_0}\frac{1}{r} \, ,$$

$$H_r = -\frac{\hat{p}^4}{8mc^2} \, , \qquad \text{(lowest − order relativistic correction to kinetic energy)}$$

$$H_{SO} = \left(\frac{e^2}{8\pi\epsilon_0}\right)\frac{1}{m^2c^2r^3}\,\vec{S}\cdot\vec{L} \, . \qquad \text{(spin − orbit interaction including the Thomas precession)}$$

The eigenvalue of H_0 is the Bohr' energy

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = -\frac{13.6 \text{ eV}}{n^2} \, ,$$

where $n \equiv j_{max}+l+1$ $(= 1, 2, 3, \cdots)$ denotes the principal quantum number. The eigenvalues of the relativistic correction to the kinetic energy operator, H_r, in first-order perturbation theory, are given by

$$E_r = -\frac{E_n^2}{2mc^2}\left[\frac{4n}{l+1/2} - 3\right] \, ,$$

and the eigenvalues of H_{SO}, also in first-order perturbation theory, are given by

$$E_{SO} = \frac{E_n^2}{mc^2}\left\{\frac{n\left[j(j+1) - l(l+1) - 3/4\right]}{l(l+1/2)(l+1)}\right\} \, .$$

a) Show that both the relativistic correction to the kinetic energy (H_r) and spin-orbit interaction (H_{SO}) are of the order of α^2 corrections to the unperturbed interaction H_0, where

$$\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c} \cong \frac{1}{137}$$

is the fine structure constant.

b) Show that the eigenvalues of the total Hamiltonian, H, in first-order perturbation theory, are given by

$$E_{nj} = -\frac{13.6 \text{ eV}}{n^2}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right] \, ,$$

Hint: Consider first the sum $E_r + E_{SO}$ for $l = j \pm 1/2$.

c) What order of magnitude would you expect for the second-order correction to the Bohr's energy, E_n, in perturbation theory?

Problem 10: (three parts)

A rocket traveling at a speed 0.8c relative to the Earth shoots forward a beam of particles with a speed 0.9c relative to the rocket. What are the particles' speed relative to the Earth?