# January 8, 2016
# Prelim Exam – Day 2

Begin each problem on a separate sheet of paper. Write your **Prelim ID #** in the top right-hand corner of each and every page you submit.

**Problem 1:**

A particle of mass $m$ in the infinite square well (of width $a$) starts out in the left half of the well, and is (at $t = 0$) equally likely to be found at any point in that region.

**(a)** What is its initial wave function, $\Psi(x, 0)$? (Assume it is real. Don't forget to normalize it.)

**(b)** What is the probability that a measurement of the energy would yield the value $\pi^2\hbar^2/2ma^2$?

**Problem 2:**

A particle of mass $m$ is in the state

$$\Psi(x,t) = A \exp\{-a[(mx^2/\hbar) + it]\}$$

where $A$ and $a$ are positive real constants.

**(a)** Find $A$.

**(b)** For what potential energy function $V(x)$ does $\Psi$ satisfy the Schrödinger equation?

**(c)** Calculate the expectation values of $x$, $x^2$, $p$, and $p^2$.

**Problem 3:**

In the Earth's reference frame, a tree is at the origin and a pole is at $x = 30$ km. Lightning strikes both the tree and the pole at $t = 10\ \mu$s. The lightning strikes are observed by a rocket traveling in the $x$-direction at $0.5c$.

**(a)** What are the spacetime coordinates for these two events in the rocket's reference frame?

**(b)** Are the events simultaneous in the rocket's frame? If not, which occurs first?

**Problem 4:**

Consider a head-on, elastic collision between a massless photon (momentum $\mathbf{p}_0$ and energy $E_0$) and a stationary free electron.

**(a)** Assuming that the photon bounces directly back with momentum $\mathbf{p}$ (in the direction of $-\mathbf{p}_0$) and energy $E$, use conservation of energy and momentum to find $p$.

**(b)** Verify that your answer agrees with that given by Compton's formula with $\theta = \pi$.

**Problem 5:**

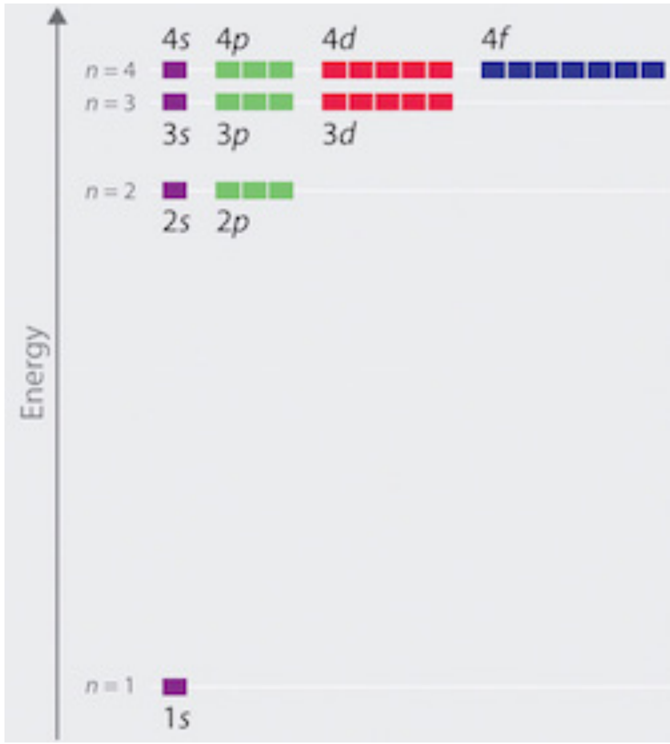


Figure Caption: Shown above is the energy level diagram for a hydrogen atom, showing the four lowest energy levels. Ignoring electron spin *s*, there is one independent state with energy = –13.6 eV, four independent states with energy = –3.4 eV, nine independent states with energy = –1.5 eV, etc.

Estimate the ratio of the probability that a hydrogen atom at warm room temperature (300 K) is in one of its first excited states ($n = 2$) relative to the probability of it being in the ground state ($n = 1$). Don't forget to take degeneracy into account. Show all steps in the calculation, even if you think that the answer will be trivial. The value of Boltzmann's constant, $k = 8.62 \times 10^{-5}$ eV/K.