Physics Prelim Exam Fall 2014

Day 1:

Mon. August 11, 2014, 9:00am-1:00pm

Department of Physics and Astronomy

Problem 1.1: Classical Mechanics

Problem 1.2: Classical Mechanics

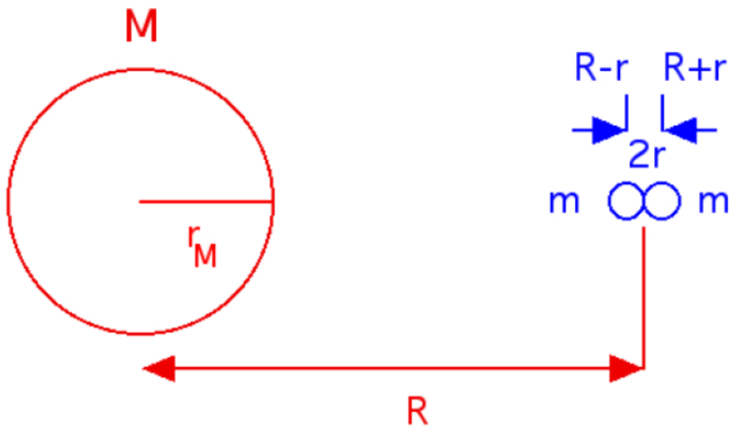
Problem 1.3: Electrodynamics

Problem 1.4: Electrodynamics

Problem 1.5: Optics

Problem 1.1: Classical Mechanics

Inside of the Roche limit (R) of a massive object (mass M and radius r_M), an orbiting small object held only by its own gravity gets disintegrated because a tidal force of M is larger than the self gravity of the smaller object. Using the simplified version of the system below (i.e., replacing the small object with two small spheres), show that the Roche Limit of the large object can be expressed in terms of the large object radius alone (R≈2.5r_M). In your derivation, use the first order approximation ($(r/R)^n = 0$ for n > 1) and assume that densities of all objects are the same.



Solution 1.1:

Gravtitational force between two small spheres is

$$\frac{Gmm}{(2r)^2}$$

Differential gravity from M to each sphere is

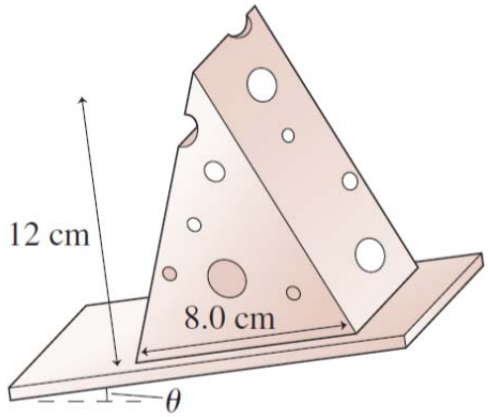
$$\begin{aligned}
\Delta F &= \frac{GMm}{(R-r)^2} - \frac{GMm}{(R+r)^2} \\
&= \frac{GMm}{R^2}\left(\frac{1}{(1-r/R)^2} - \frac{1}{(1+r/R)^2}\right) \\
&\approx \frac{GMm}{R^2}\left(\frac{1}{1-2r/R} - \frac{1}{1+2r/R}\right) \\
&= \frac{GMm}{R^2}\left(\frac{4r/R}{1-(2r/R)^2}\right) \\
&\approx \frac{GMm}{R^2}\frac{4r}{R}
\end{aligned}$$

Then

$$\begin{aligned}
\frac{Gm^2}{4r^2} &= \frac{4GMmr}{R^3} \\
\frac{m}{r^3} &= \frac{16M}{R^3} \\
r^3\rho/r^3 &= 16r_M^3\rho/R^3 \\
R^3 &= 16r_M^3 \\
R &\approx 2.5r_M
\end{aligned}$$

Problem 1.2: Classical Mechanics

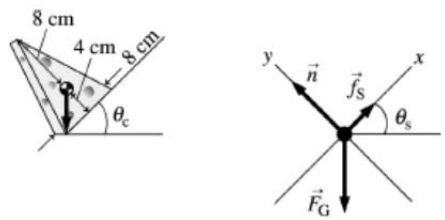
The figure below shows a triangular block of Swiss cheese sitting on a cheese board. You and your friends start to wonder what will happen if you slowly tilt the board, increasing the angle θ. Emily thinks the cheese will start to slide before it topples over. Fred thinks it will topple before starting to slide. Some quick Internet research on your part reveals that the coefficient of static friction of Swiss cheese on wood is 0.90. Calculate through both cases and determine who is right.



Solutions 1.2

12.85. Model: The Swiss cheese wedge is of uniform density—or at least uniform enough that its center of mass is at the same location as that of a solid piece. To find the angle at which the cheese starts sliding, the cheese will be treated as a particle, and the model of static friction will be used.

Visualize:



Solve: The angle at which the cheese starts sliding, θ_S, will be compared to the critical angle θ_c for stability. Use Newton's second law with the free body diagram.

$$(F_{net})_x = 0 = f_s - F_G \sin\theta_s$$
$$(F_{net})_y = 0 = n - F_G \cos\theta_s$$

With $F_G = mg$, the y-direction equation gives $n = mg\cos\theta$. The cheese starts sliding when μ_s is at its maximum value. Combining that with the x-direction equation and $f_s = \mu_s n$,

$$0 = \mu_s(mg\cos\theta_s) - mg\sin\theta_s$$
$$\Rightarrow \theta_s = \tan^{-1}(\mu_s) = \tan^{-1}(0.90) = 42°$$

The cheese will start sliding at an angle of 42°.

The center of mass of the cheese wedge can be found using the result of Problem 12.52. There, the center of mass of a triangle with the same proportions as the cheese wedge was found. So x_{cm} is at the center of the cheese wedge (by symmetry). The y_{cm} can be found by proportional reasoning.

$$\frac{y_{cm}}{12\text{ cm}} = \frac{(30\text{ cm} - 20\text{ cm})}{30\text{ cm}} \Rightarrow y_{cm} = 4.0\text{ cm}$$

Note that here we have measured y_{cm} from the base of the wedge.

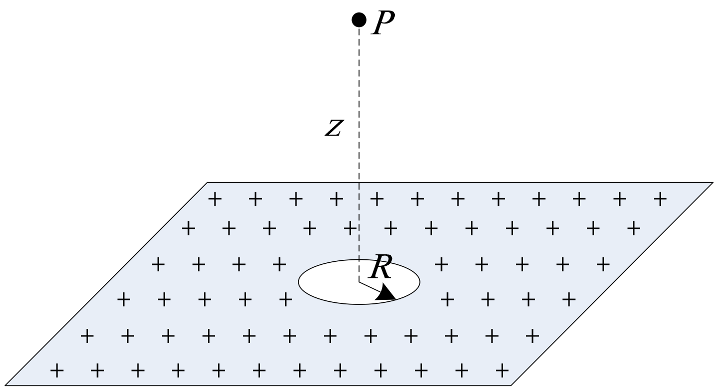
Stability considerations require that the center of mass be no farther than the left corner of the wedge. At the critical angle geometry shown in the figure above, the right triangle formed by the wedge's center of mass, lower-left corner, and center point of the base is a 45°-45°-90° triangle. So $\theta_c = 45°$.

The cheese will slide first as the incline reaches 42°. It would not topple until the angle reaches 45°. So Emily is correct.

Assess: Both Emily's and Fred's suppositions are plausible. The calculation must be done to find out which is right.

Problem 1.3: Electrodynamics

A large, flat, nonconducting surface has a uniform charge density σ. A small circular hole of radius R has been cut in the middle of the surface, as shown below. Ignoring fringing of the field lines around all edges (i.e., you can treat the surface as an infinitely large plate), calculate the electric field at point P, a distance z from the center of the hole along its axis.



Integrals:

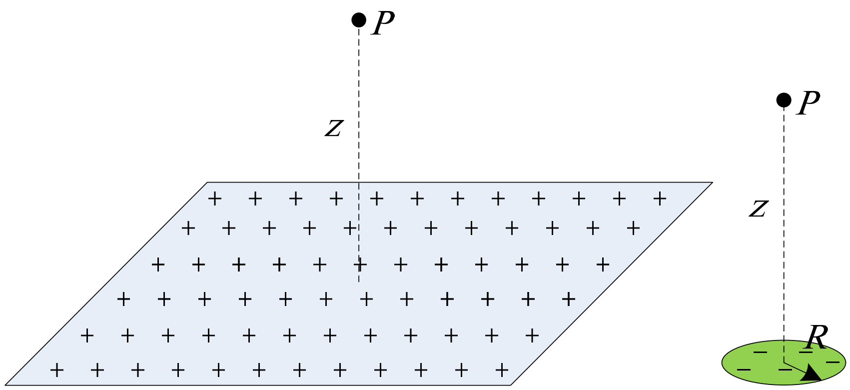
$$\int \frac{dy}{\left(a^2 + y^2\right)^{3/2}} = \frac{y}{a^2\sqrt{a^2 + y^2}}$$

$$\int \frac{dy}{(a + y)^{3/2}} = -\frac{2}{\sqrt{a + y}}$$

$$\int \frac{dy}{(a + y)^{3/2}} = -\frac{2}{\sqrt{a + y}}$$

Solution 1.3

This problem can be solved using the superposition principle: the filed at location P is the result of superposition of the e-fields generated by an infinitely uniformly charged plate (σ) and a negatively charged disk (-σ) as shown below.



The e-field generated from the uniformly charge distributed plate can be found by Gauss' law:

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{n},$$

where $\hat{n}$ is the surface normal of the plate.

The E-field along the central axis of a uniformly charged disk can be found through an integration:

First find the E-field due to a charged ring with radius *r* and width *dr*. Due to the symmetry, the E-filed is only along the z-direction, i.e., $\hat{n}$ direction. The total charge along the ring is

$$dq = -\sigma dA = -\sigma(2\pi r dr)$$

The E-field at P location can be written as

$$dE = \frac{-z\sigma 2\pi r dr}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}$$

Thus,

$$E_2 = \int dE = \frac{-\sigma z}{4\varepsilon_0}\int_0^R \frac{2r}{(z^2+r^2)^{3/2}}dr = -\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

And $\vec{E}_2 = E_2\hat{n}$. Thus, the final field at P is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\frac{z}{\sqrt{z^2+R^2}}\hat{n}$$

Problem 1.4: Electrodynamics

A solenoid has a radius of 2.00cm and 1,000 turns per meter. Over a certain time interval the current varies with time according to the expression $I = 3e^{0.2t}$, where I is in amperes and t is in seconds. Calculate the electric field 5.00cm from the axis of the solenoid at t=10.0s.

Solutions 1.4

The induced emf is

$$\overrightarrow{|\epsilon|} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2 \frac{dB}{dt} = \oint \vec{E} \cdot d\vec{l}$$

$$E(2\pi R) = \pi r^2 \frac{dB}{dt}$$

$$E = \left(\frac{\pi r^2}{2\pi R}\right)\frac{dB}{dt}$$

Here $B = \mu_0 n I$, so $\frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$

$I = 3e^{0.2t}$, so $\frac{dI}{dt} = 0.6e^{0.2t}$

At t=10.0s,

$$E = \left(\frac{\pi r^2}{2\pi R}\right)(\mu_0 n)0.6e^{0.2t} = \left(\frac{(0.200m)^2}{2 \times 0.500m}\right)\left(4\pi \times \frac{10^{-7}N}{A^2}\right)\left(1000\frac{turns}{m}\right)0.6e^{2.00} = 2.23 \times 10^{-5}N/C$$

Problem 1.5: Optics

A praying mantis preys along the central axis of a thin symmetric lens, 20 cm from the lens. The lateral magnification of the mantis provided by the lens is m = -0.25, and the index of refraction of the lens material is 1.65. (a) Determine the type of image produced by the lens, the type of lens, and the location of the image, the orientation of the image. (b) What are the two radii of curvature of the lens?

Solution 1.5

(a) Since $m = -i/p$, one obtain

$$i = -mp = 0.25\,p = 0.25 \times 20 \text{ cm} = 5.0 \text{ cm}$$

Since p is positive, i must be positive. Therefore we have a real image, only converging lens can produce real image, the image must be on the other side of the lens, and the image is inverted.

(b) Use image equation to find focal length f:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

Thus**,**

$$\frac{1}{20cm} + \frac{1}{5cm} = \frac{1}{f}$$

And we obtain: $f = 4.0$ cm

Use lens' maker equation to find r:

$$\frac{1}{f} = (\frac{n_2}{n_1} - 1)(\frac{1}{R_1} - \frac{1}{R_2}) = (n-1)(\frac{1}{+r} - \frac{1}{-r})$$

$$\frac{1}{4.0cm} = (1.65 - 1)\frac{2}{r}$$

$r = 0.65 \times 2 \times 4.0$ cm $= 5.2$ cm.

Thus, the radius of curvature is 5.2 cm.