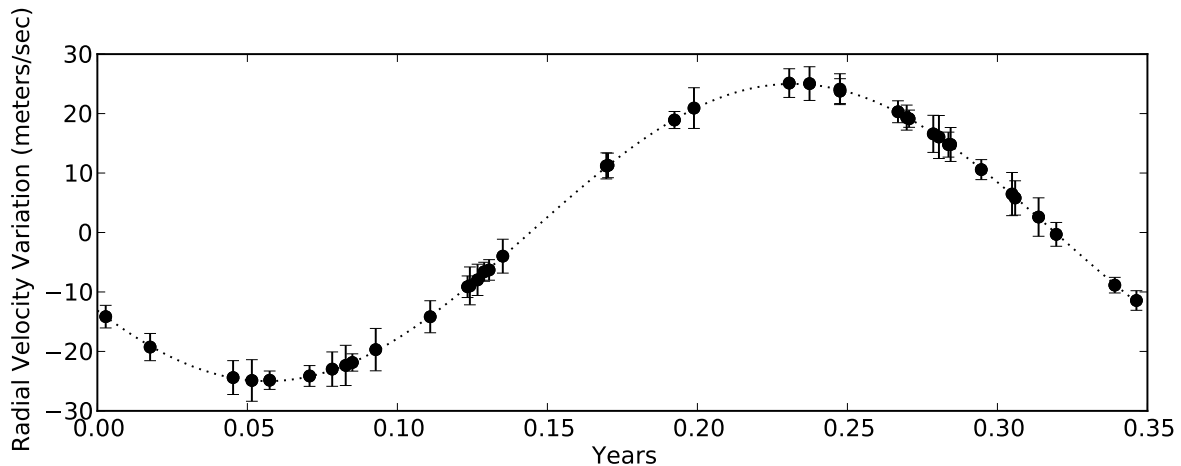


The University of Georgia  
Department of Physics and Astronomy  
Graduate Qualifying Exam – Part II  
(Dated: January 7, 2011)

**Instructions:** Attempt all problems. Start each problem on a new sheet of paper, and print your name on each sheet of paper that you submit. This is a closed-book, closed-notes exam. You may use a calculator, but only for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.). For full credit, you must show your work and/or explain your answers. This part has 6 problems.

1. A group of astronomers has monitored radial velocities of a star looking for a periodic variation caused by an unseen exoplanet. From the obtained data, the group found that the star shows a periodically varying radial velocities that can be fit by a sinusoidal curve as shown below. In your calculation, assume a circular orbit viewed edge-on, i.e., an inclination of the orbit is 90 degrees.



(a) If the star is a Sun-like star (i.e., mass of the star =  $1 M_{\odot}$ ), estimate the orbital separation of the unseen planet from the central star in terms of the Astronomical Unit (1 AU = mean separation between the Earth and Sun). Hint: Use Kepler's 3rd law

$$P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)},$$

and the orbital period (1 year) and separation of the Earth from the Sun (1 AU). Here  $a$  is the planet-star separation and  $M_1$  and  $M_2$  are the two masses.

(b) Show that the maximum reflex motion of the star due to a unseen planet can be expressed as

$$v = 2\pi \frac{M_{planet}}{M_{star} + M_{planet}} \frac{a}{P}$$

where  $a$  is the orbital size and  $P$  is the orbital period of the planet.

(c) Estimate the mass of the planet in Jupiter mass ( $1 M_J = 0.001 M_{\odot}$ ).

2. Consider a light bulb that emits light evenly in all directions. The power of the emission is  $P$ .

(a) What is the intensity,  $I$ , of the emitted light at a distance of  $d$  from the center of the light bulb?

(b) What is the rms value of the electric field of the emitted light at a distance  $d$  from the center of the light bulb?

(c) What is the rms value of the magnetic field of the emitted light at a distance  $d$  from the center of the light bulb?

(d) The power is  $P = 100$  meters and the distance is  $d = 1$  meter. What are the values of the rms electric field strength (in units of Volt/meter), and magnetic field (in units of Tesla = Volt sec / meter<sup>2</sup>)?

3. Damping is negligible for a 0.150 kg mass hanging from a light 6.30 N/m spring. The system is driven by a force oscillating with an amplitude of 1.70 N. At what frequency will the force make the mass vibrate with an amplitude of 0.440m?

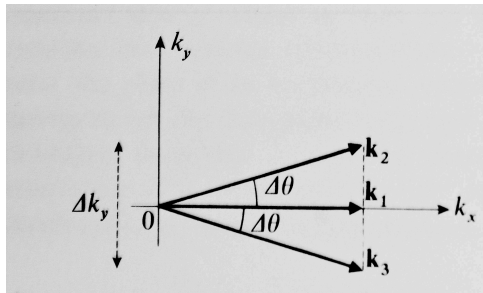
4. A light source recedes from an observer with a speed  $v_{\text{source}}$ , which is smaller compared with the speed of light  $c$ .  
 (a) Show that the fractional shift in the measured wavelength is given by the approximate expression

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v_{\text{source}}}{c}.$$

This phenomenon is known as the red shift because visible light is shifted toward the red.

(b) Spectroscopic measurement of light at  $\lambda = 397$  nm coming from a galaxy in Ursa Major reveals a red shift of 20.0 nm. What is the recessional speed of the galaxy?

5. Consider three plane waves propagating in the  $xOy$  plane, whose wave vectors  $\mathbf{k}_1, \mathbf{k}_2$ , and  $\mathbf{k}_3$  are shown in the figure below. Assume the angle  $\Delta\theta$  is small.



(a) Find the  $x$  and  $y$  components of each wavevector to first order in  $\Delta\theta$ , defining  $k \equiv k_{1x}$ .

(b) Assigning the waves real amplitudes  $g(\mathbf{k}_i)$  which satisfy the relations  $g(\mathbf{k}_2) = g(\mathbf{k}_3) = \frac{1}{2}g(\mathbf{k}_1)$ , create the superposition

$$\psi(x, y) = \sum_{j=1}^3 g(\mathbf{k}_j) e^{i\mathbf{k}_j \cdot \mathbf{r}}.$$

(c) For what value of  $y$  is  $\psi(x, y)$  peaked?

(d) Estimate the lateral extent  $\Delta y$  of  $\psi(x, y)$  around its peak to be the distance between the first nodes in the  $y$ -direction, on either side of the peak. Show that  $\Delta y$  and  $\Delta k_y \equiv 2k\Delta\theta$  obey a type of uncertainty relation:

$$\Delta y \Delta k_y = 4\pi.$$

(e) Discuss how this result is related to the diffraction of a plane wave of wavelength  $\lambda = \frac{2\pi}{k}$  incident on a slit of width  $\Delta y$ .

6. Optical computers require microscopic optical switches to turn signals on and off. One device for doing so is the Mach-Zender interferometer shown in the figure. Light from an infrared laser ( $\lambda = 1.00\mu\text{m}$ ) is split into two waves that travel equal distances around the arms of the interferometer. One arm passes through an electro-optic crystal, a transparent material that can change its index of refraction in response to an applied voltage. Suppose both arms are exactly the same length and the index of refraction with no voltage applied is 1.5222.

(a) With no voltage applied, is the output bright (switch closed, optical signal passing through) or dark (switch open, no signal)? Explain your answer.

(b) What is the first index of refraction of the crystal larger than 1.5222 that changes the optical switch to the state opposite to that of the state you found in part (a)?

