The University of Georgia
Department of Physics and Astronomy

Graduate Qualifying Exam — Part I
5 January 2010

Instructions: Attempt all problems. Start each problem on a new sheet of paper, and print your name on each sheet of paper that you submit. This is a closed-book, closed-notes exam. You may use a calculator, but *only* for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.). For full credit, you must show your work and/or explain your answers. Part I has six problems, numbered 1–6.

Problem 1: (one part)

Imagine a ball of radius R. It has a uniform charge density ρ everywhere except in a small, spherical cavity whose center is located at a distance a from the center of the ball. The radius of the cavity is b. What is the electric field $\mathbf{E}$ (magnitude and direction) in the center of the cavity?

Problem 2: (four parts)

A metal bar of mass m slides from left to right frictionlessly on two parallel conducting rails a distance l apart and laid on a long table. A resistor with a resistance R is connected across the rail, and a uniform magnetic field $\mathbf{B}$, pointing out of the table upward and perpendicular to the table, fills the entire region.

(a) If the bar moves to the right at speed v, determine the current (magnitude and direction of flow) in the resistor.

(b) Determine the magnetic force $\mathbf{F}_m$ (magnitude and direction) on the bar.

(c) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, determine its speed at a later time t.

(d) Show that the total energy dissipated by the resistor is $mv_0^2/2$, i.e., exactly the initial kinetic energy of the bar.

Problem 3: (one part)

You use a horizontal cue stick to hit a pool ball of radius R and mass M. The stick hits the ball at a distance of h above the surface. Find the value of h such that the ball rolls without slipping. (The rotational inertia of the ball around its center of mass is $I = 2MR^2/5$.)
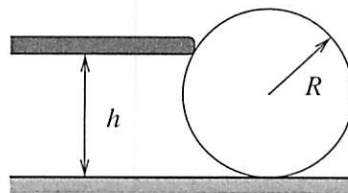
Problem 10: (three parts)

A one-dimensional quantum wavefunction has the form

$$\psi(x) = \frac{A}{\sqrt{x^2 + a^2}},$$

for length scale a and normalization constant A. It is a stationary state of an unknown potential energy function $V(x)$.

(a) Sketch a graph of the wavefunction. Is this wavefunction the ground state of the potential energy function $V(x)$? Explain why or why not.

(b) Assuming that the potential $V(x)$ asymptotically approaches 0 as $x \to \pm\infty$, find $V(x)$, and determine the energy of this stationary state.

(c) Are there any higher-energy bound states for this potential? Explain why or why not.

Problem 11: (three parts)

(a) The First Law of Thermodynamics can be written as

$$dU = dQ - dW \; ,$$

where U, Q and W stand for the energy of the system, heat absorbed by (or removed from) the system, and work done by (or on) the system, respectively. Interpret this law. What does it tell us? Does it have something to do with energy conservation?

(b) The thermal efficiency of a heat engine is defined by

$$e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} \; ,$$

where Q_h stands for the amount of heat extracted from a high-temperature source by the engine which does work W and gives up an amount of heat Q_c to a low-temperature heat reservoir.

i) What does the Second Law of Thermodynamics tells us about such a (thermodynamical) system?

ii) What is the difference between the First and Second Laws of Thermodynamics? Explain in a few words.

(c) The absolute zero temperature of a (thermodynamical) system is associated with the absence of disorder, or a perfect order of that system, rather than the absence of (its internal) motion. Is the absolute zero attainable? Explain in terms of the Law(s) of Thermodynamics.

Problem 12: (two parts)

Consider the point of view of a pet fish. The fish lives in water (index of refraction $n_w = 1.3$), inside a glass tank (index of refraction $n_g = 1.5$) whose walls are surrounded by air (index of refraction $n_a = 1.0$). A cat likes to watch the fish and a person likes to feed the fish. These three creatures are shown, from a view that looks down on the scene, in the figure. N, E, S, and W mark the north, east, south, and west walls of the tank, respectively. The walls of the tank are $d = 1$ cm thick. Measured from inside the tank, the tank is 200 cm in height, depth, and width. The fish can see through only one eye, and that eye is located 2 cm from the west wall, 198 cm from the east wall, 100 cm from the south wall, and 100 cm from the north wall of the tank. Glass naturally allows a small amount ($\sim 4\%$) of specular reflection for light that approaches it from any angle; completely ignore such reflection in this problem. The line from the fish to the wall shows a sample line of sight.
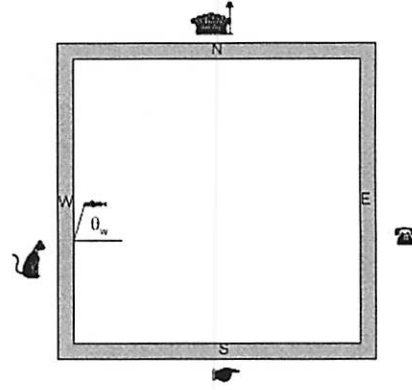
(a) What is the largest value of θ_w such that the fish is able to see through the west wall of the tank? (Work your solution in symbolic form first, and then plug in the numbers.)

(b) At angles that are larger than your answer to part (a), what does the fish see? (examples: objects beyond the tank on the west, south, east, or north side of the tank, or the interior of the tank.) In order to get full points, you must explain your logic.

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Graduate Qualifying Exam — Part II
6 January 2010

Instructions: Attempt all problems. Start each problem on a new sheet of paper, and print your name on each sheet of paper that you submit. This is a closed-book, closed-notes exam. You may use a calculator, but *only* for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.). For full credit, you must show your work and/or explain your answers. Part II has six problems, numbered 7–12.

Problem 7: (one part)

Estimate the radius of a planetoid on which a person could escape via a simple jump. (For the sake of specificity, assume that a person jumping on Earth will increase his/her center of mass by $\Delta h = 1.1$ m. Also assume that the density of the planetoid is the same as that of Earth.)

Problem 8: (three parts)

Given a normalized ket $|\alpha\rangle$, define an operator

$$\hat{P}_\alpha \equiv |\alpha\rangle\langle\alpha|.$$

(a) Explain why this is called a "projection operator".

(b) Show that this operator is idempotent, i.e., that $\hat{P}_\alpha^2 = \hat{P}_\alpha$.

(c) Assume a finite Hilbert space of dimension $n > 2$. Determine all the eigenvalues of this operator, and describe the form of its eigenvectors.

Problem 9: (one part)

Show that for a system described by the Hamiltonian

$$H = \frac{[\mathbf{p} + (e/c)\mathbf{A}]^2}{2m},$$

the probability flux $\mathbf{j}$, which satisfies the continuity equation

$$\frac{\partial}{\partial t}|\psi|^2 + \boldsymbol{\nabla} \cdot \mathbf{j} = 0,$$

is given by

$$\mathbf{j} = \mathrm{Re}\left[\psi^*\left(\frac{\mathbf{p} + (e/c)\mathbf{A}}{m}\right)\psi\right].$$

Problem 4: (two parts)

Three descriptions or models of light are commonly used in physics: (i) the electromagnetic wave model, (ii) the ray or geometric optics model, and (iii) the photon model.

(a) Briefly describe each of these models.

(b) For each model, discuss the conditions for which the model is an appropriate and accurate description of light. Compare and contrast these regimes of applicability.

Problem 5: (two parts)

(a) A phonograph record of radius R carries a uniform "static" charge density σ. If it rotates at a constant angular velocity ω, determine the surface current density $\mathbf{K}$ at a distance s from the center. Also, calculate the magnetic dipole moment of the rotating record.

(b) A uniformly charged solid sphere, of radius R and total charge Q, is centered at the origin and spinning at a constant angular velocity ω about the z-axis. Find the current density $\mathbf{J}$ at any point $\mathbf{r}$ within the sphere. Calculate also the magnetic dipole moment of the sphere.

Problem 6: (one part)

A uniform cylinder of mass M and radius R is at rest on a block of mass m, which in turn rests on a horizontal, frictionless table. If a horizontal force F is applied to the block, it accelerates and the cylinder rolls without slipping. Find the acceleration of the block in terms of the given variables.
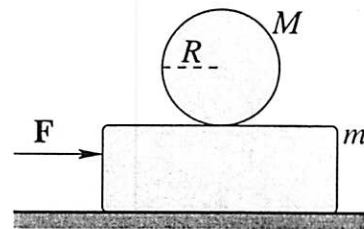
Note Sheet for Physics Qualifying Exam

Vector Identities: (a, b, c, d, and **F** are vector fields, and ψ is a scalar field)

$$\nabla \times \nabla \psi = 0$$
$$\nabla \cdot (\nabla \times a) = 0$$
$$\nabla \times (\nabla \times a) = \nabla(\nabla \cdot a) - \nabla^2 a$$
$$\nabla \cdot (\psi a) = a \cdot \nabla \psi + \psi \nabla \cdot a$$
$$\nabla \times (\psi a) = \nabla \psi \times a + \psi \nabla \times a$$
$$\nabla(a \cdot b) = (a \cdot \nabla)b + (b \cdot \nabla)a + a \times (\nabla \times b) + b \times (\nabla \times a)$$
$$\nabla \cdot (a \times b) = b \cdot (\nabla \times a) - a \cdot (\nabla \times b)$$
$$\nabla \times (a \times b) = a(\nabla \cdot b) - b(\nabla \cdot a) + (b \cdot \nabla)a - (a \cdot \nabla)b$$

$$a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b)$$
$$a \times (b \times c) = b(a \cdot c) - c(a \cdot b)$$
$$(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$$

Vector Differential Operators:

Cylindrical coordinates: (ρ, ϕ, z)

$$\nabla \psi = \frac{\partial \psi}{\partial \rho}\, \hat{\rho} + \frac{1}{\rho}\frac{\partial \psi}{\partial \phi}\, \hat{\phi} + \frac{\partial \psi}{\partial z}\, \hat{z}$$

$$\nabla^2 \psi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \psi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho}\frac{\partial(\rho F_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix}$$

Spherical coordinates: (r, θ, ϕ)

$$\nabla \psi = \frac{\partial \psi}{\partial r}\, \hat{r} + \frac{1}{r}\frac{\partial \psi}{\partial \theta}\, \hat{\theta} + \frac{1}{r \sin\theta}\frac{\partial \psi}{\partial \phi}\, \hat{\phi}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2}\frac{\partial(r^2 F_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial(\sin\theta\, F_\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{F} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\, \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta\, F_\phi \end{vmatrix}$$

$$\nabla^2 \psi = \frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \psi}{\partial \phi^2}$$

Trigonometric Identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\, \cos\beta \pm \cos\alpha\, \sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\, \cos\beta \mp \sin\alpha\, \sin\beta$$

Physical Data and Conversions:

Quantity	Symbol	Value
Speed of light in vacuum	c	2.9979×10^8 m/s
Gravitational constant	G	6.6743×10^{-11} N·m^2/kg^2
Coulomb constant ($\epsilon_0 \equiv 1/4\pi k$)	k	8.9876×10^9 N·m^2/C^2
Planck constant ($\hbar \equiv h/2\pi$)	h	6.6261×10^{-34} J·s
Boltzmann constant	k_B	1.3807×10^{-23} J/K
Avogadro number	N_A	6.0221×10^{23} /mol
Molar gas constant	$R \equiv N_A k_B$	8.3145 J/mol·K
Proton rest mass	m_p	1.6726×10^{-27} kg
Electron rest mass	m_e	9.1094×10^{-31} kg
Electron charge magnitude	e	1.6022×10^{-19} C
Electron-volt	1 eV	1.6022×10^{-19} J
Celsius scale offset	0° C	273.15 K

Solar System Physical Data:

Quantity	Value	Quantity	Value
Earth mass	5.972×10^{24} kg	Moon mass	7.348×10^{22} kg
Earth radius	6.378×10^6 m	Moon radius	1.737×10^6 m
Mean Earth-Moon distance	3.844×10^8 m	Solar mass	1.988×10^{30} kg
Mean Earth-Sun distance	1.496×10^{11} m	Solar radius	6.955×10^8 m

Pauli Spin Matrices:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Definite Integrals: (for integer $n \geq 0$)

$$\int_0^{\pi/2} \cos^{2n} x \, dx = \int_0^{\pi/2} \sin^{2n} x \, dx = \frac{\pi \, (2n)!}{2^{2n+1} \, (n!)^2}$$

$$\int_0^{\pi/2} \cos^{2n+1} x \, dx = \int_0^{\pi/2} \sin^{2n+1} x \, dx = \frac{2^{2n}(n!)^2}{(2n+1)!}$$

$$\int_0^\infty x^{2n} e^{-ax^2} \, dx = \sqrt{\frac{\pi}{a}} \, \frac{(2n)!}{2^{2n+1} a^n \, n!}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} \, dx = \frac{n!}{2a^{n+1}}$$