The University of Georgia
Department of Physics and Astronomy
Graduate Qualifying Exam – Part I

(Dated: August 9, 2010)

Instructions: Attempt all problems. Start each problem on a new sheet of paper, and print your name on each sheet of paper that you submit. This is a closed-book, closed-notes exam. You may use a calculator, but only for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.). For full credit, you must show your work and/or explain your answers. Part I has six problems.

Problem 1. A Geostationary orbit is a circular orbit of a satellite above the Earth's equator where the orbital period of the satellite equals to the rotational period of the Earth. An object in a Geostationary orbit appears motionless in the sky for a stationary observer on the surface of the Earth. Find the altitude of a Geostationary orbit in km for a satellite of mass 1,000 kg. Please pay attention to the number of significant digits used in your calculation.

Problem 2. Figure 1 shows a circuit with 6 resistors and a battery. The resistors are marked by their resistance (R_1, R_2, R_3, R_4, R_5, and R_6), while the battery is marked by its electromotive force (emf), ε. The battery is an ideal battery; it has no internal resistance. The letters **a**, **b**, **c**, **d**, **e**, and **f** mark locations in the circuit.

(a) Using only the variables R_1, R_2, R_3, R_4, R_5, and/or ε, what is the total resistance in the circuit, R_{total}?

(b) Using only the variables R_1, R_2, R_3, R_4, R_5, ε, and/or R_{total}, what is the current, i_{ab}, that travels from point **a** to point **b**, i.e., the current that passes through the resister marked R_1?

(c) Using only the variables R_1, R_2, R_3, R_4, R_5, ε, R_{total}, and/or i_{ab}, what is the voltage drop between points **a** and **b**, i.e. what is V_{ab}?

(d) What is the ratio of the current that travels through the R_5 resistor to the current that travels through the R_2 resistor?

(e) Using only the variables R_1, R_2, R_3, R_4, R_5, ε, R_{total}, i_{ab}, and/or V_{ab}, how much power is dissipated by the resister marked R_1?

Problem 3. Scientist X has built a vacuum and placed a small number of electrons in it. When the electrons are introduced into the chamber, they move horizontally, from west to east and travel at speed v. There is a weak magnetic field, $\mathbf{B_1}$ that is oriented horizontal and pointing from the north to the south. The number of electrons is so small and the chamber so big that you don't need to worry about electrons hitting each other or the walls of the chamber.

(a) What is the gyroradius of the electrons (including the orientation of their orbits)? Your answer can use B_1, v, q_e (the charge of an electron), m_e (the mass of an electron), and physical constants.

(b) What is the orbital period of each electron's orbit? Your answer can use B_1, v, q_e, m_e, and physical constants.

(c) At the center of each electron's orbit, what is the magnitude and direction of the magnetic field that is induced due to that electron's motion? Your answer can use B_1, v, q_e, m_e, and physical constants.

Problem 4. An aluminum rod 0.5 m in length and 2.50 cm^2 in cross-sectional area is inserted into a thermally insulated vessel containing liquid helium at 4.20 K. The rod is initially at 300 K. You may assume that:
(i) Aluminum has thermal conductivity of 31.0 J/s cm K at 4.2 K; ignore its temperature variation.
(ii) Aluminum has a specific heat of 0.210 cal/g ^{o}C and density of 2.70 g/cm^3.
(iii) The density of liquid helium is 0.125 g/cm^3 and its latent heat of vaporization is L=5 cal/g.

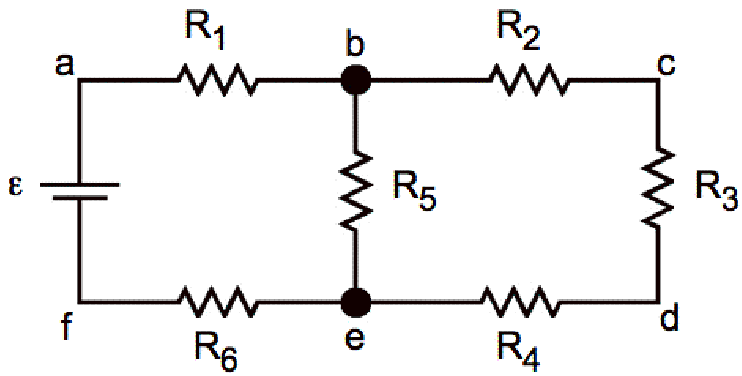

FIG. 1.

Also recall that 1 cal = 4.186 J.

(a) If one half of the rod is inserted into the helium, how many liters of helium boil off by the time the inserted half cools to 4.20 K? (Assume that the upper half does not yet cool, which means that there will be a sharp discontinuity in the temperature of the rod.)

(b) If the upper end of the rod is maintained at 300 K, what is the approximate boil-off rate of liquid helium after the lower half has reached 4.20 K?

Problem 5. The position of a particle in an infinitely deep one-dimensional potential well (of width a) is measured at $t = 0$ to be $x = a/2$ to within an uncertainty $\pm\epsilon/2$. See Fig. 2. Suppose that $\epsilon \ll a$. The normalized position wave function immediately after the measurement can be written

$$\psi(x) = \sqrt{\epsilon}\, \delta^{(\epsilon)}\left(x - \frac{a}{2}\right)$$

where

$$\delta^{(\epsilon)}(x) = \begin{cases} 1/\epsilon & \frac{a}{2} - \frac{\epsilon}{2} \le x \le \frac{a}{2} + \frac{\epsilon}{2} \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $\psi(x)$ is, in fact, properly normalized.

(b) Show that a subsequent measurement of the energy of the particle would yield the result E_n with probability

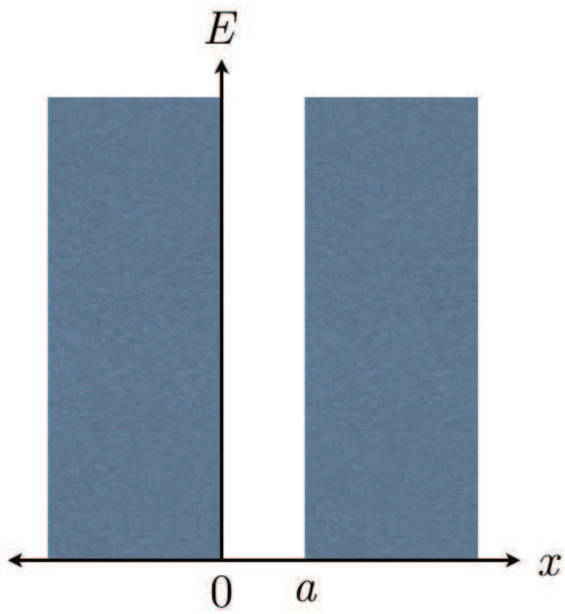

FIG. 2.

$$\mathcal{P}(E_n) = \begin{cases} \frac{8a}{\epsilon}\left(\frac{1}{n\pi}\right)^2 \sin^2\frac{n\pi\epsilon}{2a} & \text{if n is odd} \\ 0 & \text{if n is even} \end{cases}$$

(c) Make sense of this result in light of the Heisenberg uncertainty relation. Specifically, discuss the dependence of $\mathcal{P}(E_n)$ on the ratio a/ϵ.

(d) Why is $\mathcal{P}(E_n) = 0$ for all n even?

Hint: The energy levels and position wavefunctions for the stationary states of a particle in an an infinitely deep one-dimensional potential well (of width a) are given by

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

and

$$\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right),$$

respectively.

Problem 6. A fortune teller's (solid spherical) crystal ball, made from glass with index of refraction n=1.5, is 10cm in diameter. Her secret ring is placed 6cm from the edge of the ball.

(a) An image of the ring appears on the opposite side of the crystal ball. How far is the image from the center of the ball?

(b) Draw a ray diagram showing the formation of the image.

(c) The crystal ball is removed and a thin lens is placed where the center of the ball had been. If the image is still in the same position, what is the focal length of the lens?