The University of Georgia
Department of Physics and Astronomy

Graduate Qualifying Exam — Part II

11 August 2009

Instructions: Attempt all problems. Start each problem on a new sheet of paper, and print your name on each sheet of paper that you submit. This is a closed-book, closed-notes exam. You may use a calculator, but *only* for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.). For full credit, you must show your work and/or explain your answers. Part II has six problems, numbered 7–12.

Problem 7: (one part)

Consider the following Hamiltonian describing a one-dimensional oscillator in an external electric field,

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 - eEx.$$

Derive the equations of motion for the operators x and p, in the Heisenberg picture. Show that their equations of motion are operator versions of the classical equations of motion.

Problem 8: (three parts)

A given three-state quantum system has a Hamiltonian of the form

$$H_0 = \begin{bmatrix} h & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

with h a positive real number. To split the degeneracy of the excited state, the system is perturbed by a real-valued potential

$$V = \begin{bmatrix} 0 & 0 & f \\ 0 & 0 & f \\ f & f & 0 \end{bmatrix}.$$

(a) Determine *exactly* the energy levels of the perturbed system.

(b) Define the dimensionless parameter $\epsilon \equiv f/h$. Assuming the perturbation is small ($\epsilon \ll 1$), use part (a) to determine how each energy level shifts, to lowest non-trivial order in ϵ.

(c) Use the approximations from part (b) to find the eigenvectors of the perturbed system, to lowest non-trivial order in ϵ. You don't need to normalize your answers; in fact, it's strongly discouraged.

Problem 9: (four parts)

Kepler's three laws of planetary motion are as follows:

(i) *Law of Orbits:* Planets move in elliptical orbits with the Sun located at one focus.

(ii) *Law of Areas:* A straight line joining the Sun to any of the planets describes equal areas in equal times.

(iii) *Law of Periods:* The square of the period of any planet around the Sun is proportional to the cubed power of the average distance between the planet and the Sun.

Isaac Newton gave the theoretical foundation to these laws.

(a) Write down the equation that expresses Newton's universal law of gravitation.

(b) Describe what feature of Newton's law of gravitation accounts for Kepler's first law (Law of Orbits).

(c) Describe what feature of Newton's law accounts for Kepler's second law (Law of Areas).

(d) Show that Newton's law leads to Kepler's third law (Law of Periods). (You may assume a circular orbit for simplicity.)

Problem 10: (three parts)

The electromagnetic fields corresponding to the Liénard-Wiechert potentials are given, at a location $\mathbf{r}$ and time t, by

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \frac{q}{4\pi\epsilon_0}\frac{\eta}{(\boldsymbol{\eta}\cdot\mathbf{u})^3}\left[(c^2-v^2)\mathbf{u}+\boldsymbol{\eta}\times(\mathbf{u}\times\mathbf{a})\right] , \\ \mathbf{B}(\mathbf{r},t) &= \frac{1}{c}\hat{\boldsymbol{\eta}}\times\mathbf{E}(\mathbf{r},t) , \end{aligned}$$

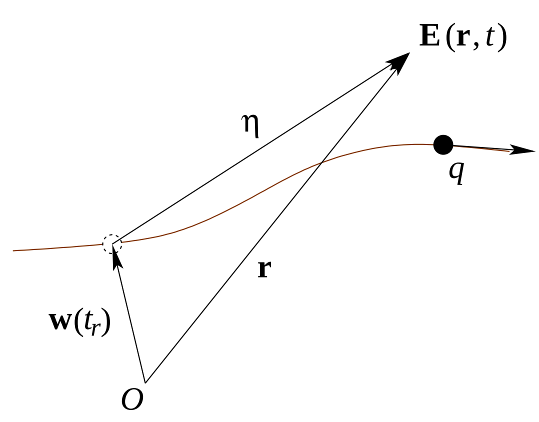
where $\boldsymbol{\eta} \equiv \mathbf{r}-\mathbf{w}(t_r)$; $\mathbf{w}(t)$ describes the trajectory of the charge and $t_r \equiv t-\frac{\eta}{c}$ stands for the retarded time, i.e., the time required for the signal to reach the position $\mathbf{r}$ from the source position $\mathbf{w}(t_r)$. The velocity and acceleration of the point charge at the retarded time are denoted by $\mathbf{v}$ and $\mathbf{a}$, respectively, and we define $\mathbf{u} \equiv c\hat{\boldsymbol{\eta}} - \mathbf{v}$.

Suppose a point charge q is constrained to move along the x-axis with a velocity $\mathbf{v}$.

(a) Calculate the fields at points on the x-axis with $|\mathbf{r}| > |\mathbf{w}(t_r)|$.

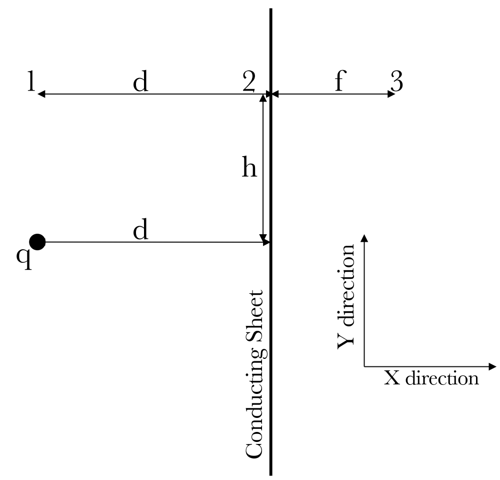
(b) Calculate the fields at points on the x-axis with $|\mathbf{r}| < |\mathbf{w}(t_r)|$.

(c) Find the radiation intensity (i.e., the square of the magnitude of the Poynting vector) of the point charge in the direction of its velocity.



Problem 11: (four parts)

There is a point charge q located a distance d from a grounded conducting plate. The plate is infinitely long and approximately infinitely thin. The remaining space is a vacuum. You can use the symbols q, d, h, f, π, and ϵ_0 in your answers below. Please use the coordinate system indicated on the figure.



(a) What is the electric field $\mathbf{E}$ (in Cartesian components) at the location marked 1 on the figure? Location 1 is a distance h above the charge and d from the plate.

(b) What is the electric potential V at the location marked 1 on the figure?

(c) What is the electric field $\mathbf{E}$ (in Cartesian components) at the location marked 2 on the figure? Location 2 is on the plate at a distance h above the charge.

(d) What is the electric potential V at the location marked 3 on the figure? Location 3 is a distance h above the charge and a distance f behind the plate.

Problem 12: (three parts)

The surface of the Sun has a temperature of $T_S = 6000$ K. The Sun is mostly ionized hydrogen and, for ease, you can make the approximation that it is composed *only* of ionized hydrogen; i.e., it is a gas of protons and electrons. You can ignore the Sun's rotation, electric field, and magnetic field. Use a Cartesian coordinate system placed anywhere on the surface of the Sun and let $\hat{\mathbf{z}}$ be the direction pointing away from the Sun.

(a) Let v_z be a proton's velocity in the $\hat{\mathbf{z}}$ direction. Write an expression for the velocity distribution of protons in the $\hat{\mathbf{z}}$ direction. Your equation can contain v_z, T_S, and other physical and mathematical constants, but no other invented variables.

(b) Consider that if a proton is traveling fast enough in the $\hat{\mathbf{z}}$ direction, it will escape the gravity of the Sun. For protons at the surface of the Sun, the required kinetic energy is $K_p = 3.2 \times 10^{-16}$ joules. What is the numerical value of the ratio of the number of protons that are traveling at the escape velocity in the z direction, to the number of protons with no velocity in the z direction?

(c) Write an expression for the fraction of protons that can escape from the surface of the Sun (assuming no further collisions), and simplify it as much as possible. Your answer can use the above variables rather than their numerical values.