The University of Georgia
Department of Physics and Astronomy

Graduate Qualifying Exam — Part I
10 August 2009

Instructions: Attempt all problems. Start each problem on a new sheet of paper, and print your name on each sheet of paper that you submit. This is a closed-book, closed-notes exam. You may use a calculator, but *only* for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.). For full credit, you must show your work and/or explain your answers. Part I has six problems, numbered 1–6.

Problem 1: (two parts)

The wave function for a traveling sinusoidal wave has the form

$$y(x,t) = A \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right).$$

(a) Prove that a crest of the wave moves with speed $v = \lambda/T$ to the right.

(b) Show that if the minus sign in the above equation is replaced by a plus sign, then the wave moves to the left with the same speed.

Problem 2: (three parts)

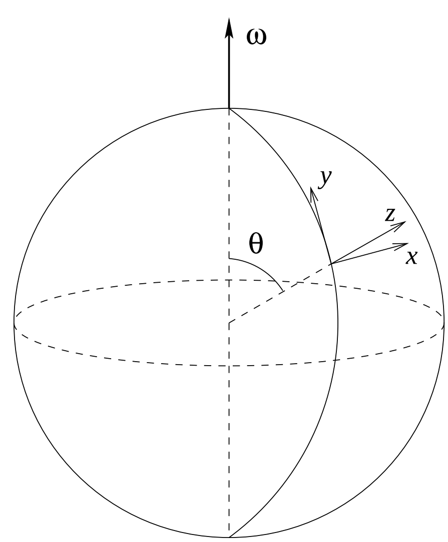
Refer to the diagram of the Earth below right. In this coordinate frame, the x-axis points East, the y-axis points North, and the z-axis points up. The angle between the Earth's angular velocity vector $\boldsymbol{\omega}$ and the z axis is θ. The equation of motion in the x direction for a particle in this rotating reference frame is

$$m\ddot{x} = -2m\omega\dot{z}\,\sin\theta.$$

(a) A particle is dropped at rest from a height h above the ground; it arrives at the ground with velocity $v_0 = \sqrt{2gh}$. Find the magnitude and direction of the Coriolis deflection. (Hint: find the z equation of motion first and substitute into the x equation of motion.)

(b) The particle is now thrown vertically upward with an initial speed v_0, so that it reaches the maximum height h as in part (a), and then falls back to the ground. Find the magnitude and direction of the Coriolis deflection.

(c) Compare your results for parts (a) and (b).



Problem 3: (one part)

A cylindrical glass cup has mass m and height h. When the cup is full, it can hold a mass of water equal to $6m$. When empty, the cup's center of mass is a height $h/4$ from the bottom. Find the height of water in the cup such that the cup is in the most stable state. (Stability is related to the position of the system's center of mass. You may ignore the thickness of the bottom of the cup.)

Problem 4: (three parts)

An electron is initially in the spin state

$$\chi = A \begin{bmatrix} 1 - 2i \\ 2 \end{bmatrix}$$

in the standard S_z basis. Here A is the normalization constant.

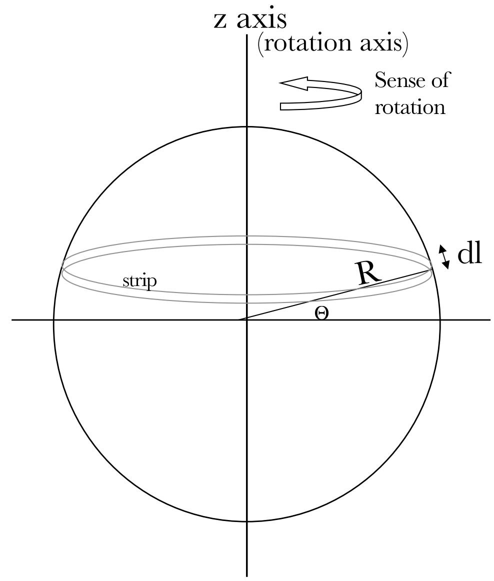
(a) If you measured S_z for this electron, what values would you get, and with what probabilities? Also, what is the expectation value $\langle S_z \rangle$?

(b) Calculate the probabilities of measuring the initial electron spin to be in the $+x$ or $-x$ direction. What is the expectation value $\langle S_x \rangle$?

(c) Suppose you initially measure the z-component of the electron's spin, and *then* measure the y-component of this electron's spin. What are the probabilities that the second measurement yields an electron spin in the $+y$ or $-y$ direction, and what is the expectation value $\langle S_y \rangle$?

Problem 5: (three parts)

The Sun has a net charge Q on its surface. Although the Sun is not perfectly round, let us approximate it as being a perfectly round, conducting sphere of radius R. Although the Sun's rotation period varies with latitude, let us approximate the rotation period as a constant τ.



(a) What is the current (magnitude and direction) in a length element dl of the Sun's surface at a latitude of θ?

(b) Use the Biot-Savart Law,

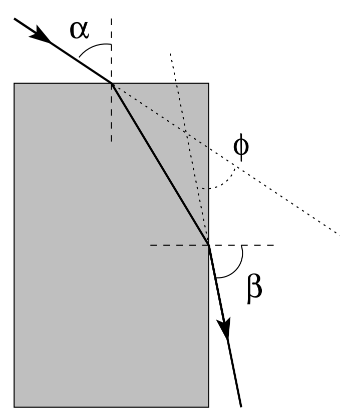
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{s} \times \mathbf{r}}{r^3},$$

in which $d\mathbf{s}$ is a differential element of current, and $\mathbf{r}$ is the displacement from the current element to the location where the field is being computed. Calculate the component of $\mathbf{B}$ along the $\hat{\mathbf{z}}$ rotation axis at the exact center of the Sun, due to the current in a loop of width dl located on the Sun's surface at a latitude θ.

(c) Use your answers from above to determine the magnitude and direction of $\mathbf{B}$ at the origin due to the current on the entire surface of the Sun. (Note that in reality, the Sun's magnetic field includes other, more interesting and more important, contributions.)

Problem 6: (two parts)

A beam of light is incident from air onto a rectangular block with a refractive index n satisfying $1 < n < \sqrt{2}$. The beam enters the top face of the block at an angle of α and leaves the side face at an angle of β, as shown in the figure.



(a) Prove that the two angles satisfy the equation

$$\sin^2\alpha + \sin^2\beta = n^2.$$

(b) There is an incident angle α_m for which the net deflection ϕ of the light beam is a minimum. Find α_m in terms of n.