KEY

PHYS 1312 Fall 2015 Test 3
Nov. 19, 2015

Name ______________ **Student ID** ______________ **Score** ________

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The magnetic dipole moment of the helium atom in its ground state is zero.

True $He(1s^2)$ no unpaired spins, $S=0$
$L = \sum l_i = 0$

(b) If the electric field inside a charged spherical conductor in equilibrium is zero, the electric potential must also be zero.

False $E_r = -\frac{\partial V}{\partial r} = 0$ tells us the slope of V is zero, but V could be a non-zero constant

(c) For a conductive wire in a circuit with a conventional current I, the magnitude of the electric field inside the wire is inversely proportional to the cross sectional area of the wire.

True $I = |q| n A u E$
or $E = \frac{I}{|q| n u}\left(\frac{1}{A}\right)$

Consider electric field outside a spherical conductor and $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}$

$\vec{E} = \frac{k_e q}{r^2}\hat{r} \quad r > R$

Problem 2. Two solid spherical conductors of radius R are separated by a distance D from center-to-center located on the x-axis (from the figure $D = 2R + 2d + L$). The left sphere is at the origin and carries a uniformly distributed charge $+Q$ and the right sphere carries a uniformly distributed charge $-Q$. (a) Calculate the potential difference ΔV from $x = R$ to $x = D - R$, giving the result in terms of $k_e, Q, R,$ and D. b) Sketch the potential from $x \to -\infty$ to $x \to \infty$ taking the potential to go to zero as $x \to \pm\infty$. (c) Show that the result from part (a) becomes $\Delta V = \cancel{4}k_eQ/D$ when $D >> R$. (30 points total)

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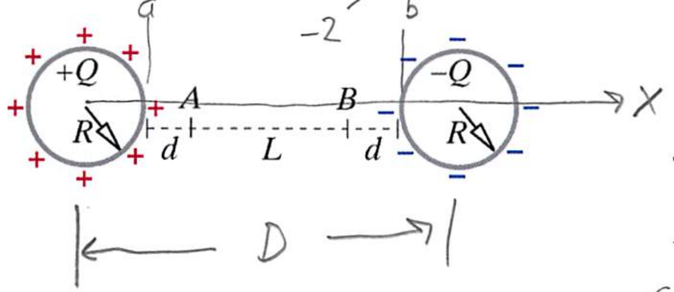


a) $\Delta V = \Delta V_{+Q} + \Delta V_{-Q}$

$$\Delta V = -\int_R^{D-R} \frac{k_e Q}{r^2}\hat{r} \cdot d\vec{x} - \int_R^{D-R} \frac{k_e(-Q)}{r^2}\hat{r} \cdot d\vec{x}$$

or

$$\Delta V = -k_e Q\left[\int_R^{D-R} \frac{dx}{x^2} - \int_{D-R}^{R} \frac{dx'}{x'^2}\right]$$

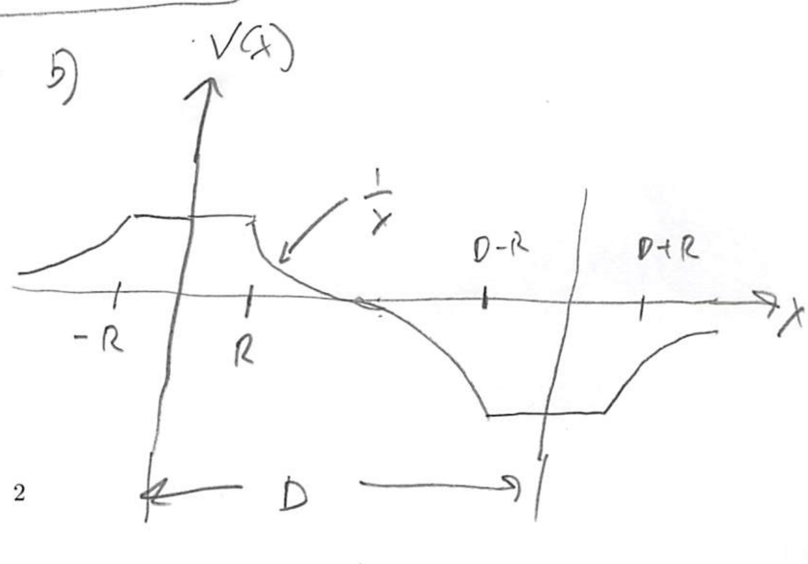
$$\Delta V = k_e Q\left[\frac{1}{x}\Big|_R^{D-R} - \frac{1}{x'}\Big|_{D-R}^{D}\right] = k_e Q\left[\frac{1}{D-R} - \frac{1}{R} - \frac{1}{R} + \frac{1}{D-R}\right]$$

$$\boxed{\Delta V = 2k_e Q\left[\frac{1}{D-R} - \frac{1}{R}\right] = 2k_e Q\left[\frac{2R-D}{(D-R)R}\right]}$$

c) $D >> R$

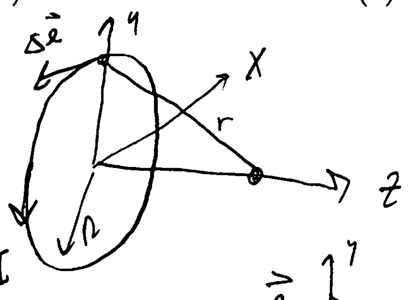
$$\Delta V \simeq 2k_e Q\left[\frac{-D}{DR}\right] = \boxed{-\frac{2k_e Q}{R}}$$

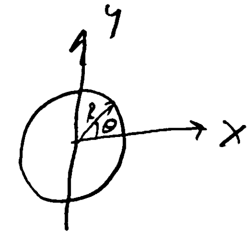
b)



$$\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{l} \times \hat{r}}{r^2} \quad , \quad \mu = IA$$

Problem 3. Consider a ring of radius R with a conventional current I flowing counter-clockwise. The ring is the $x - y$ plane with the center at the origin. Starting with the Biot-Savart equation for a current in a segment of wire $\Delta\vec{\ell}$, (a) derive the magnetic field at an arbitrary location on the z-axis (i.e., along the ring axis). (b) Find the magnitude of magnetic field at $z = 0$ and for $z >> R$. (c) Rewrite the results from (b) in terms of the magnetic dipole moment. (30 points total)



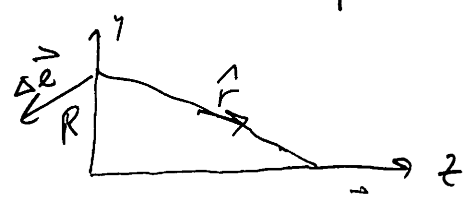


a)

$\vec{r}_{source} = \langle R\cos\theta, R\sin\theta, 0 \rangle$

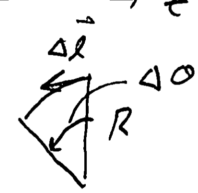
$\vec{r}_{obs} = \langle 0, 0, z \rangle$

$\vec{r} = \langle -R\cos\theta, -R\sin\theta, z \rangle$



Due to symmetry, all perpendicular components of B with respect to z-axis will cancel. So, consider only one location on ring at $\langle 0, R, 0 \rangle \Rightarrow \vec{r} = \langle 0, -R, z \rangle$

$\rightarrow \theta = \pi/2$



$|\vec{r}| = \sqrt{R^2+z^2}$, $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle 0, -R, z \rangle}{\sqrt{R^2+z^2}}$, $\Delta\vec{l} = R\Delta\theta \langle -1, 0, 0 \rangle$

put it all together

$$\Delta\vec{B} = \frac{\mu_0 I}{4\pi} R\Delta\theta \frac{\langle -1, 0, 0 \rangle \times \langle 0, -R, z \rangle}{(R^2+z^2)^{3/2}}$$

the cross product $\langle -1, 0, 0 \rangle \times \langle 0, -R, z \rangle = \langle 0, z, R \rangle$

giving

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{R}{(R^2+z^2)^{3/2}} \int d\theta \langle 0, z, R \rangle$$

As above the y-component vanishes. So, consider only z-component.

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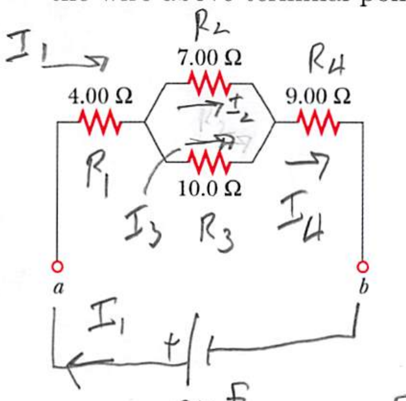
Problem 4. For the four resistors shown in the diagram, (a) find the equivalent resistor for the circuit. (b) Connect a battery with $emf = 1.5$ V across the ends of the resistor circuit (terminals a and b) and determine the conventional current in each of the four resistors. (c) If the connecting wires are copper with a radius of 1.00 mm, electron mobility of 4.5×10^{-3} (m/s)/(V/m), and number density of mobile electrons of 8×10^{23} electrons per m^3, what is the conductivity of the wire, the current density, and the magnitude of the electr~~on~~ic field in the wire above terminal point a? (30 points total)



a) $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{7} + \frac{1}{10}$

or $R_{23} = \frac{7\cdot 10}{7+10} = 4.118\,\Omega$

$R_{eq} = R_1 + R_{23} + R_4 = 4 + 4.118 + 9$

$\boxed{R_{eq} = 17.118\,\Omega}$ $\Rightarrow$

b) For equivalent circuit, use Ohm's Law, $\Delta V = I R_{eq}$

$I = \frac{emf}{R_{eq}} = \frac{1.5\,V}{17.118\,\Omega} = \boxed{8.763\times10^{-2}\,A = I_1 = I_4}$

Apply Kirchhoff's loop rule

$\sum \Delta V_i = 0 = emf - I_1 R_1 - \Delta V_{cd} - I_4 R_4 = 0$

$\Delta V_{cd} = emf - I_1(R_1 + R_4) = 1.5\,V - (8.763\times10^{-2})(4+9)$

$= 0.3608\,V$

$I_3 = \frac{\Delta V_{cd}}{R_3} = \frac{0.3608\,V}{10\,\Omega} = \boxed{3.61\times10^{-2}\,A}$

$I_2 = \frac{\Delta V_{cd}}{R_2} = \frac{0.3608}{7\,\Omega} = \boxed{5.15\times10^{-2}\,A}$

c) conductivity

$\sigma = |q| u n$

$= (1.602\times10^{-19}\,\frac{C}{e^-})(4.5\times10^{-3}\,\frac{m}{s}\frac{m}{V})\cdot(8\times10^{23}\,e^-/m^3)$

$= \boxed{576.7\,\frac{A}{Vm}}$

Problem 3 (Cont'd)

$$B_z = \frac{\mu_0}{4\pi}\frac{IR^2}{(R^2+z^2)^{3/2}}\int_0^{2\pi} d\theta = \boxed{\frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(R^2+z^2)^{3/2}}}$$

b) for $z=0$ $\quad B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{R^3} = \boxed{\frac{\mu_0 I}{2R}}$

for $z \gg R$ $\quad B_z \simeq \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{z^3} = \boxed{\frac{\mu_0 R^2 I}{2z^3}}$

c) Magnetic dipole is defined as $\mu = IA = I\pi R^2$ for the loop

$\Rightarrow B_z^{(z=0)} = \boxed{\frac{\mu_0}{4\pi}\frac{2\mu}{R^3}}$, $B_z(z \gg R) = \boxed{\frac{\mu_0}{4\pi}\frac{2\mu}{z^3}}$

Problem 4 (Cont'd)

$I = |q| n A v E = A\sigma E \Rightarrow E = \frac{I}{A\sigma} = \frac{8.763\times10^{-2}\,A}{\pi(1\times10^{-3}\,m)^2(576.\cancel{8})}$

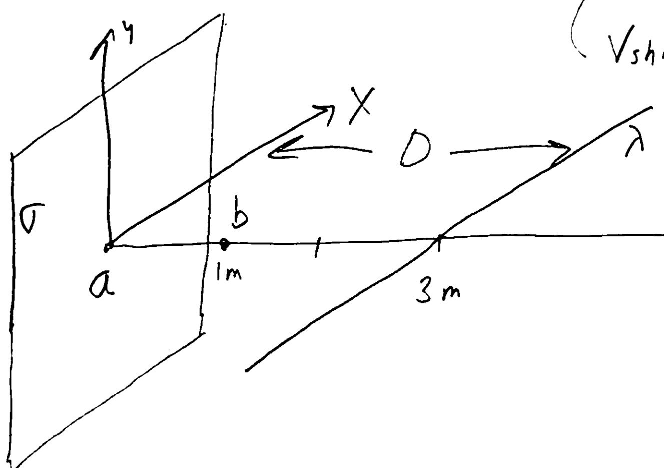
$\boxed{E = 48.4\,\frac{V}{m}}$

$J = \sigma E = \frac{I}{A} = \frac{8.763\times10^{-2}\,A}{\pi(1\times10^{-3})^2} = \boxed{27{,}890\ A/m^2}$

σ

Bonus Problem. An infinite sheet of charge that has a surface charge density of 20 nC/m^2 lies in the $x-y$ plane passing through the origin. An infinitely long wire of linear charge $\lambda =$ density 80 nC/m lies parallel to the x-axis and intersects the z-axis at $z = 3.00$ m. If the potential of the system at $\vec{r} =< 0, 0, 0 >$ is 1.50 kV, (a) what is the potential at $< 0, 0, 1.00 >$ m? (b) Make a sketch of the potential along the z-axis. (5 points total)

V_{sheet}



apply

$\Delta V = V_b - V_a$

$= -\int \vec{E} \cdot d\vec{r}$

Need electric fields due to disk and rod

Infinite sheet can be obtained from a disk with $R \gg z$

$$E_{z,disk} = \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right] \simeq \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{R}\right] \simeq \boxed{\frac{\sigma}{2\varepsilon_0}}$$ for z small

infinite wire use rod with $L \gg r$.

$$E_{rod} = \frac{k_e Q}{r\sqrt{r^2+(L/2)^2}} \simeq \frac{k_e Q}{r L/2} = \frac{2k_e \lambda}{r}$$ since $\lambda = Q/L$

$$\Delta V_{ab} = V_b - V_a = -\int_{sheet} E\,dz - \int_{wire} E\,dr + V_{sheet}$$

$$= -\int_0^z \frac{\sigma\,dz}{2\varepsilon_0} - \int \frac{2k_e\lambda}{r}\,dr + V_{sheet} = -\frac{\sigma}{2\varepsilon_0} z - 2k_e\lambda \int_D^{D-z} \frac{dz}{z} + V_{sheet}$$

$$= -\frac{\sigma}{2\varepsilon_0} z + V_{sheet} - 2k_e\lambda \ln z\Big|_D^{D-z}$$

(Cont'd on next page)

Bonus problem continued

$$\Delta V = -\frac{\sigma}{2\varepsilon_0} z + V_{sheet} - 2k_e\lambda \ln\left(\frac{D-z}{D}\right)$$

$$= -\frac{(20\times10^{-9})(1)}{2(8.854\times10^{-12})} + 1500 - 2(8.99\times10^{9})(80\times10^{-9}) \ln\left(\frac{2}{3}\right)$$

$$= -1129.4 + 1500 + 583.22 = \boxed{953.8\ V}$$

sketch