(1)

KEY

PHYS 1312 Fall 2015 Test 2

Oct. 13, 2015

Name ______________________ **Student ID** ______________________ **Score** ________

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) For a single slit, the maxima in the diffraction intensity I occur at odd integer multiples of the wavelength λ.

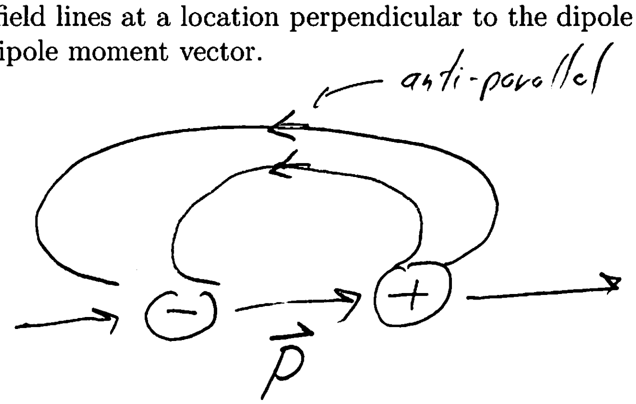
False. Only the minima can be predicted analytically.

(b) In steady-state, the electric field $|\vec{E}|$ inside of a conductor is always zero.

True.

(c) For an electric dipole, the electric field lines at a location perpendicular to the dipole axis point generally anti-parallel to the dipole moment vector.

True.



Problem 2. (a) In an experiment, monochromatic x-rays ($\lambda = 0.166$ nm) are incident on a potassium chloride (KCl) crystal surface. The spacing between planes of atoms in KCl is $d =$ 0.314 nm. At what angle (relative to the surface) should the beam be directed for a <u>second-order maximum</u> to be observed? (b) In a second experiment, a He-Ne laser beam ($\lambda = 633$ nm) is focused on a 10-μm-diameter spot 8.0 cm behind a lens ($f = 8.0$ cm). What minimum diameter must the lens have? (c) Explain why the He-Ne laser would not be useful for the experiment in part (a). (30 points total)

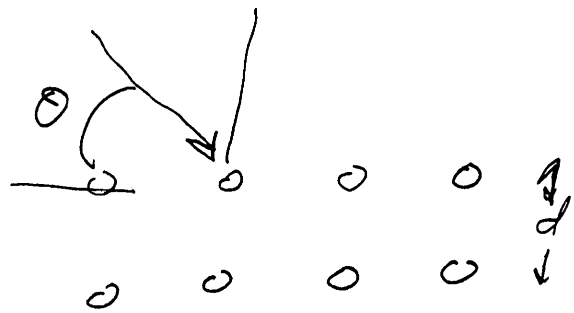
$m = 2$

a) Use Bragg's Law

$$2d\sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$$

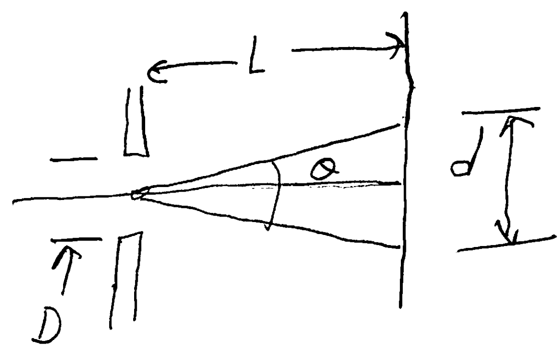
$$= \sin^{-1}\left(\frac{2(0.166\times10^{-9}\,m)}{2(0.314\times10^{-9}\,m)}\right) = \boxed{31.9^\circ}$$



b) $\lambda = 633$ nm

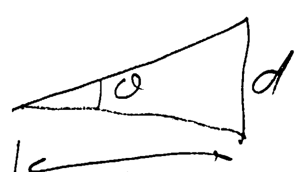
$d = 10\,\mu m$ (spot size)

$L = 8.0$ cm



use Rayleigh's criteria for circular aperture

$$\theta_{min} = 1.22\frac{\lambda}{D}$$



$\tan\theta = \frac{d}{L}$

$$D = \frac{1.22\lambda}{\theta_{min}} = \frac{1.22\lambda}{\tan^{-1}\left(\frac{d}{L}\right)} = \frac{1.22(633\times10^{-9})}{\tan^{-1}\left(\frac{1e-5}{0.08}\right)}$$

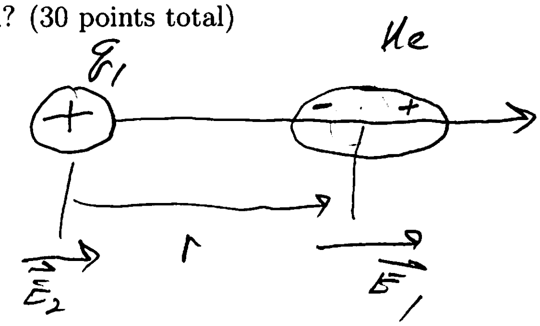
$\theta = \tan^{-1}\left(\frac{d}{L}\right)$

$= \boxed{0.618\text{ cm}}$

c) The wavelength of the "light" needs to be comparable to the scale of the feature to resolve. To resolve objects on a nm scale we need nm scale probes.

Problem 3. A proton and a neutral He atom are initially 100 nm apart and there are no other particles in the vicinity. The polarizability of neutral helium is 2.3×10^{-41} C^2m/N. (a) Starting with the definition of the electric field due to a point charge, obtain the electric field acting on the proton due to the He atom. (b) Assuming the He atom is fixed, what is the initial acceleration magnitude and direction of the proton? (30 points total)

$= r$ $\quad = \alpha$



a) The electric field at the center of the He atom due to the proton (point charge) is

$$\vec{E}_1 = \frac{k_e q_1}{r^2}\hat{r}$$

this creates induced-dipole moment of the He $\vec{P}_2 = \alpha \vec{E}_1$

Then an electric field acts on the proton due to the dipole

$$\vec{E}_2 = \frac{k_e 2 P_2}{r^3}\hat{r} = \frac{k_e 2\alpha \vec{E_1}}{r^3} = \frac{k_e 2\alpha}{r^3}\left(\frac{k_e q_1}{r^2}\right)\hat{r}$$

$$= \frac{k_e^2\, 2\alpha q_1}{r^5}\hat{r}$$ or since $q_1 = e$, the magnitude is

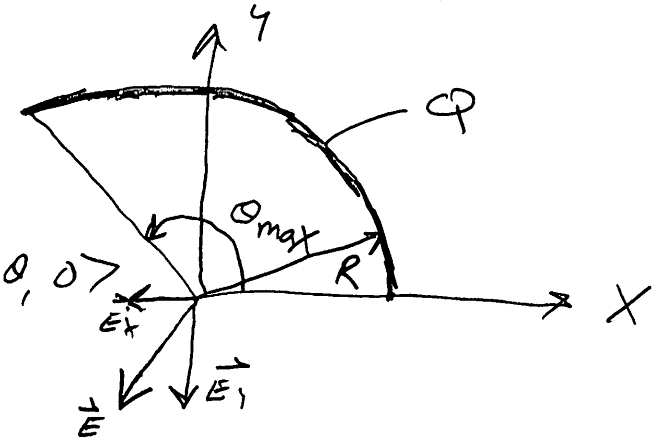
$$E_2 = \frac{\left(8.99 \times 10^9 \frac{Nm^2}{C^2}\right)^2 2\left(2.3\times 10^{-41}\frac{C^2 m}{N}\right)\left(1.602\times 10^{-19} C\right)}{(100\times 10^{-9} m)^5}$$

$$\boxed{E_2 = 5.96\times 10^{-5}\frac{N}{C}}$$

$\oplus \rightarrow F_e = eE_2 = m_p a$

b) $$\vec{a} = \frac{eE_2}{m_p}\langle 1,0,0\rangle = \frac{(1.602\times 10^{-19})(5.96\times 10^{-5}\frac{N}{C})}{1.672\times 10^{-27} kg} = \boxed{5.7\ kN \ \langle 1,0,0\rangle}$$

Problem 4. Consider a thin metal rod in the $x - y$ plane bent into an arc with radius R centered at the origin. Starting on the positive x-axis, the rod is bent through an arc of $\theta_{\max}$ and has a uniformly distributed charge Q. (a) Starting with the definition of the electric field due to a point charge, derive the electric field vector at the origin. Draw a diagram showing the components of the electric field if $\theta_{\max} = 3\pi/4$. (b) What is the electric field in the limit that $\theta_{\max} = 2\pi$? (c) What is the electric field in the limit that $\theta_{\max} = 0$? (30 points total)



a) $\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$, $\vec{r}_{obs} = \langle 0, 0, 0 \rangle$

$\vec{r}_{source} = \langle R\cos\theta, R\sin\theta, 0 \rangle$, $|\vec{r}| = R$

$\vec{r} = \vec{r}_{obs} - \vec{r}_{source} = \langle -R\cos\theta, -R\sin\theta, 0 \rangle$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \langle -\cos\theta, -\sin\theta, 0 \rangle$

the linear charge density $\lambda = \frac{Q}{L}$ where $L = R\theta_{max}$ is the arc length

or $dq = \lambda R\, d\theta$ $\qquad \lambda = \frac{Q}{R\theta_{max}}$

Therefore

$$\vec{E} = k_e \int \frac{\lambda R\, d\theta}{R^2} \langle -\cos\theta, -\sin\theta, 0 \rangle = \frac{k_e \lambda}{R} \int_0^{\theta_{max}} d\theta \langle -\cos\theta, -\sin\theta, 0 \rangle$$

Then

$$E_x = -\frac{k_e \lambda}{R} \int_0^{\theta_{max}} \cos\theta\, d\theta = -\frac{k_e \lambda}{R} \sin\theta \Big|_0^{\theta_{max}} = -\frac{k_e}{R}\left(\frac{Q}{R\theta_{max}}\right) \sin\theta_{max}$$

$$= \frac{k_e Q}{\theta_{max} R^2} (-\sin\theta_{max})$$

$$E_y = -\frac{k_e \lambda}{R} \int_0^{\theta_{max}} \sin\theta\, d\theta = \frac{k_e}{R}\left(\frac{Q}{R\theta_{max}}\right)\left[\cos\theta\right]_0^{\theta_{max}}$$

$$= \frac{k_e Q}{\theta_{max} R^2}\left[\cos\theta_{max} - 1\right]$$

$$\boxed{\vec{E} = \frac{k_e Q}{\theta_{max} R^2} \langle -\sin\theta_{max}, \cos\theta_{max} - 1, 0 \rangle}$$

$E_z = 0$

Problem 4 (Cont'd)

take $\theta_{max} = \frac{3\pi}{4}$

$$\vec{E} = \frac{4K_e Q}{3\pi R^2}\left\langle -\sin\frac{3\pi}{4},\ \cos\frac{3\pi}{4} - 1,\ 0\right\rangle$$

(see sketch)

$$\vec{E} = \frac{4K_e Q}{3\pi R^2}\langle -0.707,\ -1.707,\ 0\rangle$$

b) let $\theta_{max} = 2\pi$

$$\vec{E} = \frac{k_e Q}{2\pi R^2}\langle -\sin(2\pi),\ \cos(2\pi) - 1,\ 0\rangle$$

$$= \frac{k_e Q}{2\pi R^2}\langle 0,\ 0,\ 0\rangle = \boxed{\langle 0, 0, 0\rangle = \vec{E}}$$

c) let $\theta_{max} = 0$

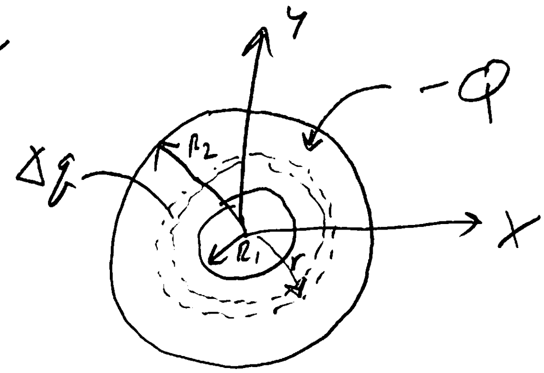
now $\sin\theta \simeq \theta$, $\cos\theta \simeq 1$ for $\theta \to 0$

$$\vec{E} \to \frac{k_e Q}{\theta_{max} R^2}\langle -\theta,\ 1 - 1,\ 0\rangle$$

$$\boxed{\vec{E} = \frac{K_e Q}{R^2}\langle -1, 0, 0\rangle}$$

Point charge located at $\vec{r}_{source} = \langle R, 0, 0\rangle$

Bonus Problem. Consider a solid thin disk in the $x - y$ plane with radius R_2, but with a concentric hole cut-out with radius R_1 (i.e., $R_2 > R_1$). If the disk carries a uniformly distributed charge $-Q$, find the initial maximum acceleration magnitude of an electron placed at a point on the z-axis. Hint, you must start with the relation for the electric field due to a thin ring of charge $-\Delta Q$ and find z where the electric field is a maximum. (5 points total)



The surface charge density $\sigma = Q/A$

So for a ring of radius r

$$dq = \sigma \pi d(r^2) = \sigma 2\pi r dr$$

Also, $A = \pi(R_2^2 - R_1^2)$

The electric field due to the ring charge element Δq is

$$\Delta E_z = \frac{K_e z \Delta q}{(r^2+z^2)^{3/2}} = \frac{K_e z \sigma 2\pi r dr}{(r^2+z^2)^{3/2}}$$

from equation sheet

or

$$E_z = K_e z \sigma 2\pi \int_{R_1}^{R_2} \frac{r dr}{(r^2+z^2)^{3/2}} = K_e z \sigma 2\pi \left[\frac{-1}{\sqrt{r^2+z^2}}\right]\Bigg|_{R_1}^{R_2}$$

$$E_z = \frac{1}{4\pi\varepsilon_0} z \left(\frac{Q \quad 2\pi}{\pi(R_2^2-R_1^2)}\right)\left[(R_1^2+z^2)^{-1/2} - (R_2^2+z^2)^{-1/2}\right]$$

$$\boxed{E_z = \left(\frac{1}{2\varepsilon_0}\right)\left(\frac{Q}{\pi(R_2^2-R_1^2)}\right)\left[\frac{z}{\sqrt{R_1^2+z^2}} - \frac{z}{\sqrt{R_2^2+z^2}}\right]} \quad -(1)$$

now to find maximum

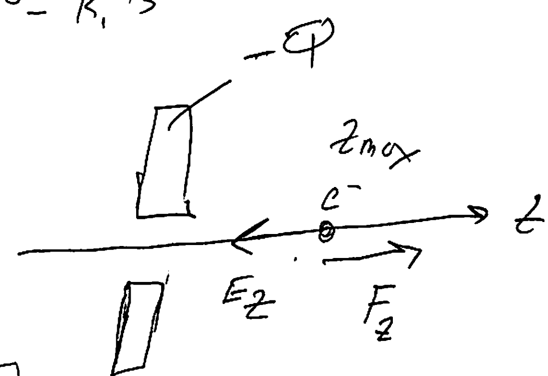
$$\frac{dE_z}{dz} = 0 = \frac{d}{dz}\left[\frac{z}{\sqrt{R_1^2+z^2}} - \frac{z}{\sqrt{R_2^2+z^2}}\right]$$

Bonus Problem (Cont'd)

Besides the trivial solutions $z \to 0$, $z \to \infty$, one gets

$$\frac{d(E_z)}{dz} = \frac{R_1^2}{(R_1^2+z^2)^{3/2}} - \frac{R_2^2}{(R_2^2+z^2)^{1/2}} = 0$$

solving for z gives (after lots of algebra) $z_{max} = \sqrt{\dfrac{R_2^{2/3} - R_1^{2/3}}{R_2^{4/3} - R_1^{4/3}}}\; R_1^{2/3} R_2^{2/3} > 0$



$E_z = E_z(1)$ evaluated at z_{max}

$\vec{E}_z = |\vec{E}_z| \langle -1, 0, 0 \rangle$

$\vec{F}_e = -e\vec{E}_z = m_e \vec{a}_e$

$$\boxed{\vec{a}_e = \frac{-e\vec{E}_z}{m_e} = \frac{|e\vec{E}_z|}{m_e}\langle 1, 0, 0\rangle}$$

limits $R_1 \to 0$, $R_2 \to R$ $\quad E_z = \dfrac{Q/A}{2\varepsilon_0}\left[1 - \dfrac{z}{\sqrt{R^2+z^2}}\right]$

solid disk result

$R_1 \to 0$, $R_2 \to R$, $\quad z_{max} \to 0$

$$E_z^{max} = \frac{Q/A}{2\varepsilon_0}\left[\frac{z_{max}}{z_{max}} - \frac{z_{max}}{\sqrt{R^2+z_{max}^2}}\right]$$

$$\to \frac{Q/A}{2\varepsilon_0}\left[1 - 0\right] = \frac{Q/A}{2\varepsilon_0}$$ solid disk near $z=0$