KEY

PHYS 1312 Fall 2015 Test 1
Sept. 10, 2015

Name ______________ **Student ID** ______________ **Score** ______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) When a light wave propagates through a medium, the dependence of its frequency f on the index of refraction n is called dispersion.

False. f remains constant; it is λ that changes

(b) Only virtual images can be formed with convex spherical mirrors.

True. They do not create real images.

(c) Chromatic aberration cannot occur with a mirror.

True. chromatic aberration occurs as light passes through a lens since n depends on λ

$q = p$

Problem 2. A thin converging spherical lens with focal length f = 25.0 cm is used to image an object. (a) If the object is placed 30.0 cm from the lens, draw a ray diagram to locate the image. Find the image distance and magnification and indicate whether it is real or virtual, upright or inverted. (b) Repeat (a) with the object 20.0 cm from the lens. (c) To image a hair follicle, one needs a magnification of $|M|$=200. If a compound microscope is composed of an objective lens (f_o = 1.00 cm) and an eyepiece (f_e = 2.00 cm) what is the required distance between the objective and eyepiece? (30 points total)

a) Use lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \text{ solve for } q$$

$$q = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{25} - \frac{1}{30}}$$

$q = 150.$ cm

$$M = -\frac{q}{p} = \frac{-150}{30} = -5$$ (inverted)

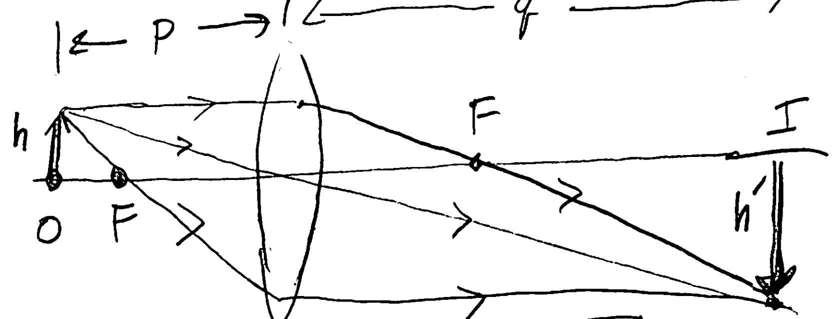


Image is real, but inverted

b)
$$q = \frac{1}{\frac{1}{25} - \frac{1}{20}} = -100.\text{ cm}$$ (virtual)

$$m = -\frac{q}{p} = \frac{-(-100)}{20} = 5$$ (upright)

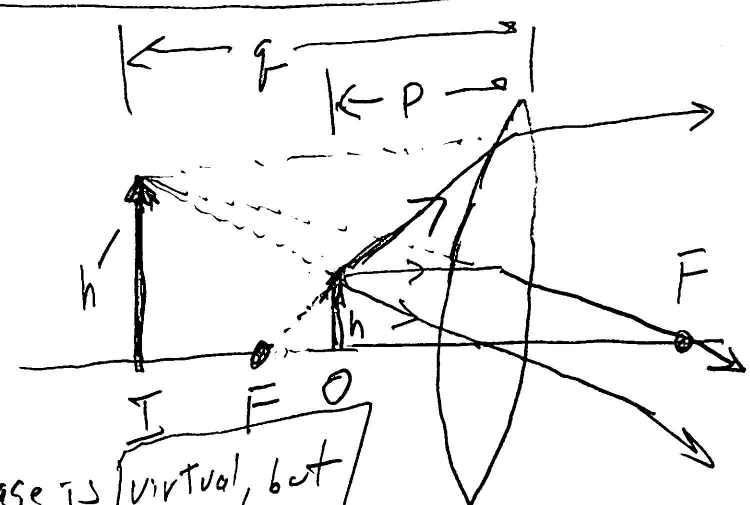


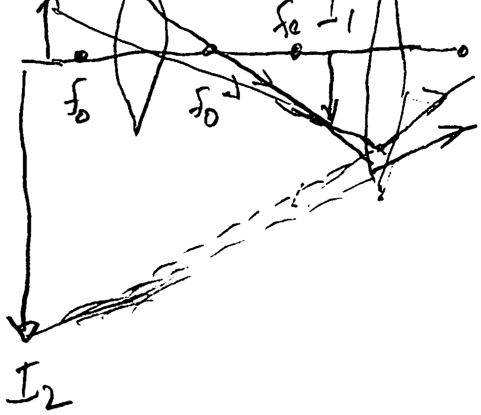
Image is virtual, but upright

c) Use compound microscope equation

$$M = m_o m_e = -\frac{L}{f_o}\frac{25\text{ cm}}{f_e}$$

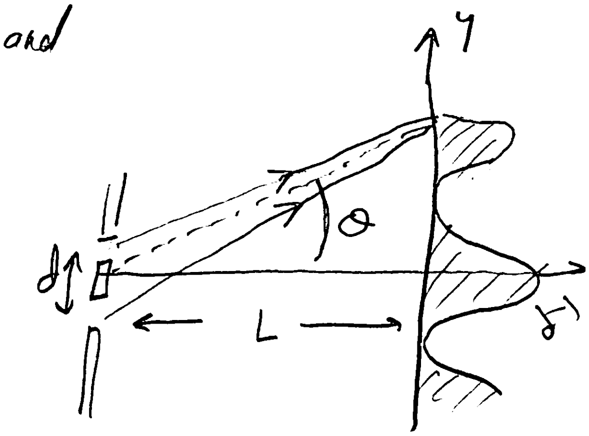
Solve for L

$$L = \frac{|M| f_o f_e}{25} = \frac{|200|(1)(2)}{25} = 16.0\text{ cm}$$



Problem 3. The intensity on the vertical screen at a certain point in a double-slit interference pattern is 64.0% of the maximum value. (a) What minimum phase difference (in radians) between sources produces this result? (b) Express this phase difference as a path difference for 486.1-nm light. (c) If the distance between slits is 1.00 μm, determine the angle to the interference point on the screen with respect to the horizontal. (d) If the horizontal distance between the slits and screen is 1.00 m, what is the vertical distance of the interference point on the screen with respect to the central maximum? (30 points total)

a) Start with the intensity equation and find phase difference ϕ



$$I = I_0 \cos^2 \frac{\phi}{2} = 0.64 I_0$$

$$\phi = 2\cos^{-1}(\sqrt{0.64}) = \boxed{1.287 \text{ rad}}$$

b) $\phi = \frac{2\pi}{\lambda}\delta$ or $\delta = \frac{\phi\lambda}{2\pi}$

$$\delta = \frac{(1.287 \text{ rad})(486.1 \text{ nm})}{2\pi \text{ rad}} = \boxed{99.57 \text{ nm}}$$

c) $\delta = d\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{\delta}{d}\right) = \sin^{-1}\left(\frac{99.57\times10^{-9}}{1\times10^{-6}}\right)$

or $\theta = 9.973\times10^{-2}$ rad $= \boxed{5.71^\circ}$

d) $\tan\theta = \frac{y}{L}$ or $y = L\tan\theta = (1\text{m})\tan 5.71^\circ$

$= \boxed{0.100\text{m}}$

Note: this point does not correspond to a minimum or maximum.

Problem 4. (a) To determine the elemental composition of a distant astronomical object (e.g., a planetary nebula), we use a telescope to focus its polychromatic light on a diffraction grating. If a first-order principal maximum is recorded to have an angle of 17.15° measured from the primary maximum for a grating with 5000 slits per cm, what element is it? Consult Table 1. (b) Does a fourth-order diffraction pattern exist? (c) If the wavelength obtained in part (a) is directed at a Michelson interferometer, interference fringes are recorded as the mirror at the end of one arm of the interferometer is moved out a distance of 0.100 mm. How many fringes (bright to dark to bright) are observed? (30 points total)

a) For grating we

$d \sin\theta = m\lambda$

with $m=1$ (first-order)

Table 1: Wavelengths for some neutral atoms.

Element	Wavelength (nm)
H	656.4
Li	661.3
Na	589.6

Find $d = \frac{1\text{ cm}}{5000\text{ slits}} = 2\times10^{-4}\text{ cm} = 2000\text{ nm}$

$\lambda = \frac{d\sin\theta}{m} = \frac{(2000\text{ nm})\sin 17.15^\circ}{1} = \boxed{589.75\text{ nm}}$

Na is the feature

b) check $m=4$, if so a line would appear for that angle

$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{4(589.75)}{2000}\right) = \sin^{-1}(1.18)$

$\Rightarrow$ not defined, argument of $\sin^{-1}$ can not be greater than 1. So $\boxed{\text{No}}$

c) For constructive interference the mirror is move $d = \frac{\lambda}{2}$ ← for one fringe

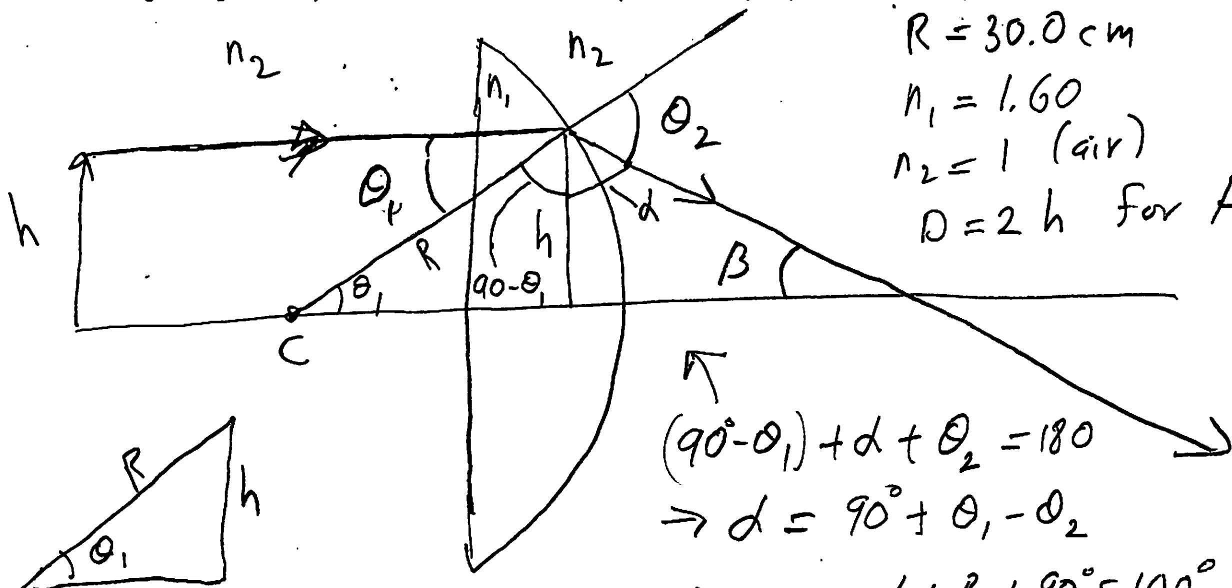
So, path difference is $\delta = 2d = 2\left(\frac{\lambda}{2}\right) = \lambda$

L = total length mirror movement $= Nd$ or $d = \frac{L}{N}$

$\Rightarrow \lambda = \frac{2L}{N} \Rightarrow N = \frac{2L}{\lambda} = \frac{2(0.1\times10^{-3})}{589.75\times10^{-9}} = \boxed{339}$

same as LON-CAPA HW #3, prob. 2

Bonus Problem. Consider a large plano-convex lens with radius of curvature 30.0 cm and index of refraction 1.60. Rays from a distant object travel parallel to the principal axis, strike the flat side of the lens and form a real image on the other side. An aperture of diameter D is placed in from of the lens to minimize spherical aberration. What minimum value of D is required so that rays crossing the principal axis to form the image make an angle (with the principal axis) of less than 0.174 rad (about 10°)? (5 points total)



$R = 30.0$ cm

$n_1 = 1.60$

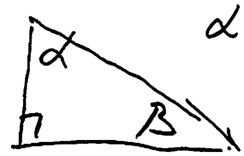
$n_2 = 1$ (air)

$D = 2h$ for $\beta \leq 0.174$ rad

$(90° - \theta_1) + \alpha + \theta_2 = 180$

$\Rightarrow \alpha = 90° + \theta_1 - \theta_2$

$\sin\theta_1 = \frac{h}{R}$



$\alpha + \beta + 90° = 180°$

$\beta = 90° - \alpha = 90° - 90° - \theta_1 + \theta_2$

$= \theta_2 - \theta_1$

or $\theta_2 = \beta + \theta_1$

apply Snell's Law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \frac{h}{R} = \sin\theta_2$

$n_1 \frac{h}{R} = \sin(\beta + \theta_1)$

$n_1 \frac{h}{R} = \sin\beta\cos\theta_1 + \cos\beta\sin\theta_1$

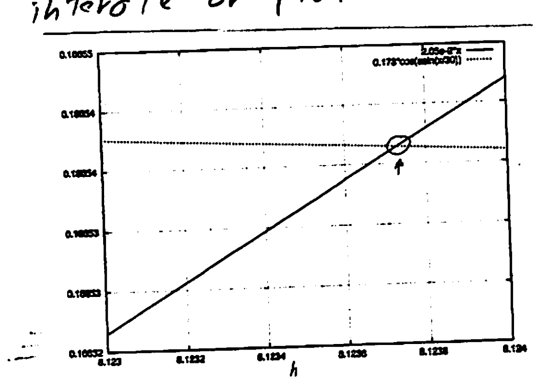
$n_1 \frac{h}{R} = \sin\beta\cos\left(\sin^{-1}\frac{h}{R}\right) + \cos\beta\frac{h}{R}$

$\left(\frac{n_1 - \cos\beta}{R}\right) h = \sin\beta\cos\left(\sin^{-1}\frac{h}{R}\right)$

↑ transcendental equation → find root

$(2.05 \times 10^{-2}) h = 0.173\cos\left(\sin^{-1}\frac{h}{30}\right)$

iterate or plot



$\Rightarrow h = 8.1237$ or

$D = 16.25$ cm