KEY

**PHYS 1312 Fall 2022 Test 3**
Nov. 17, 2022

**Name** ______________________ **Student ID** ______________________ **Score** _______

**Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you** *must show all* **of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last three pages.**

**Problem 1. Conceptual questions**. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) In steady-state, the electric field inside of a conductor is zero.

True

(b) Charges are free to move around on or in an insulator.

False

(c) An electron placed in a uniform electric field, will accelerate along an equipotential.

False    Electric field vectors point perpendicular to equipotential line.

(d) Any material placed in an electric field will become polarized.

True

$/ u$

**Problem 2.** In a particular metal, the mobility of the mobile electrons is 0.0077 (m/s)/(N/C). At some instance in time in non-equilibrium, the net electric field everywhere inside a cube of this metal is 0.053 N/C < 1, 0, 0 >. What is the average drift speed of the mobile electrons in the metal at this instant? (15 points total)

$E$

$$\bar{v} = uE = 0.0077 \frac{m/s}{N/C}\left(0.053 \frac{N}{C}\right) = \boxed{4.08 \times 10^{-4} \, m/s}$$

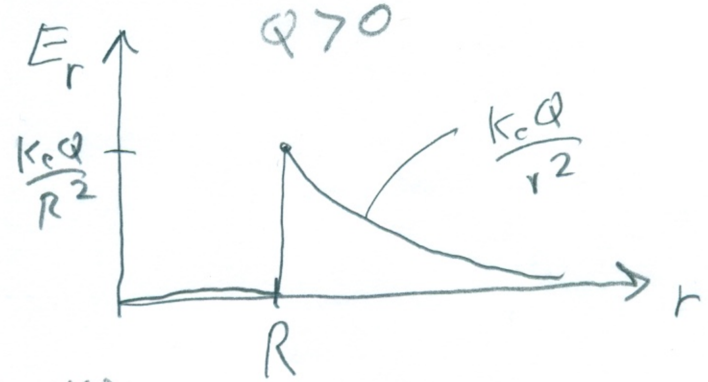
**Problem 3.** For a solid spherical conductor of radius $R$ and charge $Q$, plot for $r < R$ and $r > R$, (a) the electric field and (b) electric potential as a function of $r$. (15 points total)

a) $\vec{E} = \frac{k_e Q}{|\vec{r}|^2}\hat{r} \quad r \geq R$

$\vec{E} = 0, \; r < R$

$E(r=R) = \frac{k_e Q}{R^2}$

$Q > 0$

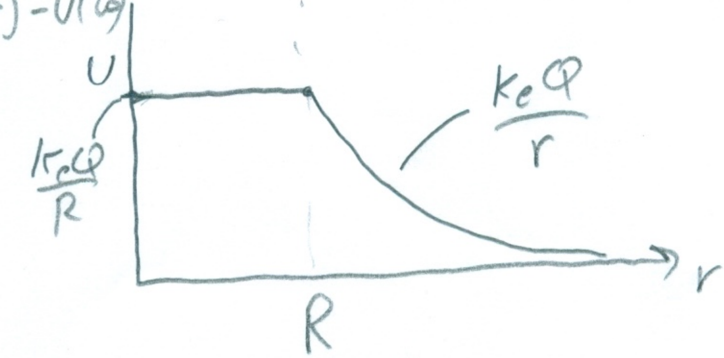



b) $\Delta U = -\int_{r>R}^{\infty} \vec{E} \cdot d\vec{r} = U(r) - U(\infty)$

$= -k_e Q \int_{r>R}^{\infty} \frac{dr}{r^2}$

$= -k_e Q \left[\frac{-1}{r}\right]_{r>R}^{\infty}$

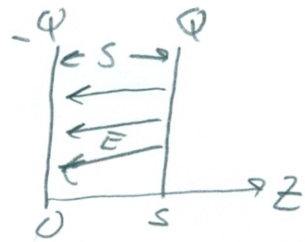
$= \frac{k_e Q}{r}, \; r > R$




$U(r=R) = \frac{k_e Q}{R}$

$U(r<R) = \frac{k_e Q}{R} = \text{constant}$

**Problem 4.** A parallel-plate capacitor with a gap of 1.0 mm ($s$) between the plates has an electric potential difference ($\Delta V$) of 36 V between the plates. What is the magnitude of the electric field ~~potential~~ between the plates? What assumption is made for the electric field to be constant between the plates? (15 points total)

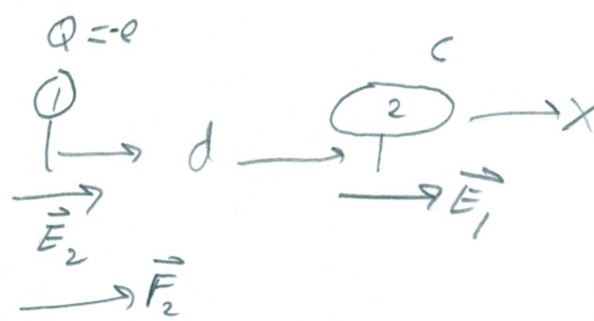


a) $\Delta V = -\int \vec{E}\cdot d\vec{s} = -E\int_0^s dz = -Es$

$|\vec{E}| = \frac{\Delta V}{s} = \frac{36V}{1\times10^{-3}m} = \boxed{36{,}000 \frac{V}{m}}$ or N/C

b) $s \ll$ Radius of the plates

**Problem 5.** An electron and a neutral carbon atom are separated by a distance of $1 \times 10^{-6}$ m ($d$). The polarizability ($\alpha$) of the carbon atom is $1.96 \times 10^{-40}$ C m/(N/C). What is the electric force on the electron? (15 points total)



the electric field experienced by the carbon atom is

$\vec{E}_1 = \frac{k_e Q}{r^2}\hat{\imath}$

The carbon atom is polarized with an induced-dipole moment of $\vec{p}_2 = \alpha\vec{E}_1$

This creates an electric field at the electron

$\vec{E}_2 = \frac{k_e 2\alpha \vec{E}_1}{r^3} = \frac{k_e 2\alpha k_e Q}{r^3 r^2}\hat{\imath} = \frac{2k_e^2 Q\alpha}{r^5}\hat{\imath}$

The force on the electron is then $\vec{F}_2 = Q\vec{E}_2 = \frac{2k_e^2 Q^2 \alpha}{r^5}\hat{\imath}$

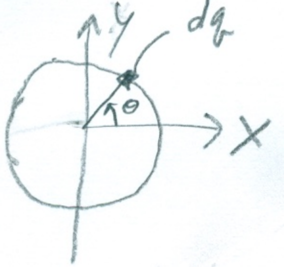
$\vec{F}_2 = \frac{2k_e^2 e^2 \alpha}{d^5}\hat{\imath} = \frac{2(8.99\times10^9)^2(1.602\times10^{-19}C)^2\, 1.96\times10^{-40}}{(1\times10^{-6})^5}\hat{\imath}$

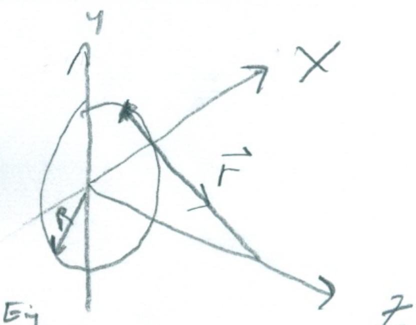
$= \boxed{8.13\times10^{-28} N\ \hat{\imath}}$ attractive

**Problem 6.** Consider a uniformly charged thin ring of radius $R$, charge $Q$, and linear charged density $\lambda$ located at the origin in the $x-y$ plane. a) Beginning with the electric field equation for uniform charge distributions

$$\vec{E} = k_e \int \frac{dq}{|\vec{r}|^2}\hat{r} \tag{1}$$

derive a relation for the electric field on the $z$-axis (i.e., perpendicular to the plane of the ring) in terms of $Q$, $R$, $k_e$, and $z$. (b) Obtain a relation for the electric potential of the same ring taking $V = 0$ as $z \to \infty$. (c) If an electron of mass $m_e$ was placed at +z, obtain a relation for its initial acceleration. (d) If the electron's initial velocity was zero, what kind of motion would it experience? Can we use standard kinematic equations to describe such motion? Why or why not? (30 points total)



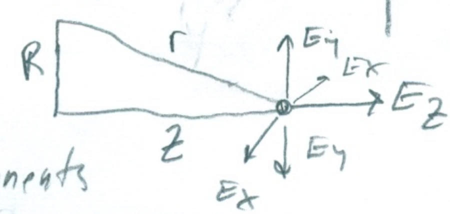




a) $r^2 = R^2 + z^2$, $|\vec r| = \sqrt{R^2+z^2}$

$\vec r_{obs} = \langle 0,0,z\rangle$, $\vec r_{source} = \langle R\cos\theta, R\sin\theta, z\rangle$

$\vec r = \langle -R\cos\theta, -R\sin\theta, z\rangle$

$$\hat r = \frac{\vec r}{|\vec r|} = \frac{\langle -R\cos\theta, -R\sin\theta, z\rangle}{(R^2+z^2)^{1/2}}$$




now, by symmetry all x and y components will sum to zero, so consider only z-component


Also $Q = \lambda L$ or $dq = \lambda ds = \lambda R d\theta$ or

$$\Delta E_z = \frac{k_e \Delta q}{(R^2+z^2)}\frac{z}{(R^2+z^2)^{1/2}} = \frac{k_e \lambda R z \Delta\theta}{(R^2+z^2)^{3/2}}$$

integrate for $\theta = 0 \to 2\pi$

$$E_z = \frac{k_e \lambda R z}{(R^2+z^2)^{3/2}}\int_0^{2\pi} d\theta = \frac{k_e \overbrace{\lambda R 2\pi}^{Q} z}{(R^2+z^2)^{3/2}} = \boxed{\frac{k_e Q z}{(R^2+z^2)^{3/2}}}$$

b) $$\Delta V = -\int_{s_1}^{s_2} \vec E \cdot d\vec s = -k_e Q \int_z^{\infty} \frac{z}{(R^2+z^2)^{3/2}}\,dz$$ (cont'd)

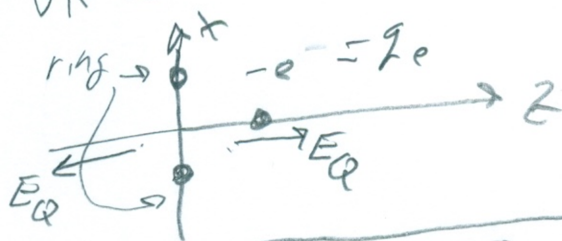
## Problem 6 (cont'd)

From the equation sheet, $\int \frac{x\,dx}{(x^2 \pm y^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm y^2}}$

so that

$$\Delta V = -k_e Q \left[ -\frac{1}{\sqrt{R^2+z^2}} \right]_z^{\infty} = \frac{k_e Q}{\sqrt{R^2+z^2}} - 0$$

or $$\boxed{V(z) = \frac{k_e Q}{\sqrt{R^2+z^2}}}$$




c) $\sum F_z = m_e a_z$

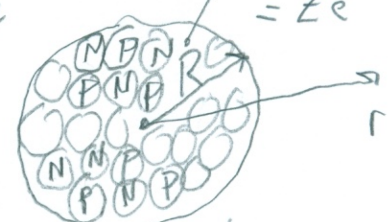
$$q_e E_z = \frac{q_e k_e Q z}{(R^2+z^2)^{3/2}} = m_e a_z \Rightarrow \boxed{a_z = -\frac{k_e e Q z}{m_e (R^2+z^2)^{3/2}}}$$

d) for $v_{iz}=0$ and $Q>0$, the electron would accelerate in the $-z$-direction (to the left), when the electron's position moved to $z<0$, the sign of the E-field and force change, hence it then accelerates to the right. This is oscillatory motion. It is not SHM, since the force is not proportional to $z$, but close if $R \gg |z|$

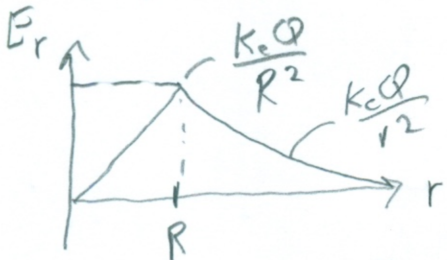
e) Since the force is not constant, kinematic equations are not valid.

**Bonus Problem.** A nucleus contains $Z$ protons that on average are uniformly distributed throughout a tiny sphere of radius $R$. (a) Calculate the electric potential (relative to infinity) at the center of the nucleus (assume there are no electrons or other charged particles near the nucleus). (b) Plot the potential as a function of $r$ from the center of the nucleus to $r > R$. (c) Now suppose that a high-energy accelerator experiment creates an anti-muon at the center of this nucleus (there is also a neutrino formed, but don't worry about that) and with a initial kinetic energy of $K_i$. The anti-muon is positively charged and has a mass of 207 times the mass of the electron. The anti-muon is repelled far away from the nucleus. If the nucleus is a gold atom (79 protons), what is the numerical value of $K_f - K_i$ in eV? (5 points)

a) Start with the electric field for an insulator




Gold nucleus moder



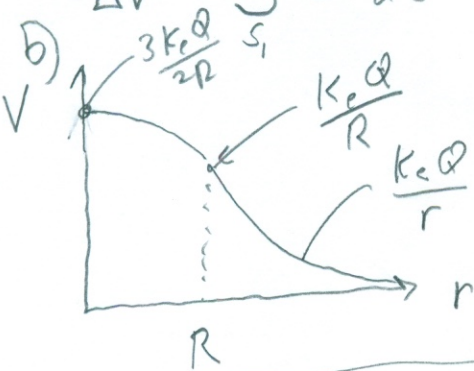

$$E_r = \frac{k_e Q}{r^2}, \quad r \geq R$$

$$E_r = \frac{k_e Q}{r^2}\left(\frac{r^3}{R^3}\right) = \left(\frac{k_e Q}{R^3}\right) r, \quad r \leq R$$

now consider the electric potential

$$\Delta V = -\int_{s_1}^{s_2} \vec{E}\cdot d\vec{s} = -\int_{r_1}^{r_2} E_r\, dr$$

b)




For $r \to r = \infty$, $r \geq R$

$$\Delta V = V_\infty - V_r = -k_e Q \int_r^\infty \frac{dr}{r^2}$$

$$= -k_e Q\left[-\frac{1}{r}\right]_r^\infty = 0 - \frac{k_e Q}{r}$$

$$V_\infty = 0, \quad V_r = \frac{k_e Q}{r}, \quad V(r=R) = \frac{k_e Q}{R}$$

For $0 \leq r \leq R$, $\Delta V = V_R - V_r = -\frac{k_e Q}{R^3}\int_r^R r\, dr = -\frac{k_e Q}{R^3}\left[\frac{r^2}{2}\right]_r^R$

or $\Delta V = -\frac{k_e Q}{2R} + \frac{k_e Q}{2R^3} r^2 = \frac{k_e Q}{2R^3}\left[r^2 - R^2\right] = -\frac{k_e Q}{2R}$ ← potential drop from $r=0$ to $R$

Therefore $V(r=0) = \frac{k_e Q}{R} + \frac{k_e Q}{2R} = \boxed{\frac{3k_e Q}{2R} = V_0}$

# Bonus Problem (Cont'd)

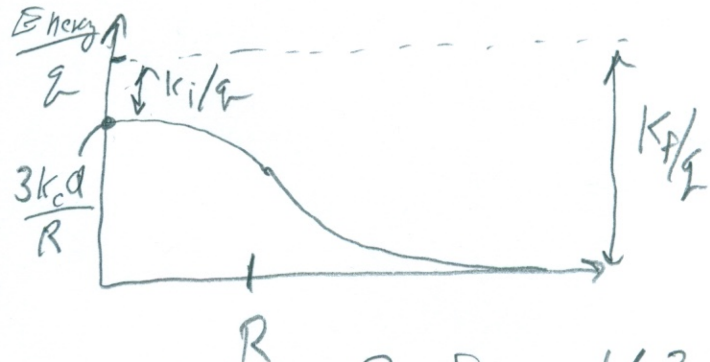
Then for $r \to 0 \to r = R$

$$V = \frac{3k_eQ}{2R} - \frac{k_eQ}{2R^3}r^2$$ (see plot) at $r = R$ $V(R) = \frac{k_eQ}{R}$

remember $E_r = -\frac{dV(r)}{dr}$ or the negative slope of the potential

c) Now use conservation of energy $K_i + U_i = K_f + U_f$

or $\frac{K_i}{q} + V_i = \frac{K_f}{q} + V_f$




where $q = e$ (charge of anti-muon), $V_f = 0$

$$K_i - K_f = qV_i = \frac{e\,3k_eZe}{2R}$$

$$\Delta K = \frac{3(8.99\times10^9)(79)(1.602\times10^{-19})^2}{2(1\times10^{-15})(160)^{1/3}}$$

$$= 5.05\times10^{-12}\ J$$

or $\Delta K = 5.05\times10^{-12}\,J \times \frac{1\,eV}{1.602\times10^{-19}\,J/eV}$

$\cong \boxed{31.5\ MeV}$

But what is $R$ for gold?

The atomic number $A = Z + N$

$Z = 79$, $N \equiv$ # of neutrons $> Z$

So $A \geq 2(79) = 158 \sim 160$

Actual number is $A = 197$

Now how many protons and neutrons can be packed into a sphere?

$$\frac{4}{3}\pi R^3 \cong A\frac{4}{3}\pi r^3$$ $r \cong$ proton radius $\cong 10^{-15}\,m$

$$R \cong rA^{1/3}$$