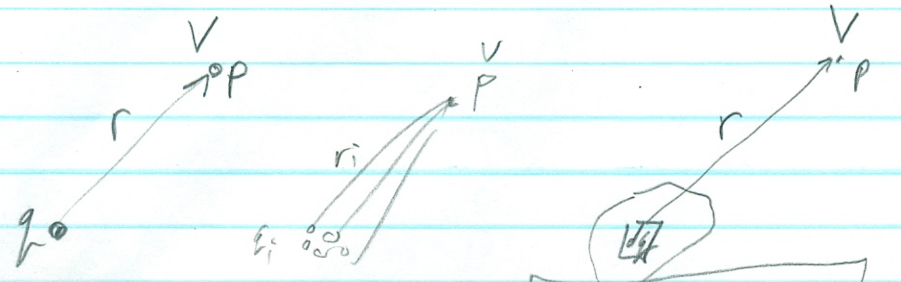
Electric Potential Due to Continuous Charge Distributions 10-24-2006

- Similar the electric field due to a charge distribution which is generalized from a lump of infinitesimal charge, we can generalize the potential due to a point charge to a distribution of charge



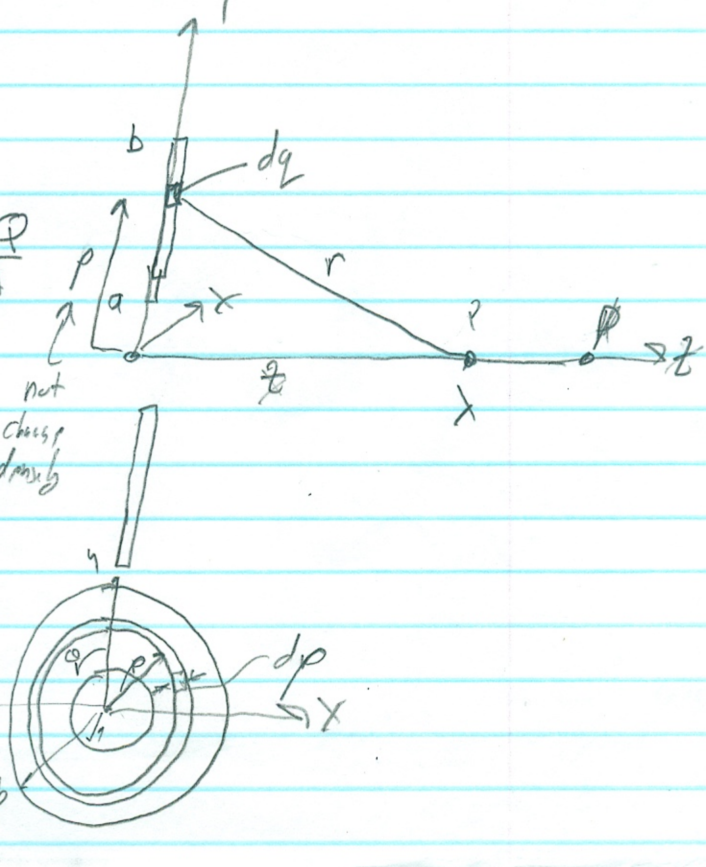
$$V = k_e \frac{q}{r} \qquad V = \frac{\sum k_e q_i}{r_i} \qquad \boxed{V = k_e \int \frac{dq}{r}}$$

- Procedure is identical to finding the electric field due to a distribution of charge

- Example Problem 25.4b

- a thin disk with hole and uniform charge surface density $\sigma = \frac{Q}{A}$

- find the electric potential along the z-axis



$$V = k_e \int \frac{dq}{r}$$

$$Q = \sigma A \Rightarrow dq = \sigma\, dA$$

$$V = k_e \sigma \int \frac{dA}{r}$$

10-24-2006

Use cylindrical polar coordinates $\rho^2 = x^2 + y^2$

$$\boxed{dA = \rho\, d\rho\, d\theta} \qquad r^2 = \rho^2 + z^2$$

$$V = k_e \sigma \int \frac{\rho\, d\rho\, d\theta}{(\rho^2+z^2)^{1/2}} = k_e \sigma \int_0^{2\pi} d\theta \int_{\rho=a}^{\rho=b} \frac{\rho\, d\rho}{(\rho^2+z^2)^{1/2}}$$

$$= k_e \sigma \left[\theta\Big|_0^{2\pi}\right]\left[\sqrt{\rho^2+z^2}\Big|_a^b\right] = \underline{\underline{2\pi k_e \sigma\left[\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right]}} = V(z)$$

limits $a \to 0$ (full disk) $\qquad \sigma = \frac{Q}{A}$

$$V = 2\pi k_e \sigma\left[\sqrt{b^2+z^2} - z\right] = \boxed{\frac{1}{2\varepsilon_0}\frac{Q}{A}\left[\sqrt{b^2+z^2} - z\right] = V}$$

limit $z >> b \Rightarrow$ use binomial expansion $\Rightarrow \frac{b}{z} << 1$

$$(b^2+z^2)^{1/2} = z\left(1 + \frac{b^2}{z^2}\right)^{1/2}$$

$$(1+x)^n \cong 1 + nx \quad \Rightarrow \quad z\left(1 + \frac{b^2}{2z^2}\right)$$

$$\underset{x>>b}{V \cong} 2\pi k_e \sigma\left[z\left(1 + \frac{b^2}{2z^2}\right) - z\right] = 2\pi k_e \sigma z\left[1 + \frac{b^2}{2z^2} - 1\right]$$

$$= 2\pi k_e \sigma z\left[\frac{b^2}{2z^2}\right] = \pi k_e \sigma \frac{b^2}{z}$$

area of a circle is $\pi r^2 = \pi b^2$

$$= k_e \frac{\sigma \pi b^2}{z} = \boxed{k_e \frac{Q}{z} = V}$$ of a point charge

$Q = \sigma A = \sigma \pi b^2$

- Far away all symmetric charge distributions look like point charges

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- Once one has the electric potential, it is straight forward to get the electric field

From our example

$$E_z = -\frac{dV(z)}{dz} = -2\pi k_e \sigma \frac{d}{dz}\left[\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right]$$

$$= -2\pi k_e \sigma \left[\frac{z}{\sqrt{b^2+z^2}} - \frac{z}{\sqrt{a^2+z^2}}\right]$$

see test 2 bonus

let $a \to 0$ (full disk)

$$E_z = 2\pi k_e \sigma \left[1 - \frac{z}{\sqrt{b^2+z^2}}\right]$$

same as example 23.9

let $x >> b \Rightarrow \frac{b}{x} << 1$

$$(b^2+x^2)^{-1/2} = x^{-1}\left(1+\frac{b^2}{x^2}\right)^{-1/2}$$

$$\simeq x^{-1}\left(1 - \frac{b^2}{2x^2}\right)$$

$$E_x \xrightarrow[x>>b]{} 2\pi k_e \sigma \left[1 - \frac{x}{x}\left(1-\frac{b^2}{2x^2}\right)\right]$$

$$= 2\pi k_e \sigma \left[1 - 1 + \frac{b^2}{2x^2}\right] = k_e \frac{\sigma \pi b^2}{x^2} = \boxed{\frac{k_e Q}{x^2}}$$

electric field due to a point charge