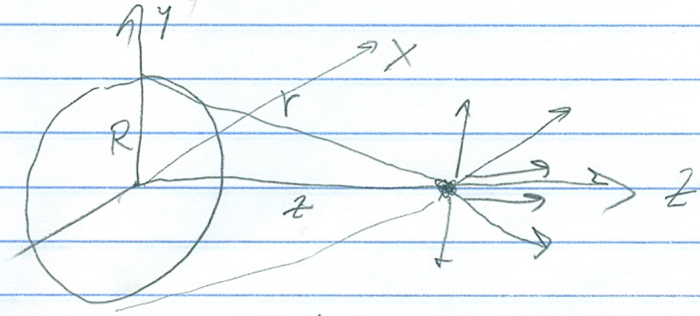
10-6-2018

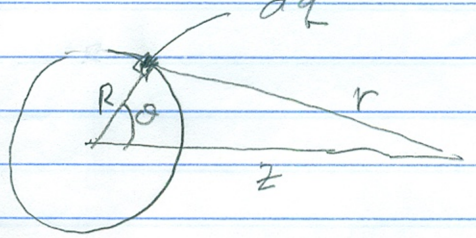
(5)

Uniformly charged ring

$\vec{E}$ on axis through ring of radius R, charge Q. Ring in x-y plane

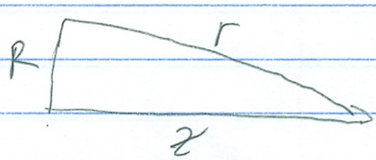


E_y's and E_z's will cancel. Resultant will be on z-axis



use point charge Δq

$$\Delta\vec{E} = \frac{k_e \Delta q}{r^2}\,\hat{r}$$



$$r^2 = R^2 + z^2$$

$$|\vec{r}| = (R^2 + z^2)^{1/2}$$

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{\Delta q}$$

$$= \langle 0, 0, z\rangle - \langle R\cos\theta, R\sin\theta, 0\rangle$$

$$= \langle -R\cos\theta, -R\sin\theta, z\rangle$$

$$\hat{r} = \frac{\langle -R\cos\theta, R\sin\theta, z\rangle}{(R^2 + z^2)^{1/2}}$$

by only E_z is non-zero

$$\Delta E_z = \frac{k_e \Delta q}{(R^2 + z^2)^{3/2}}\, z$$

$\lambda = \frac{Q}{L}$ or $Q = \lambda L$

$Q = \lambda \Theta R$

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or $\Delta q = \lambda R \Delta \theta$ since λ is uniform

$$\Delta E_z = \frac{k_e \lambda R z \Delta \theta}{(R^2+z^2)^{3/2}} \qquad \theta \to 0 \to 2\pi$$

$$E_z = \frac{k_e \lambda R z}{(R^2+z^2)^{3/2}} \int_0^{2\pi} d\theta = \frac{k_e \overbrace{\lambda\, 2\pi R}^{Q}\, z}{(R^2+z^2)^{3/2}}$$

$$\boxed{E_z = \frac{k_e Q z}{(R^2+z^2)^{3/2}}}$$

check limits

$z \to \infty$ $(z \gg R)$ $E_z \to \dfrac{k_e Q z}{z^3} = \dfrac{k_e Q}{z^2}$ point charge

$z \to 0$, $E_z \to 0$ (by symmetry)

For $\theta \not< 2\pi$, E_x, and E_y do not cancel, need to evaluate

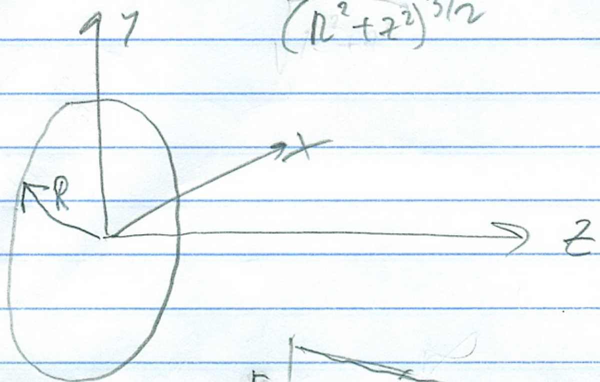
10-6-2015

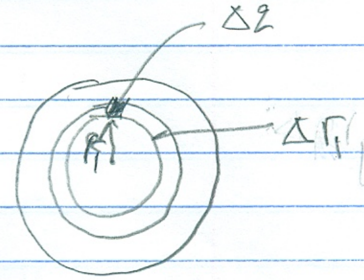
Uniformly charged Disk

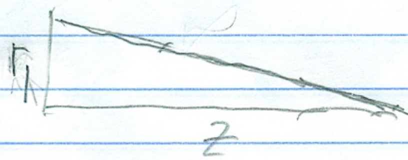
Radius R, charge Q, point on z-axis

use $\bar{E}$-field for thin ring

$$\vec{E}_z^{+} = k_e \frac{q z}{(R^2+z^2)^{3/2}} \langle 0,0,1\rangle$$







$0 \le r_1 \le R$

$\vec{r} = \langle 0,0,z\rangle$, $\hat{r} = \langle 0,0,1\rangle$, $|\vec{r}| = z$ } for ring

For each ring $\Delta\vec{E}_z = k_e \frac{\Delta q\, z}{(r_1^2+z^2)^{3/2}} \langle 0,0,1\rangle$

Surface charge density $\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$

$Q = \sigma \pi R^2$

$\Delta q = \sigma\pi\,\Delta(r_1^2) = \sigma\pi\, 2 r_1 \Delta r_1 = \sigma 2\pi r_1 \Delta r_1$

$$\Delta E_z = k_e \frac{\sigma 2\pi r_1 \Delta r_1 z}{(r_1^2+z^2)^{3/2}} \Rightarrow E_z = k_e \sigma 2\pi z \int_0^R \frac{r_1\, dr_1}{(r_1^2+z^2)^{3/2}}$$

$$\int_0^R \frac{r_1\, dr_1}{(r_1^2+z^2)^{3/2}} = \left[\frac{-1}{(r_1^2+z^2)^{1/2}} \right]_0^R$$

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(11)

$$\int \rightarrow -\left[\frac{1}{(R^2+z^2)^{1/2}} - \frac{1}{z}\right]$$

$$E_z = k_e \sigma 2\pi \left[\frac{z}{z} - \frac{z}{(R^2+z^2)^{1/2}}\right]$$

$$E_z = \frac{1}{\cancel{4\pi}\varepsilon_0} \frac{Q \cancel{2\pi}}{A}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]$$

$$\boxed{E_z = \left(\frac{1}{2\varepsilon_0}\right)\left(\frac{Q}{A}\right)\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]}$$

no approx.

limits $0 \ll z \ll R$

$$E_z \longrightarrow \left(\frac{Q/A}{2\varepsilon_0}\right)\left[1 - \frac{z}{R}\right]$$

Also, does not have to be a disk

$z/R \ll 1$ $\boxed{E_z \longrightarrow \frac{Q/A}{2\varepsilon_0}}$ $\neq f(z)$ ~~MAX~~

$z \gg R$ use $(1+\varepsilon)^n \simeq (1+n\varepsilon)$

$$(R^2+z^2)^{-1/2} = z^{-1}\left(1+\frac{R^2}{z^2}\right)^{-1/2} \simeq z^{-1}\left(1+\left(-\frac{1}{2}\right)\frac{R^2}{z^2}\right)$$

$$= \frac{\left(1-\frac{R^2}{2z^2}\right)}{z}$$

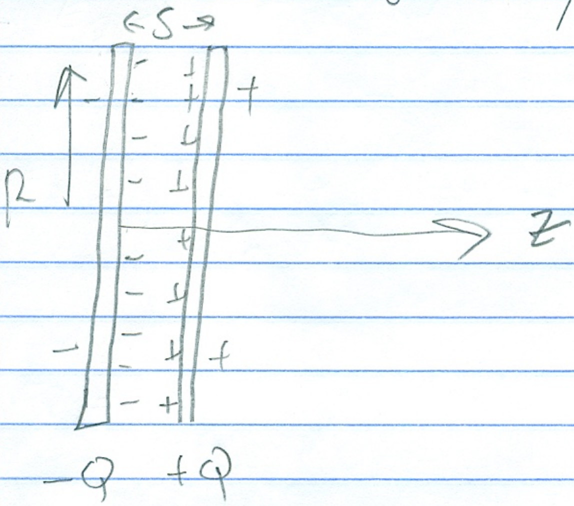
$$E_z \rightarrow \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z\left(1-\frac{R^2}{2z^2}\right)}{z}\right] = \frac{Q/A}{2\varepsilon_0}\frac{R^2}{2z^2}$$

$$= \boxed{\frac{Q/A}{4\varepsilon_0}\,\frac{R^2}{1}\,\frac{1}{z^2}} = \frac{k_e Q}{z^2}$$

Electric Field near a capacitor

10-8-2015

Two uniformly charged disks radius R, but close together, $s << R$

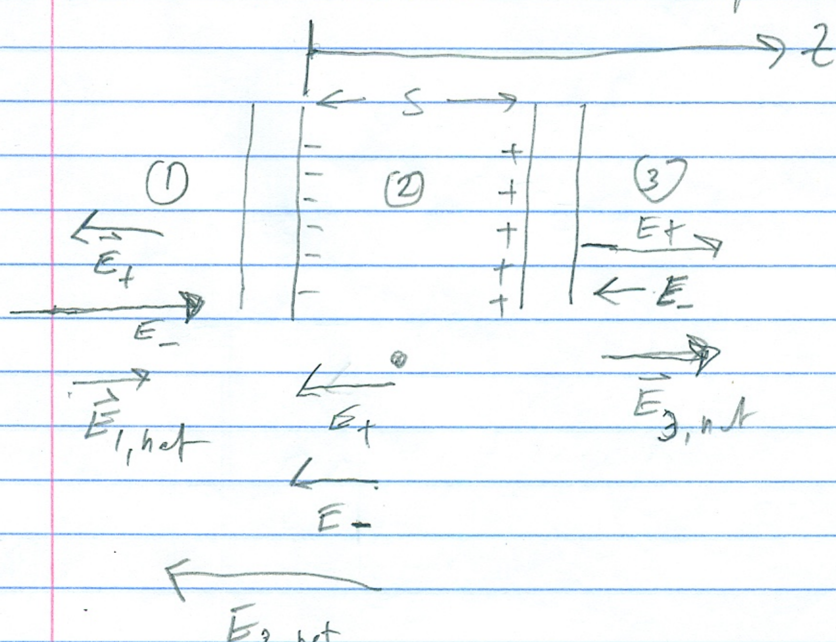


- Assume disks are conductors
- forces between disks will keep charges evenly distributed

Use single disk equation for $0 \le z << R$

$$E_z = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \qquad \frac{Q}{A} = \sigma$$

- assume uniform σ on inside surfaces ($-\sigma$)
- Find directions of all E components



- E-fields in inner region add (point to left)
- in outer region almost cancel, point to right

10-8-2018

(1) $\vec{E}_{1,net} = \vec{E}_+ + \vec{E}_- = \frac{-Q/A}{2\varepsilon_0}\left[1 - \frac{(s+z)}{R}\right]$

$+ \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{R}\right]$

$= \frac{Q/A}{2\varepsilon_0}\left[-1 + 1 + \frac{s}{R} - \frac{z}{R} + \frac{z}{R}\right]$

$= \boxed{\frac{Q/A}{2\varepsilon_0}\frac{s}{R} \to 0}$ for $s \ll R$

→ (3) $\vec{E}_{3,net} = \vec{E}_+ + \vec{E}_- = \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z-s}{R}\right]$ X

$- \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{R}\right]$

$= \frac{Q/A}{2\varepsilon_0}\left[1 - 1 - \frac{z}{R} + \frac{s}{R} + \frac{z}{R}\right] = \boxed{\frac{QA}{2\varepsilon_0}\frac{s}{R}}$

same

(2) $\vec{E}_{2,net} = \vec{E}_+ + \vec{E}_- = -\frac{Q/A}{2\varepsilon_0}\left[1 - \frac{s-z}{R}\right]$

$- \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{R}\right] = -\frac{Q/A}{2\varepsilon_0}\left[2 - \frac{s}{R} + \cancel{\frac{z}{R}} - \cancel{\frac{z}{R}}\right]$

$= -\frac{Q/A}{\varepsilon_0}\left[1 - \frac{s}{2R}\right] \simeq \boxed{-\frac{Q/A}{\varepsilon_0}}$

$\vec{E}_{2,net} = \frac{Q/A}{\varepsilon_0}\langle 0, 0, -1\rangle$ ← almost constant

$\simeq 2\vec{E}_{one\ plate}$

later we will put material between plates