PHYS 1312: In Class Problem Solution
Chapter 14, set 1
Oct. 12, 2017

Problem P39. In Fig. 14.84 from the textbook, there is a permanent dipole on the left (due to a heteronuclear molecule) with a dipole moment of Qs_1 and a neutral atom on the right with polarizability α. They interact such that an induced-dipole is created on the atom with dipole moment $qs_2 = \alpha E_1$, where E_1 is the magnitude of the electric field produced by the permanent dipole at the center of the atom. Find the force (if it exists) that the permanent dipole exerts on the atom. Note, both objects are neutral.

Solution. The electric field due to the permanent dipole at the center of the atom is given by

$$E_1 = \frac{k_e 2p_1}{r^3}, \tag{1}$$

where all particles are on a line (take to be the r direction) and $p_1 = Qs_1$. E_1 polarizes the atom creating an induced-dipole $p_2 = \alpha E_1$. The permanent dipole then experiences an electric field due to the induced-dipole of the atom,

$$E_2 = \frac{k_e 2p_2}{r^3} = \frac{k_e 2\alpha E_1}{r^3}. \tag{2}$$

Substituting eq. 1 into 2 gives

$$E_2 = \frac{k_e 2\alpha}{r^3}\frac{ke2p_1}{r^3} = \frac{4k_e^2\alpha p_1}{r^6}. \tag{3}$$

Now, the force on the permanent dipole due to E_2 acts on each of the permanent dipole charge centers which are $s_1/2$ away from the center. So, the force is

$$F_2 = -\frac{Q4k_e^2\alpha p_1}{(r+s_1/2)^6} + \frac{Q4k_e^2\alpha p_1}{(r-s_1/2)^6}. \tag{4}$$

Since, $r >> s_1$, the denominators can be rewritten and expanded with the binomial theorem, or

$$F_2 = \frac{Q4k_e^2\alpha p_1}{r^6}\left[\frac{1}{(1-s_1/2r)^6} - \frac{1}{(1+s_1/2r)^6}\right]. \tag{5}$$

Then

$$\left(1+\frac{-s_1}{2r}\right)^{-6} \approx 1 - \left(\frac{-6s_1}{2r}\right) = 1 + \frac{3s_1}{r}, \tag{6}$$

and

$$\left(1+\frac{s_1}{2r}\right)^{-6} \approx 1 - \left(\frac{6s_1}{2r}\right) = 1 - \frac{3s_1}{r}, \tag{7}$$

which gives

$$F_2 = \frac{Q4k_e^2\alpha p_1}{r^6}\left[1 + \frac{3s_1}{r} - 1 + \frac{3s_1}{\mathrm{r}}\right], \tag{8}$$

or

$$F_2 = \frac{Q4k_e^2\alpha p_1}{r^6}\frac{6s_1}{r}. \tag{9}$$

Finally

$$F_2 = \frac{k_e^2 24\alpha p_1^2}{r^7}, \tag{10}$$

since $p_1 = Qs_1$.

Extra Problem. Can an electric force exist between two neutral atoms? Do not try to derive it, but reason out why or why not and, if so, how?

Solution. Even though the electron cloud of an atom is spherically symmetric, small departures (or perturbations) result when two neutral atoms interact. This sets-up small dipole moments in the atoms which in turn produce electric fields on the other atom. As in the above problem, induced-dipoles are created, but in both atoms with polarizabilities α_1 and α_2. The atoms experience an attractive induced-dipole–induced-dipole force of the form

$$|\vec{F}| \propto \frac{\alpha_1\alpha_2}{r^7}, \tag{11}$$

but which is much weaker than the permanent-dipole–induced-dipole force derived above. This is known as a van der Waals force.