KEY

PHYS 1312 Fall 2022 Test 2
Oct. 18, 2022

Name ________________________ **Student ID** ________________________ **Score** ________

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) X-ray diffraction is a useful method to study the structure of matter (e.g., a crystal) because the wavelength of the X-ray is so much larger than that of the interatomic spacing in a solid.

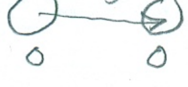
False The wave length should similar to the interatomic spacing (~1 Å)

(b) A proton can never be at rest since it creates a very large electric field near itself that accelerates it.

False The electric field acts on other particles. No self-interaction

(c) The dipole moment of molecular oxygen O_2 is zero.

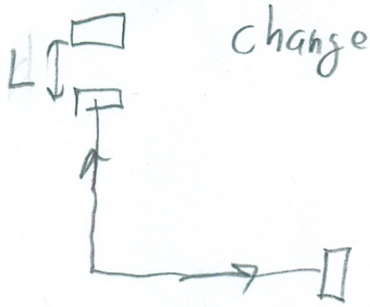
True All homonuclear molecules have $\vec{p} = 0$
$= q\vec{s}$



charge on each atom remains zero

Problem 2. Moving one mirror of a Michelson interferometer a distance of 100 μm causes 500 bright-dark-bright fringe shifts. What is the wavelength of the light? (15 points total)

$L = 100 \times 10^{-6}$ m, $N = 500$ (B → D → B), $L = Nd$, $d = L/N$



change in path length $\delta = 2d = \lambda$ dark-bright-dark

or $\lambda = \dfrac{2L}{N} = \dfrac{2(100 \times 10^{-6}\,m)}{500}$

$= \boxed{400\,nm}$

Problem 3. You look through a transmission diffraction grating at a sodium-vapor lamp which emits monochromatic light with a wavelength of 588 nm. The manufacturer of the grating states that it is etched with 10,000 lines (or slits) per centimeter. In addition to zero degrees, at what other angles will you observe bright fringes? (15 points total)

$\lambda = 588 \times 10^{-9}$ m

$d = \dfrac{1}{10{,}000\ \frac{lines}{cm}} = 10^{-4}$ cm (distance between slits)

$d \sin\theta = m\lambda$ bright fringes

$\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right) = \sin^{-1}\left(m \dfrac{588 \times 10^{-9}\,m}{10^{-6}\,m}\right)$

$= \sin^{-1}(m \cdot 0.588)$

$= 0°$ $m = 0$

$= \boxed{36°}$ $m = 1$

$m = 2, \ldots$ do not exist

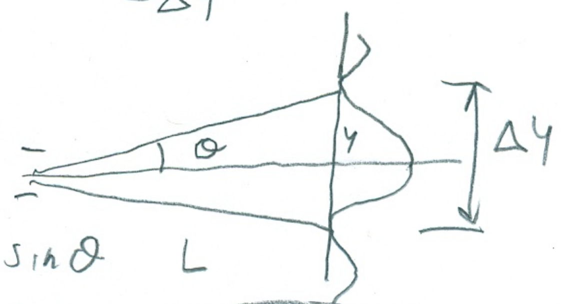
$= -36°$ $m = -1$

Problem 4. A beam of laser light is diffracted by a single slit with a width of 0.600 mm. The diffraction pattern forms on a screen 2.50 m beyond the slit. The total distance between the positions of zero intensity on either side of the central bright fringe is 4.00 mm. Determine the wavelength of the laser light. (15 points total)

L   a $\leftarrow \Delta y$

$a \sin\theta_{dark} = m\lambda$, $\tan\theta = \frac{y}{L}$

$m = 1$ $y = \frac{\Delta y}{2}$



$\lambda = a \sin\theta_{dark}$ $\theta \simeq \frac{\Delta y}{2L} \simeq \sin\theta$

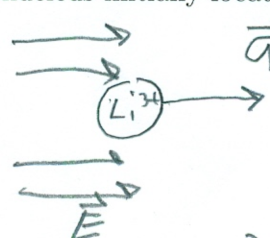
$= \frac{a\Delta y}{2L} = \frac{(0.6\times10^{-3})(4\times10^{-3})}{2(2.50)} = \boxed{480\ nm}$

Problem 5. A lithium nucleus consisting of three protons and four neutrons accelerates to the right due to an electric force and the initial magnitude of the acceleration is 3×10^{13} m/s^2. (a) What is the direction of the electric field that acts on the lithium nucleus, $\hat{r}$. (b) What is the magnitude of the electric field $|\vec{E}|$ that acts on the lithium nucleus? (c) If this acceleration is due solely to a single helium nucleus (an alpha particle with two protons and two neutrons), where is the helium nucleus initially located? (15 points total) $= a_x$

$Li^{3+} \to q_1 = 3e$

$m_{Li} = m_1 = 3m_p + 4m_n$
$\simeq 7m_p$

a)



$\vec E$ must point to the right

$\hat r = \hat i$

b) Newton's 2nd Law

$\Sigma F_x = m_1 a_x$

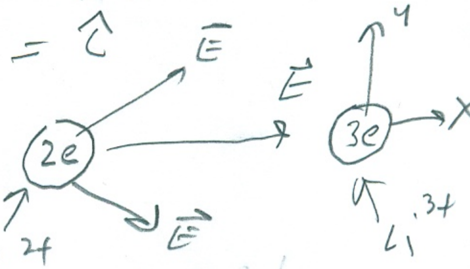
$E q_1 = m_1 a_x$

$|\vec E| = \frac{m_1 a_x}{q_1} = \frac{7 m_p a_x}{3e}$

$= \frac{7}{3}\frac{(1.6726\times10^{-27})(3\times10^{13}\ \frac{m}{s^2})}{1.602\times10^{-19}\ C}$

$= \boxed{7.31\times10^{5}\ N/C}$

c) He^{2+}
$q_2 = 2e$



Electric field at r from He^{2+}

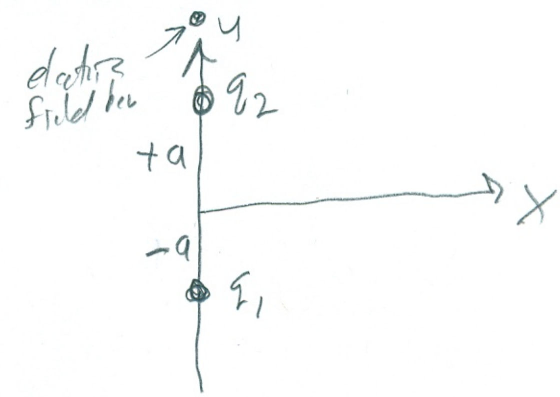
$E = \frac{k_e q_2}{r^2}$ or

$|\vec r| = \sqrt{\frac{k_e q_2}{E}} = \sqrt{\frac{8.99\times10^{9}\cdot 2\cdot 1.602\times10^{-19}}{7.31\times10^{5}}}$

$= \boxed{6.3\times10^{-8}\ m}$ away

$\vec r = 6.3\times10^{-8}\langle -1, 0, 0\rangle\ m$ Li^{3+} at origin

Problem 6. A dipole is centered at the origin and is composed of particles with charges $q_1 = +2e$ and $q_2 = -2e$, separated by a distance of 7×10^{-10} m along the y-axis. q_1 is located at $< 0, -a, 0 >$ and q_2 at $< 0, a, 0 >$ with $a = 3.5 \times 10^{-10}$ m. (a) Making no approximations, determine a relation for the electric field on the y-axis in terms of y, a, e, and k_e. (b) Compute a value for the electric field at $< 0, 1 \times 10^{-8}, 0 >$ m. (c) Using a Taylor series expansion, derive a relation for the electric field from part (a) if $|y| >> a$. (d) Using the relation from part (c) compute the electric field at $< 0, 1 \times 10^{-8}, 0 >$ m. Is the approximation reasonable? (e) Now place a proton at $< 0, 1 \times 10^{-8}, 0 >$ m and compute the force vector acting on the dipole using the relation from part (c). (30 points total)



a) $s = 2a = 7 \times 10^{-10}$ m

$$E_y = E_{q_1} + E_{q_2} = \frac{k_e q_1}{(a+y)^2} + \frac{k_e q_2}{(y-a)^2}$$

$$= k_e \left[\frac{2e}{(a+y)^2} + \frac{-2e}{(a-y)^2} \right]$$

$$\boxed{E_y = 2k_e e \left[\frac{1}{(y+a)^2} - \frac{1}{(y-a)^2} \right]}$$

b) $$E_y = 2(8.99\times10^{9})(1.602\times10^{-19}) \left[\frac{1}{(1\times10^{-8} + 3.5\times10^{-10})^2} - \frac{1}{(1\times10^{-8} - 3.5\times10^{-10})^2} \right]$$

$$= \boxed{-4.04 \times 10^{6} \text{ N/C}} \rightarrow \boxed{\vec{E}_y = -4.04\times10^{6} \text{ N/C } \hat{j}}$$

c) Now $$\frac{1}{(y+a)^2} = \frac{1}{y^2\left(1+\frac{a}{y}\right)^2} = y^{-2}\left(1+\frac{a}{y}\right)^{-2} \simeq y^{-2}\left(1 - 2\frac{a}{y}\right)$$

Taylor series $(1+x)^n = 1 + nx + \cdots$, $x \ll 1$, here $x = \frac{a}{y} \ll 1$

also $$\frac{1}{(y-a)^2} = \frac{1}{y^2}\left(1-\frac{a}{y}\right)^{-2} \simeq y^{-2}\left(1 + \frac{2a}{y}\right)$$

or $$E_y \simeq 2k_e e\left[\frac{1}{y^2}\left(1-\frac{2a}{y}\right) - \frac{1}{y^2}\left(1+\frac{2a}{y}\right)\right] =$$ (cont'd) next page

Problem 6 cont'd

$$E_y = \frac{2k_e e}{y^2}\left[\cancel{x} - \frac{2a}{y} - x - \frac{2a}{y}\right] = \frac{2k_e}{y^2}\left[-\frac{4a}{y}\right]$$

$$\boxed{E_y = -\frac{8k_e e a}{y^3}}$$

d) $$E_y = -\frac{8(8.99\times10^9)(1.602\times10^{-19})(3.5\times10^{-10})}{(1\times10^{-8})^3}$$

$$= \boxed{-4.03\times10^6\ N/C} \rightarrow \boxed{\vec{E}_y = -4.03\times10^6\ \frac{N}{C}\,\hat{j}}$$

e) $\vec{F}_y = q_3\vec{E}_y$ $\quad q_3 = -1e$

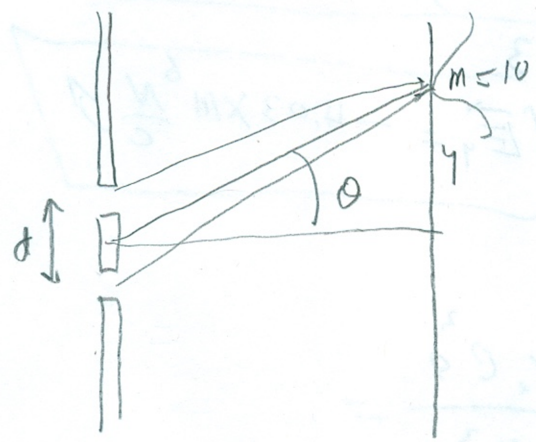
$$= \frac{-e\,8k_e e a}{y^3} = \frac{-8k_e e^2 a}{y^3} =$$

$$= \frac{-8(8.99\times10^9)(1.602\times10^{-19})^2(3.5\times10^{-10})}{(1\times10^{-8})^3}$$

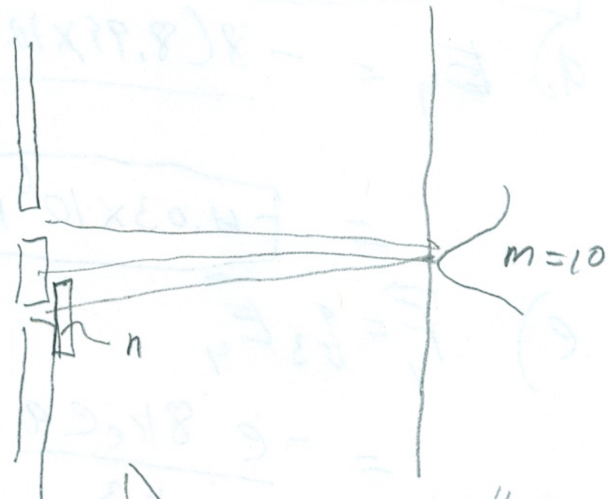
$$= \boxed{-6.46\times10^{-13}\ N} \rightarrow \boxed{\vec{F}_y = -6.46\times10^{-13}\ N\,\hat{j}}$$

Bonus Problem. A double slit experiment is set up using a helium-neon laser ($\lambda = 633$ nm). Then a very thin piece of glass (index of refraction $n = 1.50$) is placed over one of the slits. Afterward, the central point on the screen is occupied by what had originally been the $m = 10$ dark fringe. How thick is the glass? (5 points total)

usual case

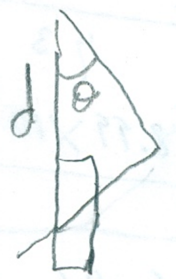


new case



$\delta = d\sin\theta = (m+\frac{1}{2})\lambda$ dark fringe

or $\boxed{d\sin\theta = \frac{21}{2}\lambda}$



assume small θ → there is an extra phase shift due to the time for wave to travel through glass ϕ_g

$t' = \frac{D}{V}$ ←D→ n

$t = \frac{D}{c}$ n=1

$\Delta t = \frac{D}{V} - \frac{D}{c} = \frac{Dn}{c} - \frac{D}{c} = \frac{D}{c}(n-1)$

$\phi_g = \omega\Delta t$, $V = f\lambda$, $n = \frac{c}{V}$

$\phi_g = \omega\frac{D}{c}(n-1)$

since $\varphi = \frac{2\pi x}{\lambda} - \omega t = 0$

$x = \frac{\lambda}{2\pi}\omega t = \frac{\lambda}{2\pi}\omega\frac{D}{c}(n-1) = \frac{\lambda f}{c}D(n-1) = D(n-1) \equiv$ extra path length

$\delta' = d\sin\theta + D(n-1) = \frac{21}{2}\lambda$, but $\theta = 0$, $D(n-1) = \frac{21}{2}\lambda$

or $D = \frac{21}{2}\frac{\lambda}{(n-1)} = \frac{21}{2}\frac{633\times10^{-9}\,m}{0.5} = \boxed{13.3\ \mu m}$