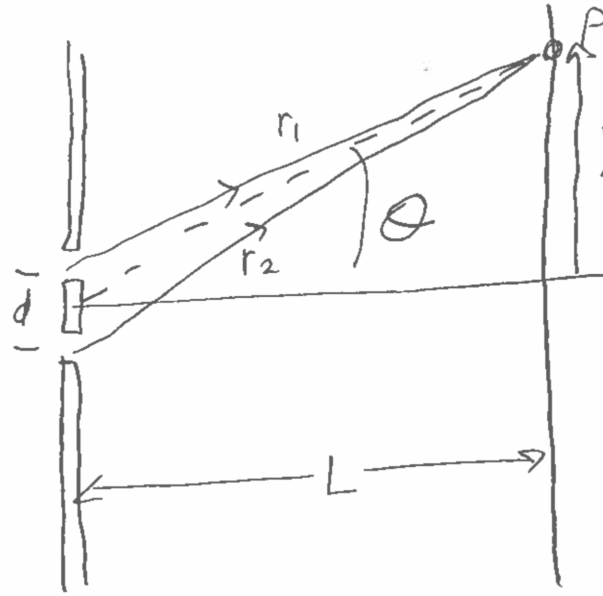
PHYS 1312 10-5-2022

Intensity Distribution of the Interference Pattern for the Double Slit



- We found previously that the bright and dark fringe locations can be found from

$d\sin\theta = m\lambda$ bright

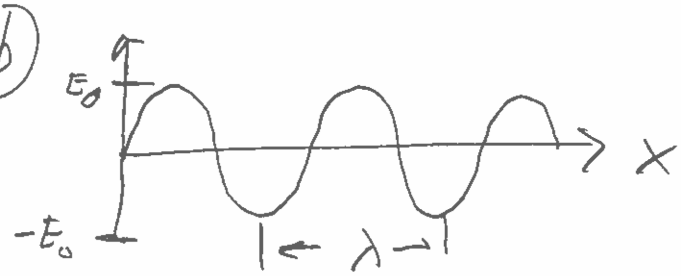
$d\sin\theta = (m+\frac{1}{2})\lambda$ dark

with $m = 0, \pm 1, \pm 2, \ldots$
$\equiv$ the order

and $y \cong \frac{\lambda L}{d}\begin{Bmatrix} m \\ m+\frac{1}{2}\end{Bmatrix}$ if θ is small ($< 10°$)

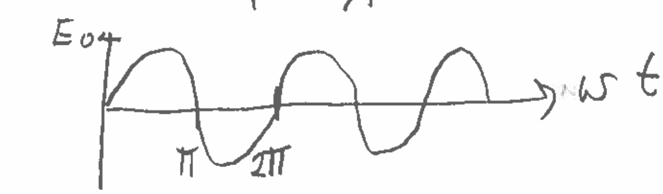
- Now the light is an electronmagnetic wave, but with the oscillating property the electric field $E(x,t)$ [topic of chapter 13]
- As before we can use the general wave equation

$E(x,t) = E_0 \sin(kx \mp \omega t + \phi)$



$E_0 \equiv$ amplitude [N/C] coulombs

k, ω, ϕ the usual quantities,



- Now consider the wave at point P and drop the sign, $x = 0$

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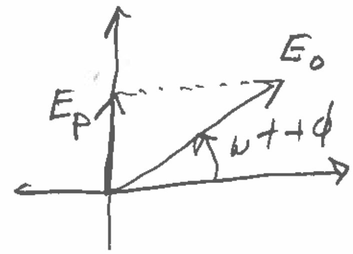
→ $E_p(0,t) = E_0 \sin(\omega t + \phi)$



- Plot this wave on a phasor diagram at time t
  - → Since E_0 is the hypotenuse and E_p the opposite of the angle
  - → the vector E_0 rotates around the origin ccw
- Now consider the two waves interfering at point P originating from the upper slit $E_1 = E_0 \sin(\omega t)$ and lower slit
  $E_2 = E_0 \sin(\omega t + \phi)$ – both have the same magnitude
- while the two waves are in phase at the slits, they travel different path lengths, $\delta = r_2 - r_1$, and therefore have a phase difference at other points in space

$$\Delta\varphi = \frac{2\pi}{\lambda}\delta + \Delta\phi$$

- If we consider constructive interference at point P $\frac{2\pi\delta}{\lambda} + \Delta\phi = 0$ (or $m2\pi$)

  or $|\Delta\phi| = \phi = \frac{2\pi}{\lambda}\delta$ to account for the path length difference.
- For simplicity, take $t = 0$

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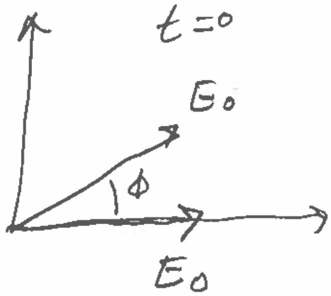
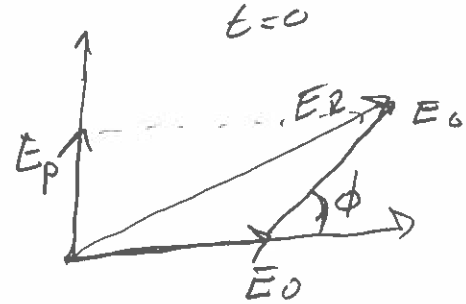
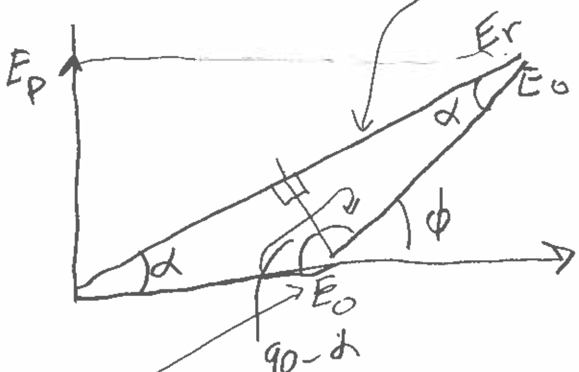
$\rightarrow E_1 = E_0 \sin\theta^\circ, \; E_2 = E_0 \sin\phi$

- phase plot for each is now add





- At any time $E_p = E_1 + E_2 = E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi)$
- E_R is the resultant



$2(90-\alpha) + \phi = 180^\circ$

$180^\circ - 2\alpha + \phi = 180^\circ$

or $\alpha = \dfrac{\phi}{2}$

$\dfrac{E_R}{2} = E_0 \cos\alpha = E_0 \cos\dfrac{\phi}{2}$

$E_R = 2E_0 \cos\dfrac{\phi}{2} \neq F(t)$

also

$E_p = E_R \sin\alpha = E_R \sin\left(\dfrac{\phi}{2}\right)$

$$\boxed{E_p = 2E_0 \cos\frac{\phi}{2} \sin\left(\frac{\phi}{2}\right)}$$

But E_p is a function of tim at $t=0$

$\Rightarrow E_p = 2E_0 \cos\dfrac{\phi}{2} \sin\left(\omega t + \dfrac{\phi}{2}\right)$

$\rightarrow$ Derived before with trig identities

- Now, we want the Intensity of light I at point P — it is proportional to E_p^2 (some constants)

$I_p = (\text{constants}) E_p^2$ or $E_p^2 = 4E_0^2 \cos^2\left(\dfrac{\phi}{2}\right) \sin^2\left(\omega t + \dfrac{\phi}{2}\right)$

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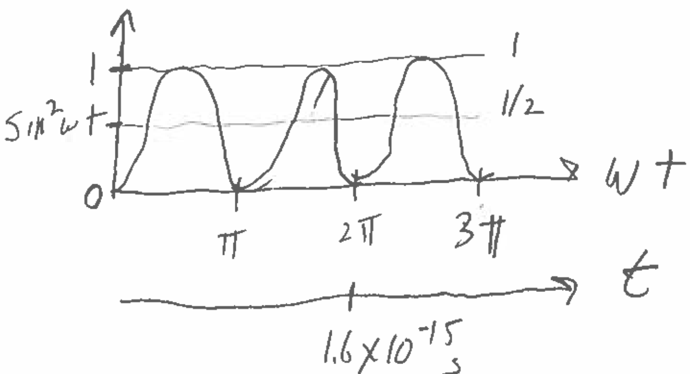
- For optical photons, the intensity pattern does not depend on time (typical ligth beam)

$\longrightarrow$ take as an exampl $\lambda = 500\,nm$

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{500\times10^{-9}\,m} = 6\times10^{14}\,Hz, \text{ or } T = 1.6\times10^{-15}\,s$$

- Plot $\sin^2(\omega t+\phi)$



oscillates rapidly

- make very good approximation $\sin^2(\omega t+\phi) = \frac{1}{2}$
- To give $\langle I_p\rangle \propto 4E_0^2\cos^2\left(\frac{\phi}{2}\right)$ average intensity

$$\text{or } \langle I_p\rangle = I_{max}\cos^2\left(\frac{\phi}{2}\right) = I_{avg}$$

- Since $\phi = \frac{2\pi\,\delta}{\lambda} = \frac{2\pi}{\lambda}d\sin\theta$
- We get $\boxed{I_{avg} = I_{max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)}$

- Since

$$d\sin\theta = \begin{cases} m\lambda \\ (m+\frac{1}{2})\lambda \end{cases}$$