KEY

PHYS 1312 Fall 2022 Test 1

Sept. 20, 2022

Name ________________________ **Student ID** ________________________ **Score** _______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) Generally, the speed of sound increases with the density of the medium it is propagating through.

True

(b) The near point corresponds to the closest an object can me located with respect to the cornea of the eye such that an image is formed on the person's retina.

True

(c) Light is an example of a transverse, periodic wave.

True

(d) To remove chromatic aberrations, an aperture is used in a typical camera.

False $\longrightarrow$ spherical aberrations

Problem 2. A wave is give by the relation

$$y(x,t) = A\sin(2.20 \times 10^7 x + 4.40 \times 10^{15} t + \pi). \qquad (1)$$

Determine the (a) frequency, (b) wavelength, (c) speed, (d) phase constant, and (e) direction of the wave. (f) If this is a light wave, what must be the index of fraction of the material it is traveling through? t is in seconds and x in m. (15 points total)

General wave equation $y(x,t) = A\sin(kx \mp \omega t + \phi)$

a) $\omega = 2\pi f$, $f = \frac{\omega}{2\pi} = \frac{4.40\times10^{15}\,\text{rad/s}}{2\pi\ \text{rad}} = \boxed{7.00\times10^{14}\,\text{Hz}}$

b) $k = \frac{2\pi}{\lambda}$, $\lambda = \frac{2\pi}{k} = \frac{2\pi\,\text{rad}}{2.20\times10^7\,\text{rad/m}} = 285.6\times10^{-9}\,\text{m} = \boxed{285.6\,\text{nm}}$

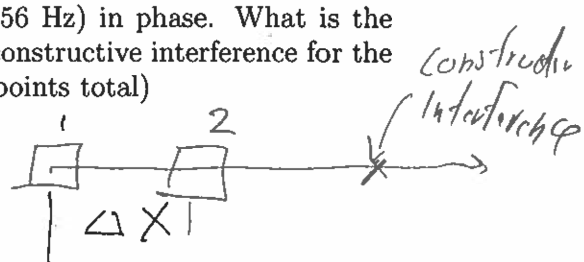
c) $v = f\lambda = (7\times10^{14})(285.6\times10^{-9}) = \boxed{2.00\times10^8\,\text{m/s}}$

d) $\phi = \boxed{\pi\ \text{rads}}$

f) $n = \frac{c}{v} = \frac{3\times10^8}{2\times10^8} = \boxed{1.5}$

e) $+ \rightarrow$ $\boxed{\text{travels to left}}$

Problem 3. ~~Two~~ violinist, one directly behind the other, play for a listener directly in front of them. Both violinist produce the sound for middle C (256 Hz) in phase. What is the smallest separation between the violinists that will produce constructive interference for the listener? Take the speed of sound in air to be 343 m/s. (15 points total)



Use phase difference relation

$\Delta\phi = \frac{2\pi}{\lambda}\Delta X + \Delta\phi_0 = n2\pi$ ← constructive

in phase → $\Delta\phi_0 = 0$, $v = \lambda f$, $\lambda = \frac{v}{f}$

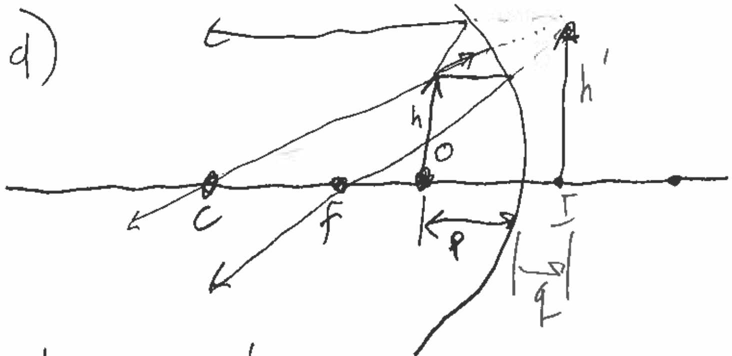
$\Rightarrow \Delta\phi = \frac{\cancel{2\pi} f}{v}\Delta X = \cancel{2\pi}$ (1st interference point for smallest separation)

$\Delta X = \frac{v}{f} = \frac{343\,\text{m/s}}{256\,1/\text{s}} = \boxed{1.34\,\text{m}}$

f h p

Problem 4. A pencil of height 5 cm is 10 cm in front of a concave mirror with a focal length of 20 cm. (a) Where is the location of the image? (b) What is the magnification of the image? (c) Is the image of the pencil inverted or upright, virtual or real? (d) Make a rough ray diagram showing the object, the image, and the mirror. (15 points total)

d)



b) $M = -\frac{q}{p} = \frac{-(-20)}{10} = \boxed{2}$

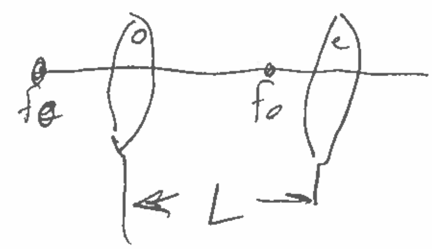
c) upright, virtual

a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$q = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{20} - \frac{1}{10}} = \boxed{-20\text{ cm}}$

Problem 5. A microscope has a 20 cm tube length. What focal-length objective will give total magnification of 100 (absolute value) when used with an eyepiece having a focal length of 5.0 cm? (15 points total)

M L f_e



objective lens $M_o = -\frac{L}{f_o}$

eyepiece, $m_{max} = 1 + \frac{25\text{cm}}{f_e}$

or $m_{min} = \frac{25\text{cm}}{f_e}$

$M = M_o m_e = -\frac{L}{f_o}\left(1 + \frac{25\text{cm}}{f_e}\right)$

$f_o = -\frac{L}{M}\left(1 + \frac{25}{f_e}\right) = -\frac{20}{-100}\left(1 + \frac{25}{5}\right)$

$= +\frac{1}{5}(6) = \boxed{1.2\text{ cm}}$

$f_o = -\frac{L}{M}\left(\frac{25}{f_e}\right)$

$= -\frac{20}{-100}\left(\frac{25}{5}\right) = \left(\frac{1}{5}\right)(5)$

$= \boxed{1\text{ cm}}$

accept either

Problem 6. Two waves of equal amplitude A and frequency f propagate on a string of length L, but in opposite directions. (a) Starting with the general wave equation, derive a relation for the superposition of the two waves (a standing wave) as a function of x and t. (b) Prove that nodes (or zeros) occur on the string for $x = n\lambda/2$, where $n = 0, 1, 2, \ldots$. (c) Find the harmonic frequencies for the standing wave on a string. (30 points total)

a) $y_1 = A_1 \sin(k_1 x - \omega_1 t + \phi_1)$, $y_2 = A_2 \sin(k_2 x + \omega_2 t + \phi_2)$ to the left

let $A_1 = A_2 = A$, $f_1 = f_2 = f$, or $\omega_1 = \omega_2 = \omega$, the $k_1 = k_2 = k$, $\phi_1 = \phi_2 = 0$

Add the two waves

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

use the identity $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$$a = kx, \; b = \omega t$$

$$y = A[\sin kx \cos \omega t - \cancel{\cos kx \sin \omega t} + \sin kx \cos \omega t + \cancel{\cos kx \sin \omega t}]$$

$$\boxed{y(x,t) = 2A \sin(kx) \cos(\omega t)}$$

b) Nodes occur for any (or all) time when $y(x,t) = 0$

or $0 = \sin(kx)$ when $kx = n\pi$, $n = 0, 1, 2, 3$

or $x = n\dfrac{\pi}{k} = n\pi\left(\dfrac{\lambda}{2\pi}\right) = \boxed{n\dfrac{\lambda}{2}}$ $n = 0, 1, 2, 3$

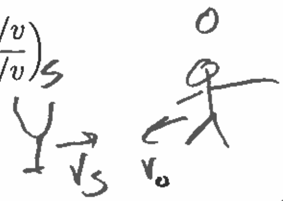
$\lambda = \dfrac{2x}{n} = \dfrac{2L}{n}$ end node

c) $f = \dfrac{v}{\lambda} = \dfrac{v}{(2L/n)} = n\left(\dfrac{v}{2L}\right)$

or $\boxed{f_n = n\left(\dfrac{v}{2L}\right) \quad n = 1, 2, 3, \ldots}$ not $n = 0$

Bonus Problem. Doppler equation derivation for sound with sound speed v, the frequency of a sound wave emitted by the source as f_s, and that heard by an observer as f_o. If the source has a speed v_s and the observer speed v_o, derive the Doppler shift in frequency for (a) $v_o = 0$ with the source moving toward the stationary observing, (b) $v_s = 0$ with the observer moving toward the stationary source, and (c) combine these relations to get the general Doppler shift equation

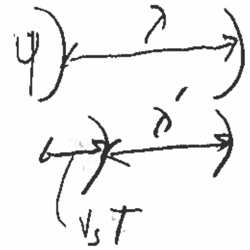
$$f_o = f_s\left(\frac{1 \pm v_o/v}{1 \mp v_s/v}\right) \tag{2}$$



(5 points total).

a) $V_0 = 0$, $V_s \neq 0$, $V =$ sound speed

$f_s = V/\lambda_s$ = frequency of source $\rightarrow \lambda_s = \frac{V}{f_s}$



source travels $d = V_s T$ in one period
wavelength is compressed
$\lambda' = \lambda - V_s T$

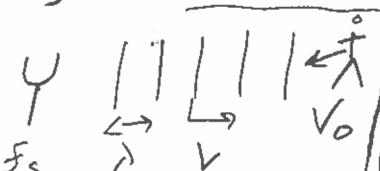
$f_0 \equiv$ frequency heard by observer

$$= \frac{V}{\lambda'} = \frac{V}{\lambda - V_s T} = \frac{V}{\frac{V}{f_s} - \frac{V_s}{f_s}} = \frac{V}{\frac{1}{f_s}(V - V_s)} = f_s\left(\frac{1}{1 - \frac{V_s}{V}}\right) = f_0$$

if source moves away from observer, $V_s \rightarrow -V_s \rightarrow f_0 = f_s\left(\frac{1}{1 + \frac{V_s}{V}}\right)$

$$\boxed{f_0 = f_s\left(\frac{1}{1 \mp \frac{V_s}{V}}\right)}$$

b) $V_0 \neq 0$, $V_s = 0$, observer travels a distance $d = V_0 t$ and encounters $\frac{V_0 t}{\lambda}$ extra wave cycles or at a frequency $\frac{V_0}{\lambda} \equiv f'$



frequency is shifted $f_0 = f_s + f' = f_0 + \frac{V_0}{\lambda} = f_s\left(1 + \frac{V_0}{f_s \lambda}\right)$

or $f_0 = f_s\left(1 + \frac{V_0}{V}\right)$ since $V = f_s \lambda$, for observer running away $V_0 \rightarrow -V_0$

$$f_0 = f_s\left(1 - \frac{V_0}{V}\right) \Rightarrow \boxed{f_0 = f_s\left(1 \pm \frac{V_0}{V}\right)}$$

c) sub a) into b) $\rightarrow$

$$\boxed{f_0 = f_s\left(\frac{1 \pm V_0/V}{1 \mp V_s/V}\right)}$$