KEY

PHYS 1312 Fall 2018 Test 3
Nov. 13, 2018

Name ______________________ **Student ID** ______________________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) At a long distance (say, a mile), the electric field due to a uniformly charged cow decreases with distance as r^{-2}.

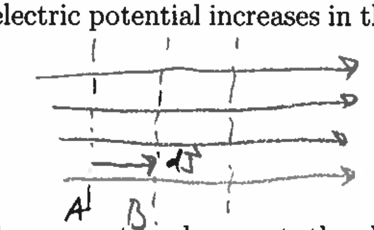
True $\quad \vec{E} \xrightarrow[r\to\infty]{} \dfrac{k_e q_{cow}}{r^2}\hat{r}$ (i.e. looks like a point charge)

(b) In a region with a uniform electric field, the electric potential increases in the direction of the electric field vector.

False $\quad \Delta V = -\int \vec{E}\cdot d\vec{s}$

$V_B - V_A = -Ex$

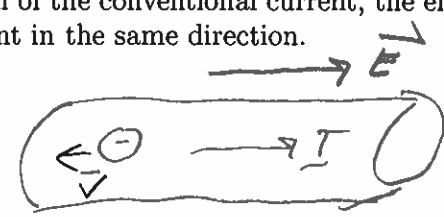
$\Rightarrow$ decreases

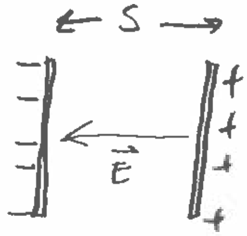


(c) In a current-carrying wire, the direction of the conventional current, the electric field, and the drift velocity of the electrons, all point in the same direction.

False

Electrons drift in opposite direction of $\vec{E}$





cm

Problem 2. The electric field in the interior of a parallel plate capacitor is found to be 5.0 V/cm. (a) What is its energy density? (b) What is its surface charge density? (c) If a dielectric is inserted between the plates completely filling the gap and the electric field magnitude is found to be reduced to 2.0 V/m, what is the dielectric constant of the material? (15 points total)

a) $\frac{\text{Energy}}{\text{Vol}} = \text{Energy density} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\left(8.854\times10^{-12}\frac{C^2}{Nm^2}\right)\left(5.0\frac{V}{cm}\times\frac{100\,cm}{1\,m}\right)^2$

$= \boxed{1.1\times10^{-6}\ \frac{J}{m^3}}$

c) $E_{insulator} = \frac{E_{vacuum}}{K}$ | or $\frac{5\,V/cm}{0.02\,V/cm} = 250$

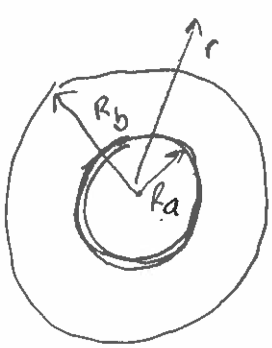
$K = \frac{E_{vacuum}}{E_{insulator}} = \frac{5\,V/cm}{2\,V/cm} = \boxed{2.5}$

b) $E_{capacitor} = \frac{E_{plate}}{2} = \frac{Q/A}{\varepsilon_0}$

$= \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 E = (8.854\times10^{-12})\left(5.0\frac{V}{cm}\times\frac{100\,cm}{1\,m}\right) = \boxed{4.43e^{-9}\ \frac{C}{m^2}}$

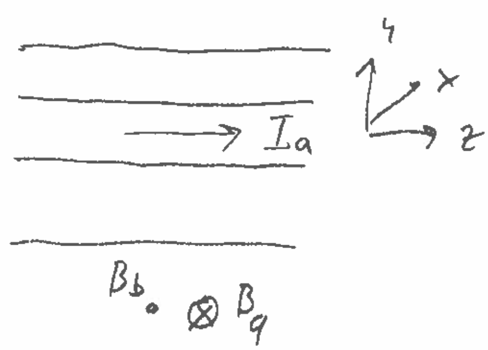
Problem 3. Consider a long coaxial cable which is composed of a conductor of radius R_a, surrounded by a very thin insulator, which is surrounded by another conductor with outer radius R_b. (a) If $R_a = 1.0$ mm, $R_b = 2.0$ mm, and the current in the inner conductor is 1.0 A, what is the magnetic field magnitude at $r = R_a$? (b) If the magnetic field at $r = 3.0$ mm is zero, what must be the current, and its direction, in the outer conductor? (15 points total)

a) For infinitly long wire $|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2I}{r}$



$|\vec{B}| = \left(1.0\times10^{-7}\frac{Tm}{A}\right)\frac{2(1A)}{1\times10^{-3}m} = \boxed{2.0\times10^{-4}\ T}$

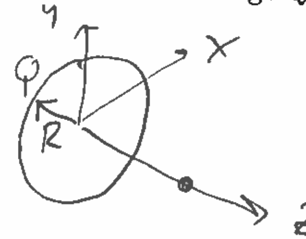
b)



$\vec{B}_a + \vec{B}_b = 0 \Rightarrow \vec{B}_a = -\vec{B}_b \Rightarrow I_a = -I_b$

$I_b = I$ but flows in opposite direction

Problem 4. The electric potential of a ring of charge with radius R and total charge Q at the origin in the $x-y$ plane is given by



$$V(z) = \frac{k_e Q}{\sqrt{R^2 + z^2}}. \quad (1)$$

Derive the electric field $\vec{E}$ of the ring. (15 points total)

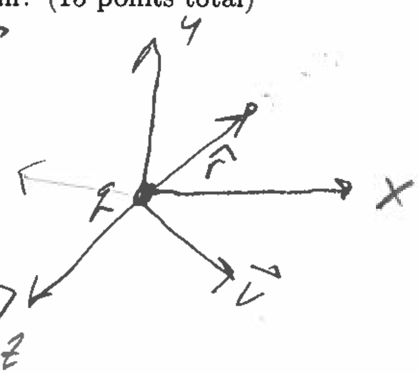
$$E_z = -\frac{\partial V(z)}{\partial z} = -\frac{\partial}{\partial z}\left[\frac{k_e Q}{(R^2+z^2)^{1/2}}\right] = -k_e Q\left[\frac{(-\frac{1}{2})(2z)}{(R^2+z^2)^{3/2}}\right]$$

$$\boxed{E_z = \frac{k_e Q \, z}{(R^2+z^2)^{3/2}}}$$

Problem 5. A proton is located at the origin, but has a velocity of $< 1.0 \times 10^4, 0.0, 1.0 \times 10^4 >$ m/s. What is the magnetic field vector at $< 1.0, 1.0, 0 >$ m? (15 points total)

$= \vec{V}$

$\| \vec{r}$



$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{V} \times \hat{r}}{|\vec{r}|^2}, \quad q = e$$

$$\vec{r} = <1, 1, 0>$$

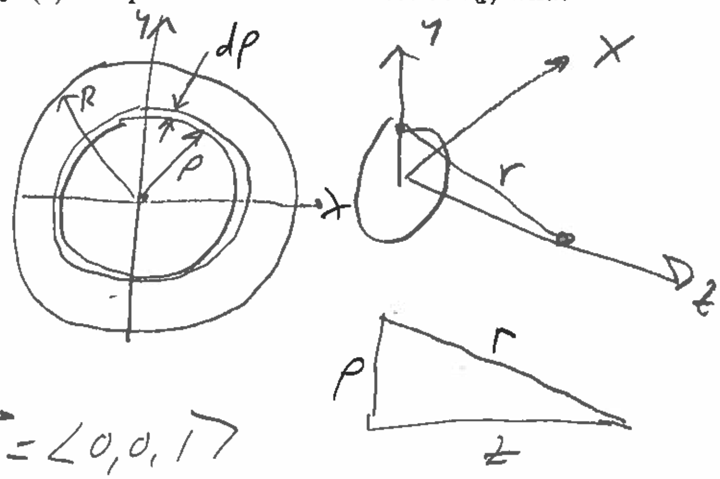
$$|\vec{r}| = \sqrt{1+1} = \sqrt{2}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \left<\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right>$$

$$\vec{V} \times \hat{r} = <10^4, 0, 10^4> \times \left<\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right> = <-1, 1, 1> \frac{10^4}{\sqrt{2}} \frac{m}{s}$$

$$\vec{B} = \frac{\left(1.0 \times 10^{-7} \frac{Tm}{A}\right)(1.602 \times 10^{-19} C) <-1, 1, 1> \frac{10^4}{\sqrt{2}} m/s}{2 \, m^2} = \boxed{<-1, 1, 1> 5.66 \times 10^{-23} T}$$

Problem 6. A uniformly-charged disk of radius R with total charge Q is located at the origin in the $x-y$ plane. (a) Starting with the relation for the electric field $\vec{E}_z$ of a ring of radius R and charge Q, find the electric field due to a disk at the location $< 0, 0, z >$. (b). Given the result in part (a), find the electric potential difference $\Delta V = V_b - V_a$ for locations $\vec{r}_a =< 0, 0, z_a >$ and $\vec{r}_b =< 0, 0, z_b >$ for $z_a < z_b$. (c) If a proton starts from rest at $\vec{r}_a$, what is its velocity at $\vec{r}_b$?



a) $\Delta E_{ring} = \dfrac{k_e \Delta q \; z}{(\rho^2+z^2)^{3/2}}$

$\vec{r}_{obs} = \langle 0,0,z \rangle$

$\vec{r}_{source} = \langle 0,0,0 \rangle$ (center of ring)

$\vec{r} = \langle 0,0,z \rangle, \; |\vec{r}| = z, \; \hat{r} = \langle 0,0,1 \rangle$

$\sigma = \dfrac{Q}{A} \Rightarrow dq = \sigma \pi \, d(\rho^2) = \sigma 2\pi \rho \, d\rho \qquad 0 \leq \rho \leq R$

$Q = \pi R^2 \sigma$

$\Rightarrow \Delta E_{ring} = \dfrac{k_e (\sigma 2\pi \rho \, d\rho) \, z}{(\rho^2+z^2)^{3/2}}$

$E_{disk} = k_e \sigma 2\pi z \displaystyle\int_0^R \frac{\rho \, d\rho}{(\rho^2+z^2)^{3/2}} = k_e \sigma 2\pi z \left[-\frac{1}{(\rho^2+z^2)^{1/2}} \right]_0^R$

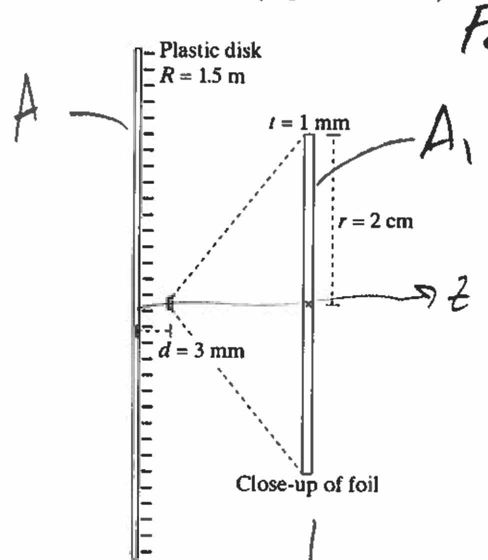
$= k_e \sigma 2\pi z \left[\dfrac{1}{z} - \dfrac{1}{\sqrt{R^2+z^2}} \right] = \dfrac{1}{4\pi\varepsilon_0} (Q/A) \, 2\pi \left[1 - \dfrac{z}{\sqrt{R^2+z^2}} \right]$

$$\boxed{E_{disk} = \frac{Q/A}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{R^2+z^2}} \right]}$$

b) $\Delta V = -\int \vec{E} \cdot d\vec{s}$

$\Delta V = -\displaystyle\int_{z_a}^{z_b} E_{disk} \, dz = -\frac{Q/A}{2\varepsilon_0} \int_{z_a}^{z_b} \left[1 - \frac{z}{\sqrt{R^2+z^2}} \right] dz$

Bonus Problem. A large, thin plastic disk with radius $R = 1.5$ m carries a uniformly distributed charge of $Q = -5 \times 10^{-5}$ C as shown in the figure. A circular piece of conducting foil is placed at $d = 3.0$ mm from the disk, parallel to the disk. The foil has a radius of $r = 2.0$ cm and a thickness of $t = 1.0$ mm. What is the force acting on the foil due to the plastic disk? (5 points total)



For $R >> z$ $\quad E_{disk} = \dfrac{Q/A}{2\varepsilon_0}\left[1 - \dfrac{z}{\sqrt{R^2+z^2}}\right]$

$$\rightarrow \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{z}{R}\right]$$

First find charge q on foil

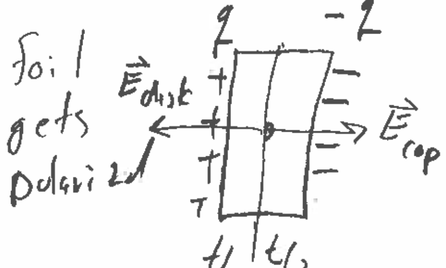
Electric field in foil must be zero.

At center

$$\vec{E} = 0 = \vec{E}_{disk} + \vec{E}_q + \vec{E}_{-q}$$

$$= \frac{Q/A}{2\varepsilon_0}\left[1 - \frac{d}{R}\right] + \frac{q/A_1}{\varepsilon_0} = 0$$

(parallel plate capacitor)



$$\Rightarrow q = \frac{(Q/A)A_1}{2}\left[1 - \frac{d}{R}\right] = \frac{Q}{2}\left[\frac{\pi r^2}{\pi R^2}\right]\left[1 - \frac{d}{R}\right]$$

$$= \frac{Q}{2}\left(\frac{r}{R}\right)^2\left[1 - \frac{d}{R}\right] = \frac{5\times10^{-5}\,C}{2}\left(\frac{2\times10^{-2}}{1.5}\right)^2\left[1 - \frac{3\times10^{-3}}{1.5}\right]$$

$$= 4.435\times10^{-9}\,C$$

$$\vec{F}_{on\ foil} = \vec{F}_q + \vec{F}_{-q} = q\vec{E}_{disk} - q\vec{E}_{disk}$$

$$= q\frac{Q/A}{2\varepsilon_0}\left[1 - \frac{(d - t/2)}{R}\right] - q\frac{Q/A}{2\varepsilon_0}\left[1 - \frac{(d + t/2)}{R}\right]$$

Problem 6 (cont'd)

$$\Delta V = -\frac{Q/A}{2\varepsilon_0}\left[\int_{z_a}^{z_b} dz - \int_{z_a}^{z_b} \frac{z}{\sqrt{R^2+z^2}}\,dz\right]$$

$$= -\frac{Q/A}{2\varepsilon_0}\left[(z_b - z_a) - \sqrt{R^2+z^2}\,\Big|_{z_a}^{z_b}\right]$$

$$\boxed{\Delta V = -\frac{Q/A}{2\varepsilon_0}\left[z_b - \sqrt{R^2+z_b^2} - \left(z_a - \sqrt{R^2+z_a^2}\right)\right]}$$

c) By conservation of energy $\Delta K = -\Delta U = -q\Delta V$

or $\frac{1}{2} m_p v_f^2 = -e\Delta V$

or $v_f = \sqrt{-\frac{2e}{m_p}\Delta V}$

or $$\boxed{v_f = \sqrt{+\frac{2e}{m_p}\frac{Q/A}{2\varepsilon_0}\left[z_b - \sqrt{R^2+z_b^2} - \left(z_a - \sqrt{R^2+z_a^2}\right)\right]}}$$

Bonus (cont'd)

$$F = q\frac{Q/A}{2\varepsilon_0}\left[1 - 1 - \frac{d}{R} + \frac{t/2}{R} + \frac{d}{R} + \frac{t/2}{R}\right] = q\frac{Q/A}{2\varepsilon_0}\frac{t}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{R^2}\frac{t}{R} = 2k_e qQ\frac{t}{R^3} = 2(8.99\times10^9)(4.435\times10^{-9})(-5\times10^{-5}) \times (1\times10^{-3})/(1.5)^3$$

$$\boxed{F = -1.18\times10^{-6}\ \text{N}}$$