KEY

PHYS 1312 Fall 2018 Test 2
Oct. 11, 2018

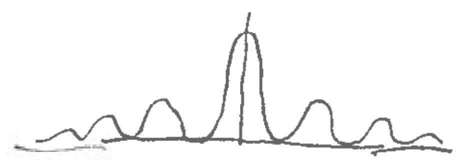
Name ______________________ **Student ID** ______________________ **Score** _______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

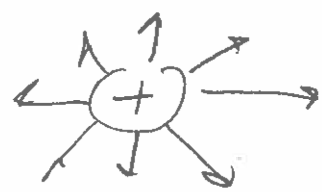
(a) The intensities of the secondary maxima for a single slit diffraction pattern decreases as the distance from the central maximum increases.

True



(b) Electric field lines point toward a positive charge.

False



(c) If excess charge in the form of 100 electrons is added to the top of a solid spherical plastic ball, after some time the charges will distribute themselves uniformly over the surface.

False

$m = 1$ $\quad = \theta$

Problem 2. A diffraction grating produces a first-order maximum at an angle of 20.0°. What is the angle of the second-order maximum? (15 points total)

$d \sin\theta = m\lambda$

or $\frac{\lambda}{d} = m \sin\theta$

$\frac{\lambda}{d} = (1)\sin 20^\circ$

$= 0.342$

For $m = 2$

$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$

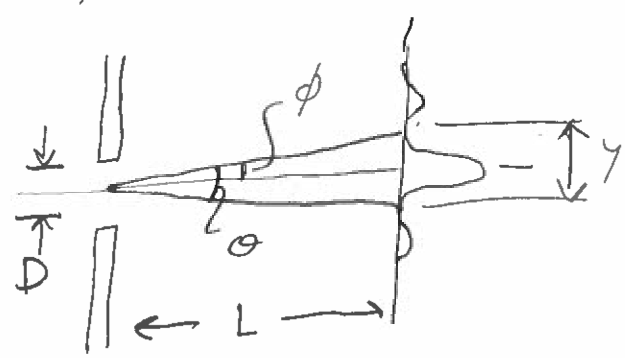
$= \sin^{-1}\left(2(0.342)\right)$

$= \boxed{43.16^\circ}$

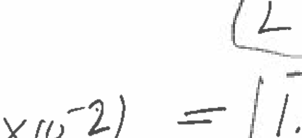
Problem 3. Light from a helium-neon laser ($\lambda = 633$ nm) passes through a circular aperture and is observed on a screen 4.0 m in front of the aperture. The width of the central maximum is 2.5 cm. What is the diameter of the hole? (15 points total)

For a circular aperture

$\theta_{min} = 1.22\frac{\lambda}{D}$, $L = 4m$



$y = 2.5 \times 10^{-3}$ m



$\tan\phi \cong \phi = \frac{y/2}{L}$

$\theta_{min} = \frac{y}{L} = 1.22\frac{\lambda}{D}$

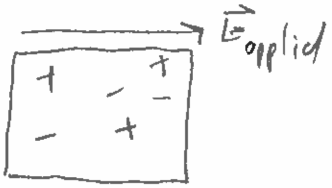
$\theta = 2\phi = \boxed{\frac{y}{L} = \theta}$

or $D = 1.22\lambda\frac{L}{y}$

$= 1.22(633 \times 10^{-9})(4.0)/(2.5 \times 10^{-2}) = \boxed{1.24 \times 10^{-4} m = 0.124 mm}$

Problem 4. An ionic solution of water, K^+, and Cl^- is initially at equilibrium. (a) If an electric field $E =< 2.0, 0.0, 0.0 >$ N/C is applied to the solution, which is the initial drift speed of the ions at $t = 0$. (b) After a reasonably long time (say 10.0 s), what is the net electric field in the solution? (c) What is the polarization electric field due to the dipole created by the ions? Take the mobility of the ions in the water to be $u = 8 \times 10^{-8}$ (m/s)/(N/C). (15 points total)

a)

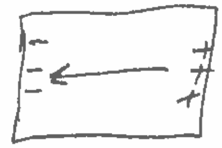


$\bar{V} = uE$ (only in x-direction)

$= \left(8 \times 10^{-8} \frac{m/s}{N/C}\right)\left(2\ N/C\right)$

$= \boxed{1.6 \times 10^{-7}\ m/s}$

b)



after long time

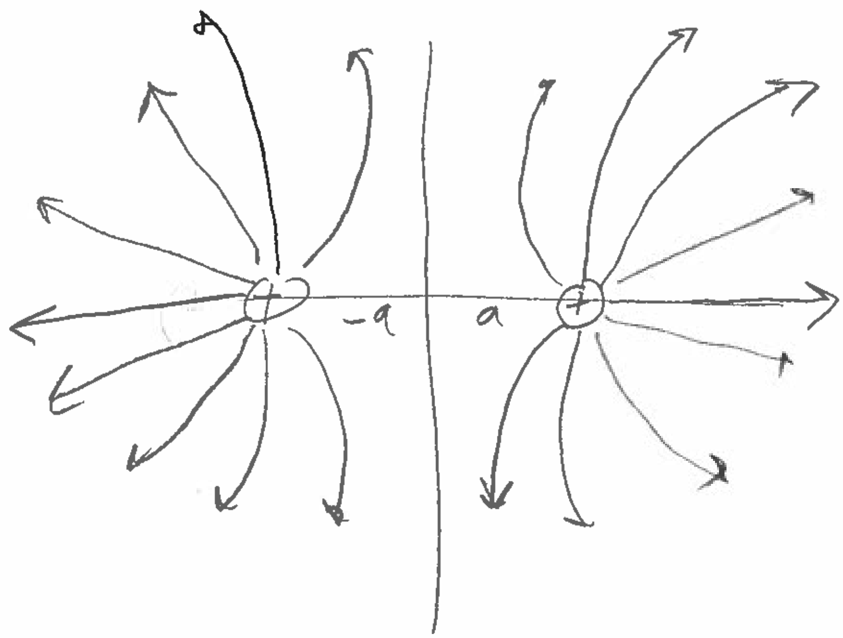
$\vec{E}_{net} = \vec{E}_{applied} + \vec{E}_{pol} = 0$

$\boxed{\vec{E}_{net} = 0}$

c) $\vec{E}_{pol} = -\vec{E}_{applied} = \boxed{\langle -2.0, 0, 0 \rangle\ N/C}$

Problem 5. Given two positive charges q located at $< -a, 0, 0 >$ and $< a, 0, 0 >$, make a sketch of the electric field lines near the charges. (15 points total)

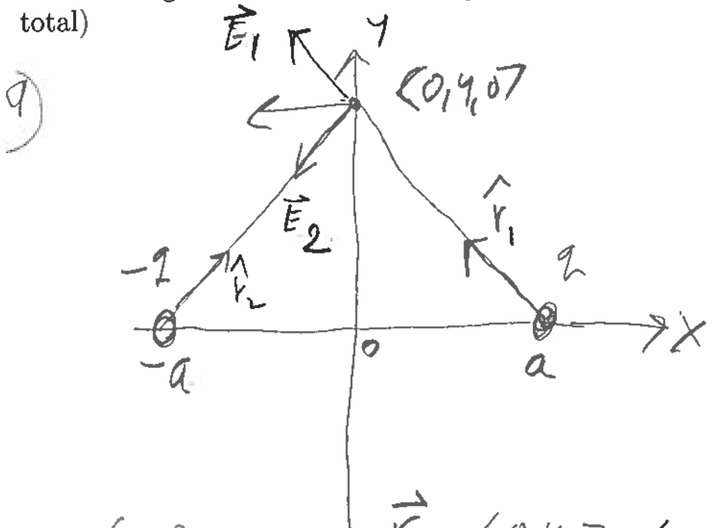
b)



Field

Problem 6. Consider two charges $-q$ and q located at $< -a, 0, 0 >$ and $< a, 0, 0 >$. (a) Derive the electric field at the observation point $< 0, y, 0 >$ from the electric of the two point charges. (b) Now place a charge Q at the point $< 0, y, 0 >$ and compute the force acting on the charge due to the electric dipole field derived in part (a) but for $y >> a$. (30 points total)

a)



$$\vec{E}_1 = \frac{k_e q}{|\vec{r}_1|^2}\hat{r}_1$$

$$\vec{r}_1 = \langle 0,y,0\rangle - \langle a,0,0\rangle = \langle -a,y,0\rangle$$

$$|\vec{r}_1| = \sqrt{a^2+y^2} = |\vec{r}_2|$$

$$\hat{r}_1 = \frac{\langle -a,y,0\rangle}{\sqrt{a^2+y^2}}$$

$$\vec{E}_2 = \frac{k_e(-q)}{|\vec{r}_2|^2}\hat{r}_2,$$

$$\vec{r}_2 = \langle 0,y,0\rangle - \langle -a,0,0\rangle = \langle a,y,0\rangle$$

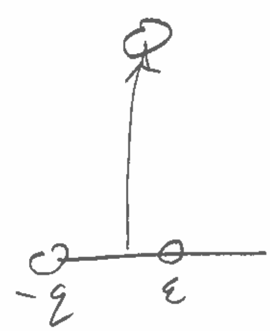
$$\hat{r}_2 = \frac{\langle a,y,0\rangle}{\sqrt{a^2+y^2}}$$

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 = \frac{k_e q}{(a^2+y^2)}\left[\frac{\langle -a,y,0\rangle}{\sqrt{a^2+y^2}} - \frac{\langle a,y,0\rangle}{\sqrt{a^2+y^2}}\right] = \boxed{\frac{2k_e q a}{(a^2+y^2)^{3/2}}\langle -1,0,0\rangle}$$

b) $y >> a$

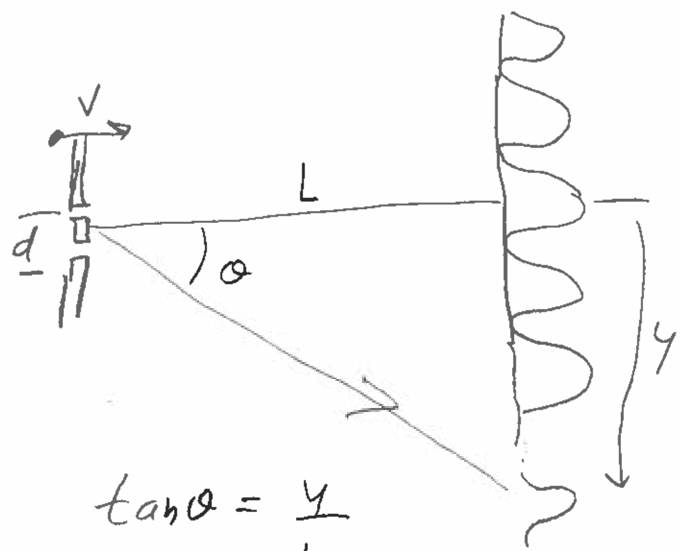
$$\vec{E} \to \frac{2k_e q a}{y^3}\langle -1,0,0\rangle$$

$$\vec{F} = Q\vec{E} = \boxed{\frac{2k_e q Q a}{y^3}\langle -1,0,0\rangle}$$



$= d$ $\quad \lambda$

Bonus Problem. A student holds a laser that emits light of wavelength 632.8 nm. The laser beam passes through a pair of slits separated by 0.300 mm, in a glass plate attached to the front of the laser. The beam then falls perpendicularly on a screen, creating a typical interference pattern. The student then begins to walk directly toward the screen at a constant speed of 3.00 m/s. The central maximum on the screen is stationary. Find the speed of the 50th-order maxima on the screen. (5 points total)



$(m$

$d \sin\theta = m\lambda$

bright

$\sin\theta = \dfrac{m\lambda}{d}$

or for small θ

$\tan\theta \simeq \dfrac{m\lambda}{d}$

$\tan\theta = \dfrac{y}{L}$

$y = L \tan\theta$

take derivative, recognizing that θ remains constant

$$\frac{dy}{dt} = \frac{dL}{dt}\tan\theta \qquad \frac{dL}{dt} = V_{student}$$

$$= (V_{student})\frac{m\lambda}{d} = \frac{(3\,m/s)(50)(632.8\times10^{-9}\,m)}{0.3\times10^{-3}\,m}$$

$$= \boxed{0.316\ m/s}$$