KEY

PHYS 1312 Fall 2018 Test 1
Sept. 13, 2018

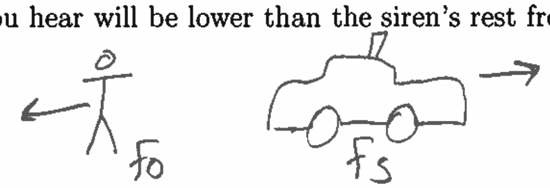
Name ____________________ **Student ID** ____________________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) If you are running in the opposite direction from a speeding police car, the frequency of its siren that you hear will be lower than the siren's rest frequency.

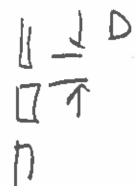
True



$f_o < f_s$ (Doppler shift)

(b) For Young's double slit experiment, an interference pattern is formed if the width of the slits is less than the wavelength of the incident light.

True

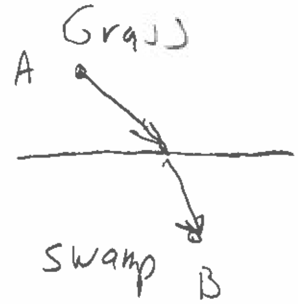


$D \lesssim \lambda$

(c) If I want to run from Point A on a just-mowed grass field to Point B in a swamp, Fermat's principle implies that I should maximize the distance I run in the swamp.

False

(travel shorter distance in swamp)



$\Delta\phi_0 = \pi$

Problem 2. Two out-of-phase radio antennas at $x = \pm 300$ m are emitting 3.0 MHz radio waves. Is the point $\vec{r} = \langle 300, 300, 0 \rangle$ m a point of maximum constructive interference, maximum destructive interference, or something in between? (15 points total)

f; electro-magnetic

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0 = m 2\pi$ (constructive)

$= (m+\frac{1}{2}) 2\pi$ (destructive)

in between

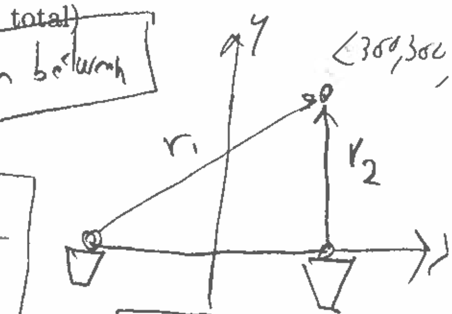
$c = \lambda f$

$\Delta\phi = 2\pi \frac{f}{c}\Delta x + \pi$

$= 2\pi \left[\frac{3\times10^6 \text{ Hz}}{3\times10^8 \text{ m/s}}(670.82 - 300) + \frac{1}{2}\right]$

$= 2\pi(4.21)$ rads

not a multiple of 2π or $\frac{1}{2}(2\pi)$

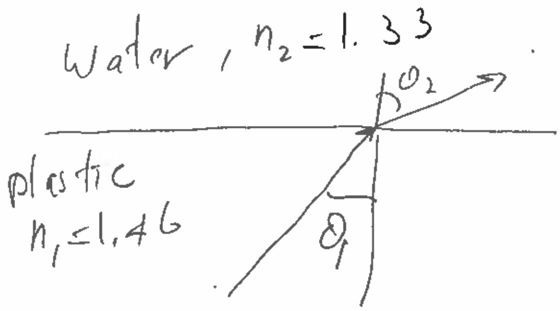


$r_1 = \sqrt{(600)^2 + 300^2}$

$= 670.82$ m

$\Delta x = r_1 - r_2$

Problem 3. To get the total internal reflection at the interface of water (refractive index 1.33) and a plastic whose index of refraction is 1.46. (a) Which material must the light start in? (b) What is the critical angle? (15 points total)



a) Ray starts in plastic

From Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

total internal reflection occurs for $\theta_2 = 90^\circ$ $\Rightarrow n_1 \sin\theta_1 = n_2$

$\Rightarrow$ or $\sin\theta_1 = \frac{n_2}{n_1}$

or $\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.33}{1.46}\right) = 65.6^\circ$

Problem 4. Starting with the magnification relation for the eyepiece $m_e = (25\ cm)/f_e$, derive the relation for the magnification of the microscope with an objective lens of focal length f_o. Take the distance between the objective and eyepiece lens to be L. (15 points total)

Make approximations

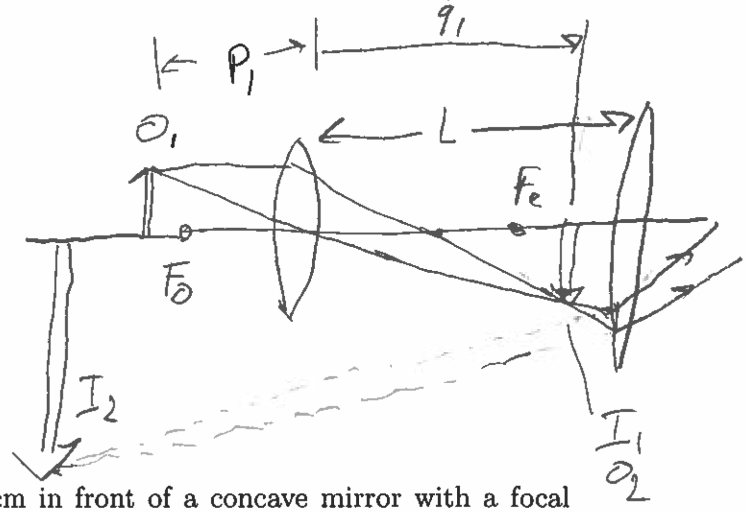
$f_o \approx p_1, \; q \approx L$

Objective lens magnification

$M_o = -\frac{q_1}{p_1} \approx -\frac{L}{f_o}$

Total magnification

$M = M_o m_e = \left(-\frac{L}{f_o}\right)\left(\frac{25cm}{f_e}\right)$



Problem 5. A pencil of height 5 cm is 40 cm in front of a concave mirror with a focal length of 20 cm. (a) Where is the location of the image? (b) What is the magnification of the image? (c) Is the image of the pencil inverted or upright, virtual or real? (d) Make a rough ray diagram showing the object, the image, and the mirror. (15 points total)

$h = 5cm$

$p = 40cm$

$f = 20cm \Rightarrow R = 2f = 40cm$

a) Apply mirror equation

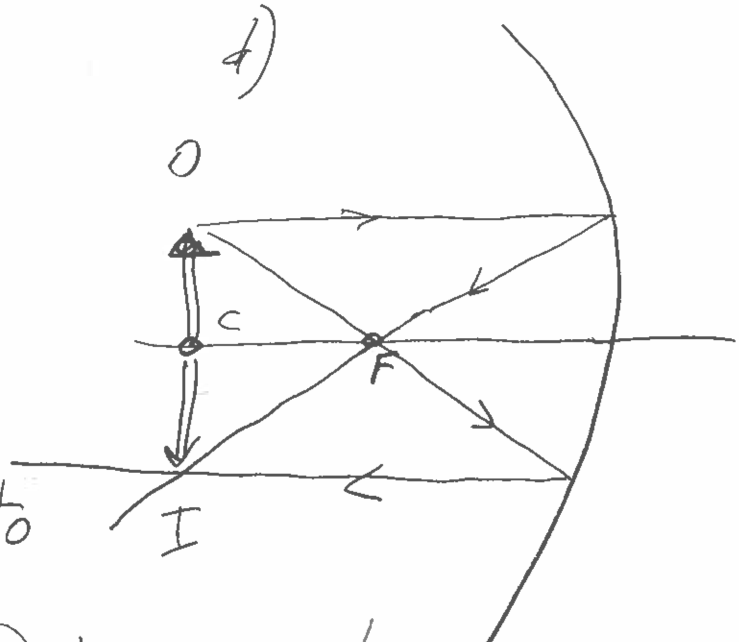
$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$

$\Rightarrow \boxed{q = 40cm}$

b) $M = -\frac{q}{p} = \frac{-40}{40} = \boxed{-1}$

d)

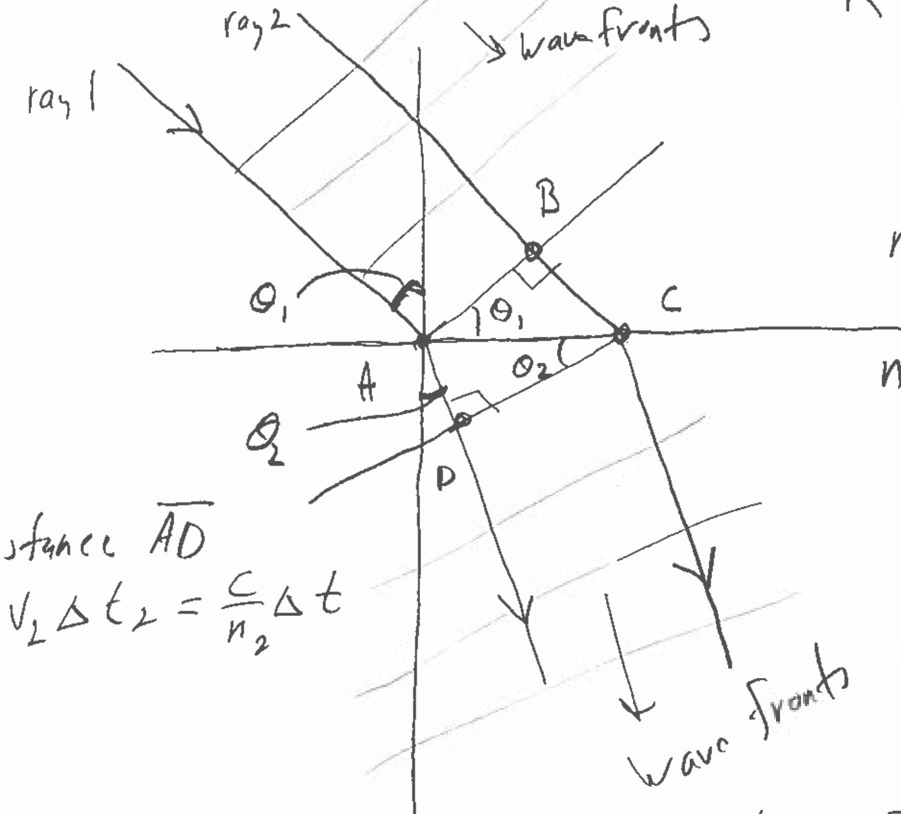


c) Image is real and inverted

Refraction

Problem 6. Derive Snell's Law of ~~Reflection~~ using Huygen's principle. Make a diagram showing appropriate rays, angles, and lengths. (30 points total)

← Not Fermat's principle



$n_1 = \frac{c}{V_1} \Rightarrow V_1 = \frac{c}{n_1}$

$n_2 = \frac{c}{V_2} \Rightarrow V_2 = \frac{c}{n_2}$

Distance $\overline{AD}$ $= V_2 \Delta t_2 = \frac{c}{n_2}\Delta t$

The distance $\overline{BC}$ $= V_1 \Delta t_1 = \frac{c}{n_1}\Delta t$

according Huygen's principle the time for ray 1 to travel from point A to D, Δt_2, is the same time for ray 2 to travel from B to C, $\Delta t_1 = \Delta t_2$

From the right triangles

$\sin\theta_1 = \frac{\overline{BC}}{\overline{AC}} = \frac{c\Delta t}{n_1 \overline{AC}} \Rightarrow \Delta t = \frac{n_1 \sin\theta_1 \overline{AC}}{c}$ set equal

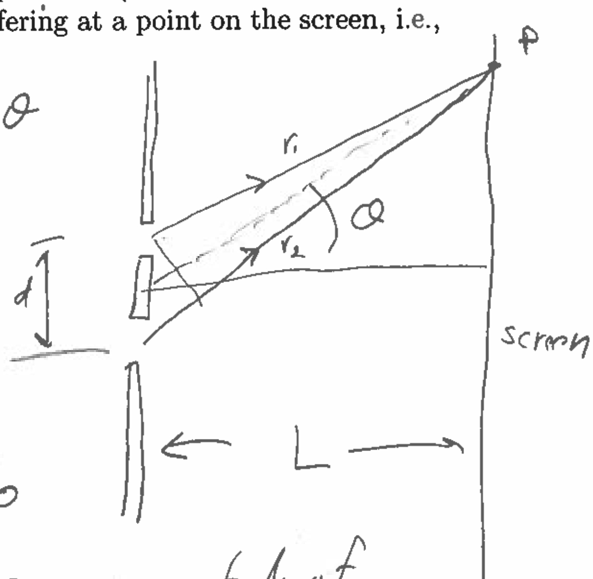
$\sin\theta_2 = \frac{\overline{AD}}{\overline{AC}} = \frac{c}{n_2}\frac{\Delta t}{\overline{AC}} \Rightarrow \Delta t = \frac{n_2 \sin\theta_2 \overline{AC}}{c}$

$\Rightarrow \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$

Bonus Problem. Derive the double slit intensity equation,

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right), \tag{1}$$

using either the phase diagram method or trig. identities. Take the distance between slits to be d, the distance from the slits to a screen to be L, and the interference pattern located at y on the screen. The slits are illuminated by monochromatic light of wavelength λ. Draw a diagram of the slit-screen arrangement and intensity pattern. (Hint: consider the wave equations for two rays emanating from the slits and interfering at a point on the screen, i.e., take $x = 0$). (5 points total)



Path length difference, $\delta = r_2 - r_1 = d\sin\theta$

For constructive interference

$\delta = d\sin\theta = m\lambda$, $m = 0, \pm 1, \ldots$

consider electrics at point P

$E_1 = E_0 \sin(\omega t)$ taking $x=0$, $\phi_1 = 0$

$E_2 = E_0 \sin(\omega t + \phi)$, $\phi = \frac{2\pi}{\lambda}\delta = 2\pi d\sin\theta/\lambda$, E_0 is magnitude of waves

$E_p = E_0[\sin(\omega t) + \sin(\omega t + \phi)] = 2E_0 \cos\left(\frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$

using the $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $A = \omega t + \phi$, $B = \omega t$

The intensity is proportional to the square of the electric field

$I = CE_p^2 = CE_0^2 4\cos^2\left(\frac{\phi}{2}\right)\sin^2\left(\omega t + \frac{\phi}{2}\right)$ → $\frac{1}{2}$

$= 2E_0^2 C\cos^2\left(\frac{\phi}{2}\right) = I_0\cos^2\left(\frac{\phi}{2}\right)$

$= I_0\cos^2\left(\frac{\frac{2\pi}{\lambda}d\sin\theta}{2}\right)$

$I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$

5 $C \equiv$ some constant

$I_0 \equiv$ maximum intensity

but $\omega = 2\pi f = 2\pi\frac{c}{\lambda}$ or $f = \frac{c}{\lambda} = \frac{3\times10^8\,m/s}{500\times10^{-9}\,m} = 6\times10^{14}$ Hz

← very rapid, take average of $\sin^2$