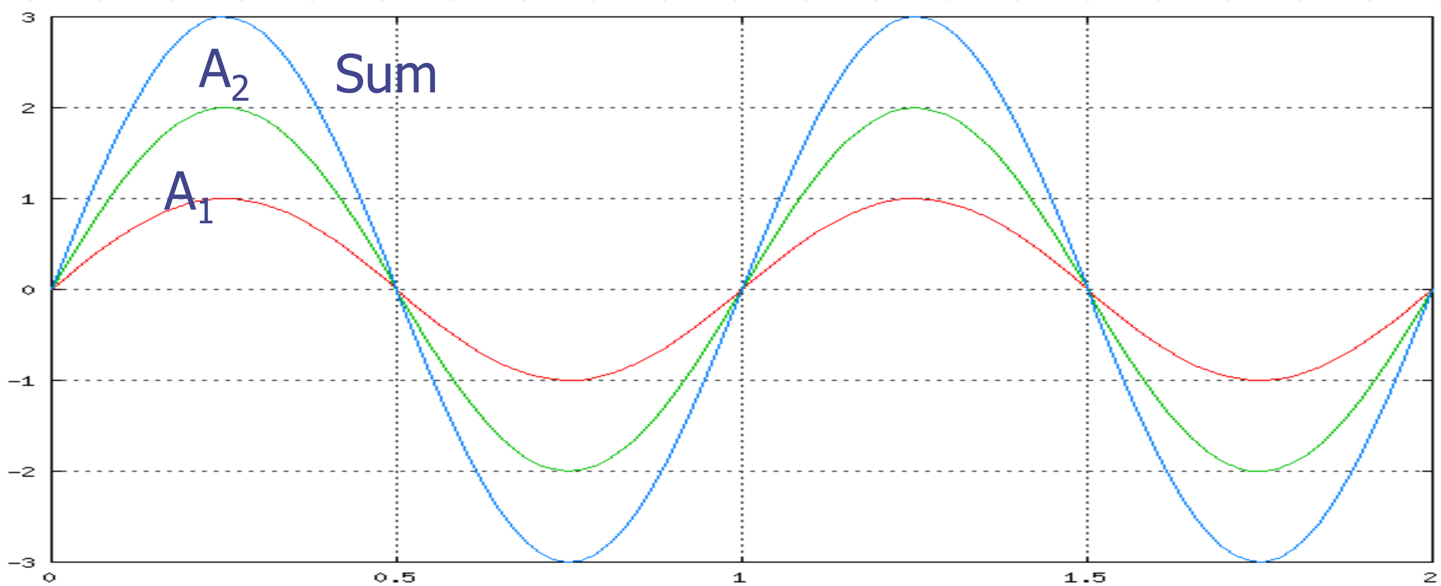
Chapter 23: Superposition, Interference, and Standing Waves

- ❑ Previously, we considered the motion of a single wave in space and time
- ❑ What if there are two waves present simultaneously – in the same place and time
- ❑ Let the first wave have λ_1 and T_1, while the second wave has λ_2 and T_2
- ❑ The two waves (or more) can be added to give a resultant wave → this is the Principle of Linear Superposition

• Consider the simplest example: $\lambda_1 = \lambda_2$

- Since both waves travel in the same medium, the wave speeds are the same, then $T_1=T_2$
- We make the additional condition, that the waves have the same phase – i.e. they start at the same time → <u>Constructive Interference</u>
- The waves have $A_1=1$ and $A_2=2$. Here the sum of the amplitudes $A_{sum}=A_1+A_2 = 3$ $(y=y_1+y_2)$



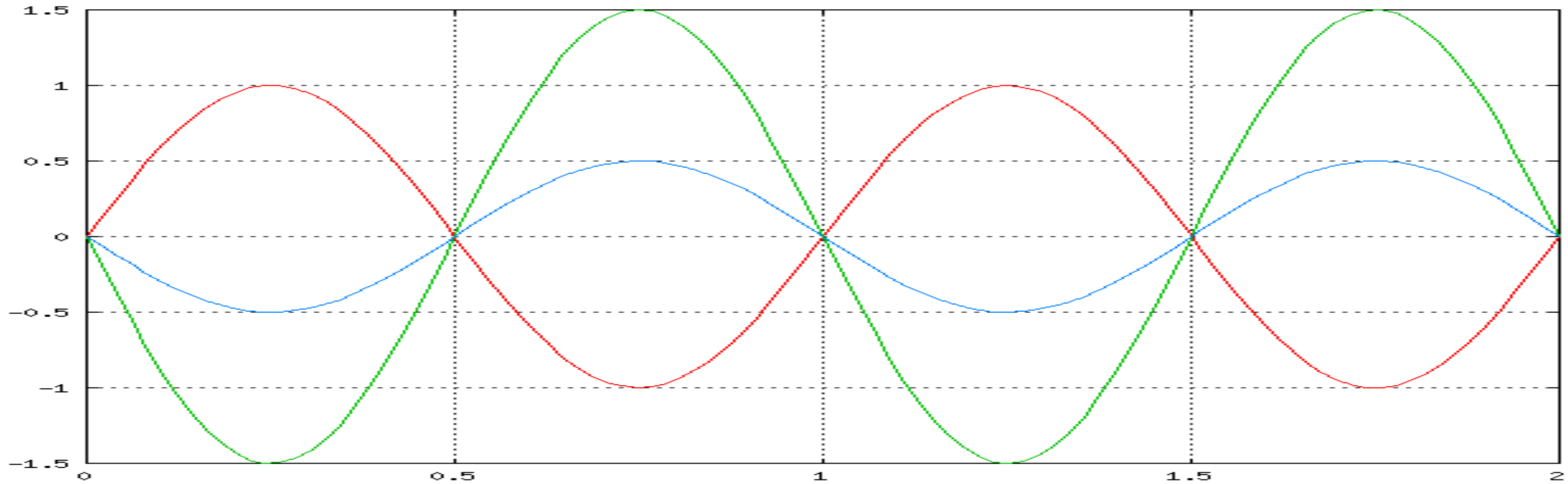
- If the waves ($\lambda_1 = \lambda_2$ and $T_1 = T_2$) are exactly out of phase, i.e. one starts a half cycle later than the other → Destructive Interference

  - If $A_1 = A_2$, we have complete cancellation: $A_{sum} = 0$

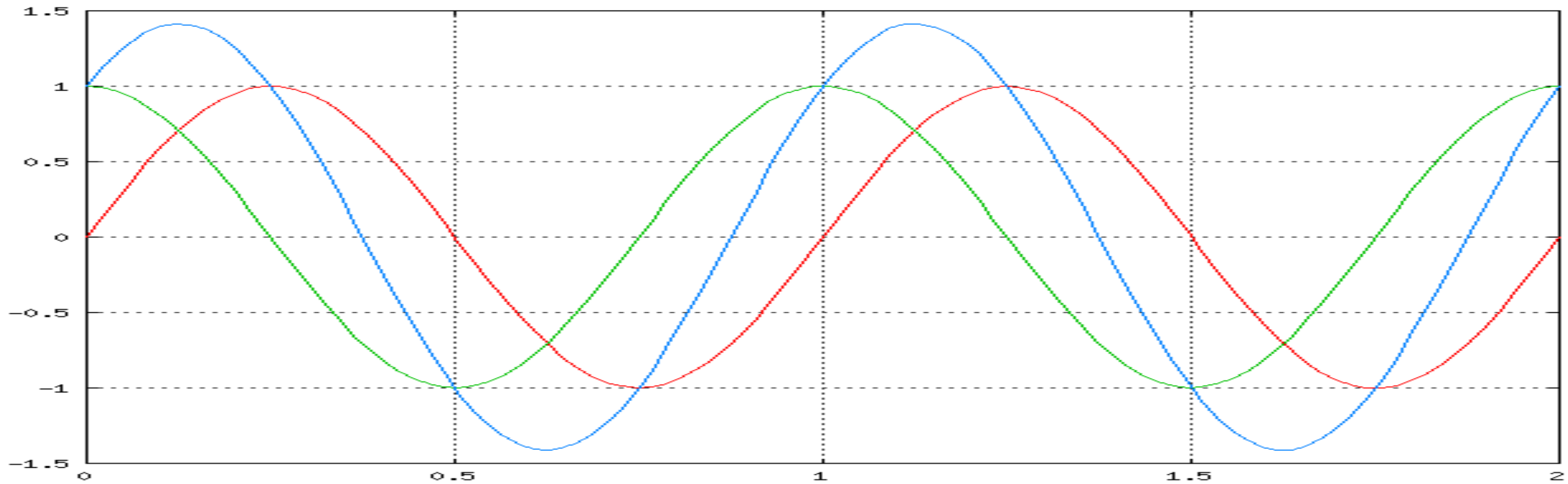
$y = y_1 + y_1 = 0$

- These are special cases. Waves may have different wavelengths, periods, and amplitudes and may have some fractional phase difference.

❑ Here are a few more examples: exactly out of phase (π), but different amplitudes



❑ Same amplitudes, but out of phase by (π/2)



Example Problem

Speakers A and B are vibrating in phase. They are directed facing each other, are 7.80 m apart, and are each playing a 73.0-Hz tone. The speed of sound is 343 m/s. On a line between the speakers there are three points where constructive interference occurs. What are the distances of these three points from speaker A?

Solution:

Given: $f_A = f_B = 73.0$ Hz, $L = 7.80$ m, $v = 343$ m/s

$$\lambda = vT = \frac{v}{f} = \frac{343\text{ m/s}}{73.0\text{ Hz}} = 4.70\text{ m}$$

$$L = \lambda + 2x$$

x is the distance to the first constructive interference point

$$x = \frac{L}{2} - \frac{\lambda}{2}$$

The next point (node) is half a wave-length away.

$$x = \frac{L}{2} - n\frac{\lambda}{2}$$

Where n=0,1,2,3,... for all nodes

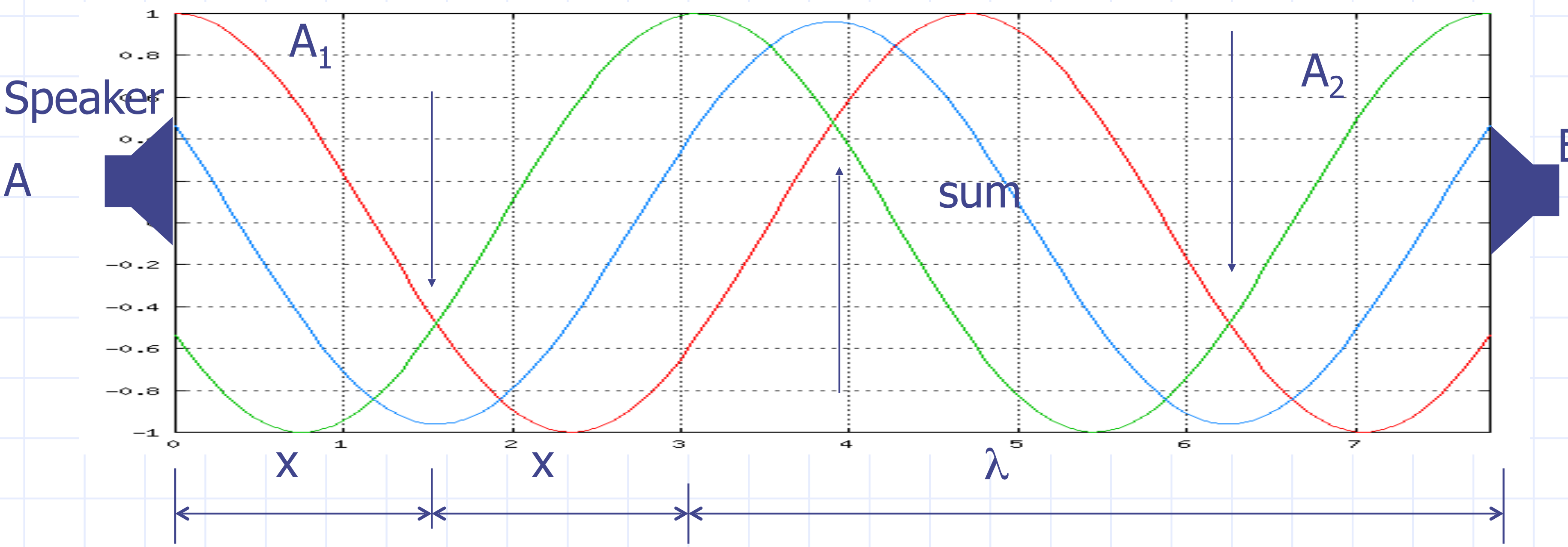
$$n = 0:\ x = \frac{7.8}{2} - 0\frac{4.70}{2} = \boxed{3.9\text{ m}}$$

$$n = 1:\ x = \frac{7.8}{2} - 1\frac{4.70}{2} = \boxed{1.55\,\text{m}}$$

$$n = 2:\ x = \frac{7.8}{2} - 2\frac{4.70}{2} = -0.8\,\text{m}$$

Behind speaker A

$$n = -1:\ x = \frac{7.8}{2} - (-1)\frac{4.70}{2} = \boxed{6.25\,\text{m}}$$



Beats

- Different waves usually don't have the same frequency. The frequencies may be much different or only slightly different.
- If the frequencies are only slightly different, an interesting effect results $\rightarrow$ the <u>beat</u> <u>frequency.</u>
- Useful for tuning musical instruments.
- If a guitar and piano, both play the same note (same frequency, $f_1=f_2$) $\rightarrow$ constructive interference
- If f_1 and f_2 are only slightly different, constructive and destructive interference occurs

- The beat frequency is

$$f_b = |f_1 - f_2| \quad \textbf{or}$$

$$\frac{1}{T_b} = \left|\frac{1}{T_1} - \frac{1}{T_2}\right|$$ In terms of periods

$$\textbf{as} \;\; f_2 \to f_1, \;\; f_b \to 0$$

- The frequencies become ``tuned"

Example Problem

When a guitar string is sounded along with a 440-Hz tuning fork, a beat frequency of 5 Hz is heard. When the same string is sounded along with a 436-Hz tuning fork, the beat frequency is 9 Hz. What is the frequency of the string?

Solution:

Given: f_{T1}=440 Hz, f_{T2}=436 Hz, f_{b1}=5 Hz, f_{b2}=9 Hz

But we don't know if frequency of the string, f_s, is greater than f_{T1} and/or f_{T2}. Assume it is.

$$f_{b1} = f_s - f_{T1} \text{ and } f_{b2} = f_s - f_{T2} \Rightarrow$$

$$f_s = f_{b1} + f_{T1} = 5 + 440 = \boxed{445\text{ Hz}}$$

$$f_s = f_{b2} + f_{T2} = 9 + 436 = \boxed{445\text{ Hz}}$$

If we chose f_s smaller

$$f_{b1} = f_{T1} - f_s \text{ and } f_{b2} = f_{T2} - f_s \Rightarrow$$

$$f_s = f_{T1} - f_{b1} = 440 - 5 = \cancel{435\text{ Hz}}$$

$$\neq$$

$$f_s = f_{T2} - f_{b2} = 436 - 9 = \cancel{427\text{ Hz}}$$

Standing Waves

- A standing wave is an interference effect due to two overlapping waves
  - transverse – wave on guitar string, violin, …
  - longitudinal – sound wave in a flute, pipe organ, other wind instruments,…
- The length (dictated by some physical constraint) of the wave is some multiple of the wavelength
- You saw this in lab last semester
- Consider a transverse wave (f_1, T_1) on a string of length L fixed at both ends.

- If the speed of the wave is v (not the speed of sound in air), the time for the wave to travel from one end to the other and back is $2L/\mathrm{v}$
- If this time is equal to the period of the wave, T_1, then the wave is a standing wave
- $$T_1 = \frac{1}{f_1} = \frac{2L}{\mathrm{v}} \Rightarrow \boxed{f_1 = \frac{\mathrm{v}}{2L}} = \frac{\mathrm{v}}{\lambda_1} \Rightarrow \lambda_1 = 2L$$
- Therefore the length of the wave is half of a wavelength or a half-cycle is contained between the end points
- We can also have a full cycle contained between end points $\lambda_2 = L \Rightarrow f_2 = \frac{\mathrm{v}}{\lambda_2} = \boxed{\frac{\mathrm{v}}{L} = f_2}$

- Or three half-cycles

$$\lambda_3 = \tfrac{2}{3}L \Rightarrow f_3 = \frac{v}{\lambda_3} = \frac{v}{\tfrac{2}{3}L} = \boxed{\frac{3v}{2L} = f_3}$$

- Or n half-cycles

- Some notation: $\boxed{f_n = n\left(\frac{v}{2L}\right),}$ $\quad n = 1, 2, 3, 4, \ldots$

For a string fixed at both ends

- The zero amplitude points are called nodes; the maximum amplitude points are the antinodes

f_1	1st harmonic or fundamental	
$f_2 = 2f_1$	2nd	1st overtone
$f_3 = 3f_1$	3rd	2nd overtone
$f_4 = 4f_1$	4th	3rd overtone

Longitudinal Standing Waves

- Consider a tube with both ends opened
- If we produce a sound of frequency f_1 at one end, the air molecules at that end are free to vibrate and they vibrate with f_1
- The amplitude of the wave is the amplitude of the vibrational motion (SHM) of the air molecule – changes in air density
- Similar to the transverse wave on a string, a standing wave occurs if the length of the tube is a half-multiple of the wavelength of the wave

- For the first harmonic (fundamental), only half of a cycle is contained in the tube

$$f_1 = \frac{v}{2L}$$

- Following the same reasoning as for the transverse standing wave, all of the harmonic frequencies are

$$\boxed{f_n = n\left(\frac{v}{2L}\right), \quad n = 1, 2, 3, ...}$$

Open-open tube

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- Identical to transverse wave, except number of nodes is different

$$\#\,\mathrm{nodes} = n - 1$$

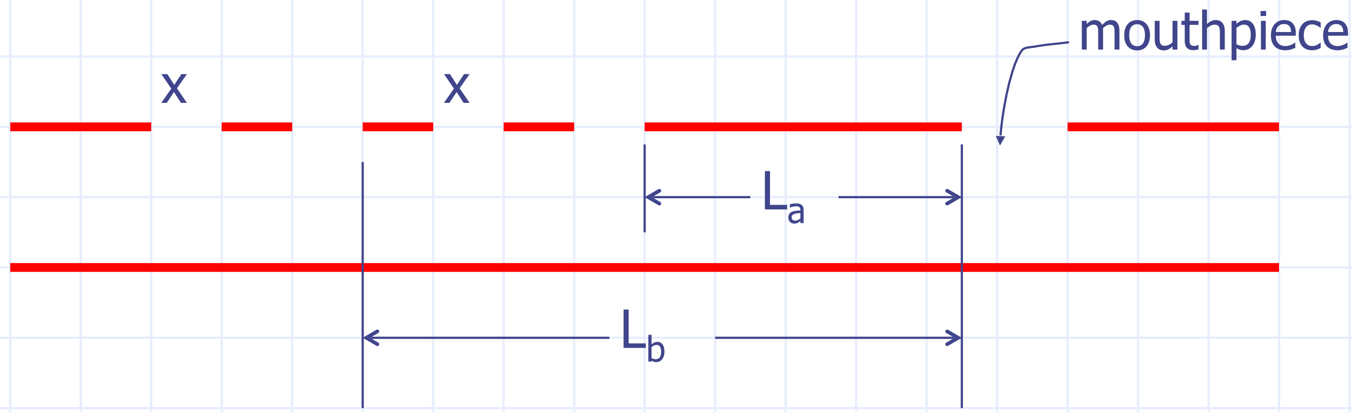
string

$$\#\,\mathrm{nodes} = n$$

Open-open tube

- An example is a flute. It is a tube which is open at both ends.

$$f_a = \frac{v}{2L_a},$$



$$f_b = \frac{v}{2L_b} < f_a$$

- We can also have a tube which is closed at one end and opened at the other (open-closed)

- At the closed end, the air molecules can not vibrate – the closed end must be a ``node”

- The open end must be an anti-node

- The ``distance" between a node and the next adjacent anti-node is 1/4 of a wavelength. Therefore the fundamental frequency of the open-closed tube is

$$f_1 = \frac{v}{4L} \quad \text{since} \quad L = \lambda / 4 \ \text{or}\ \lambda = 4L$$

- The next harmonic does not occur for 1/2 of a wavelength, but 3/4 of a wavelength. The next is at 5/4 of a wavelength – every odd 1/4 wavelength

- $$\boxed{f_n = n\left(\frac{v}{4L}\right), n = 1,3,5,\ldots}$$ Open-closed

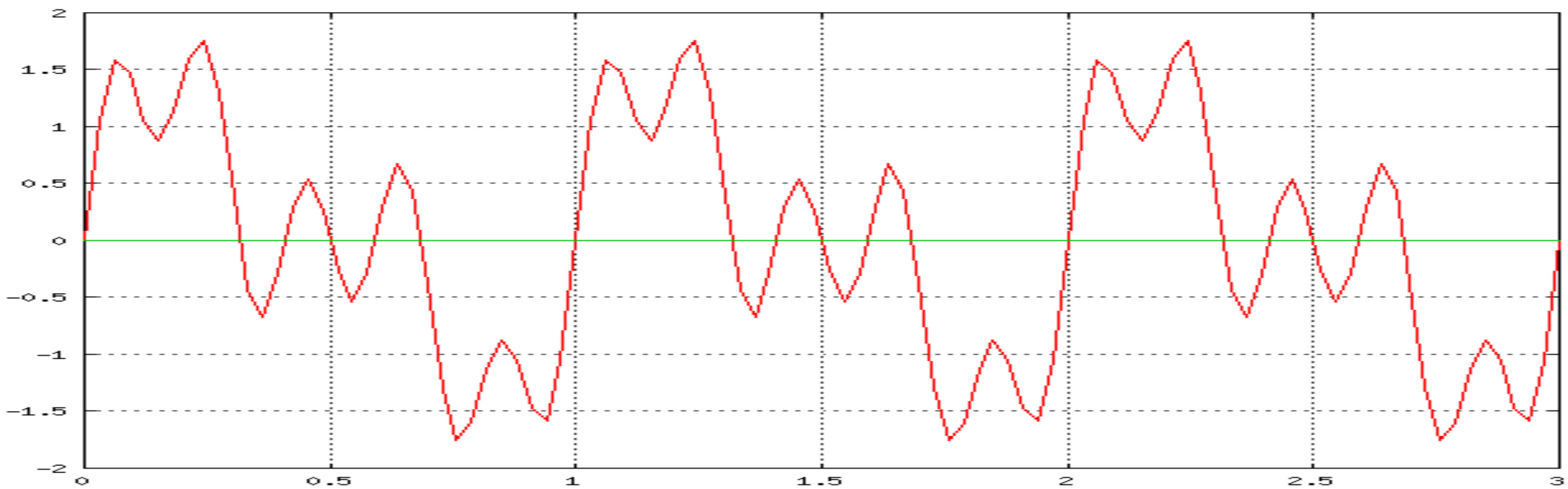
- Note that the even harmonics are missing. Also,

$$\#\text{nodes} = \frac{n-1}{2}$$

Complex (Real) Sound Waves

- Most sounds that we hear are not pure tones (single frequency – like the fundamental f_1 of a standing wave)
- But are superpositions of many frequencies with various amplitudes
- For example, when a note (tone, frequency) is played on a musical instrument, we actually hear all of the harmonics (f_1, f_2, f_3, …), but usually the amplitudes are decreased for the higher harmonics
- This is what gives each instrument it's unique sound

- For example, the sound of a piano is dominated by the 1st harmonic while for the violin, the amplitudes of the 1st, 2nd, and 5th harmonic are nearly equal – gives it a rich sound



Violin wave form

Summary

String fixed at both ends and the open-open tube

$$f_n = n\left(\frac{v}{2L}\right), \quad n = 1, 2, 3, \ldots.$$

Open-closed tube

$$f_n = n\left(\frac{v}{4L}\right), \; n = 1,3,5,\ldots$$

Example Problem

A tube with a cap on one end, but open at the other end, produces a standing wave whose fundamental frequency is 130.8 Hz. The speed of sound is 343 m/s. (a) If the cap is removed, what is the new fundamental frequency? (b) How long is the tube?

Solution:

Given: f_1^{oc}=130.8 Hz, n=1, v=343 m/s

$$f_n^{oc} = n\left(\frac{\mathrm{V}}{4L}\right) \qquad f_n^{oo} = n\left(\frac{\mathrm{V}}{2L}\right)$$

(a) We don't need to know v or L, since they are the same in both cases. Solve each equation for v/L and set equal

$$\frac{v}{L} = 4 f_1^{oc}, \quad \frac{v}{L} = 2 f_1^{oo} \quad \Rightarrow \quad 4 f_1^{oc} = 2 f_1^{oo}$$

$$f_1^{oo} = 2 f_1^{oc} = 2(130.8\ \mathrm{Hz}) = \boxed{261.6\ \mathrm{Hz}}$$

(b) Can solve for L from either open-open or open-closed tubes

$$f_1^{oc} = 1\left(\frac{v}{4L}\right) \Rightarrow$$

$$L = \frac{v}{4 f_1^{oc}} = \frac{343\ \mathrm{m/s}}{4(130.8\ \mathrm{Hz})} = \boxed{0.6556\ \mathrm{m}}$$

$$L = \frac{v}{2 f_1^{oo}} = \frac{343\ \mathrm{m/s}}{2(261.6\ \mathrm{Hz})} = 0.6556\ \mathrm{m}$$

Example problem

- Two violinists, one directly behind the other, play for a listener directly in front of them. Both violinists sound concert A (440 Hz). (a) What is the smallest separation between the violinists that will produce destructive interference for the listener? (b) Does the smallest separation increase or decrease if the violinists produce a note with higher frequency? (c) Show this for 540 Hz.

Example problem

- A pair of in-phase speakers are placed side by side, 0.85 m apart. You stand directly in from of one speaker, 1.1 m from the other speaker. What is the lowest frequency that will produce constructive interference at your location?