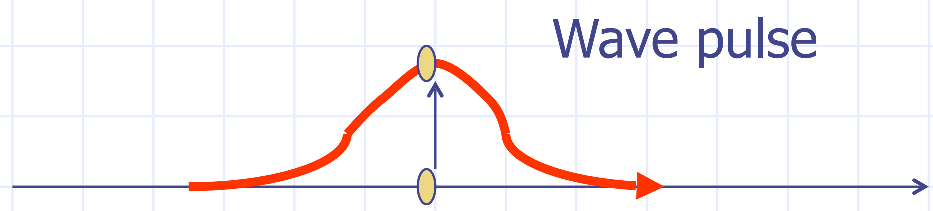
Chapter 23: Waves

- We now leave our studies of mechanics and take up the related topic – wave motion (though it is similar to SHM)

- Wave – a traveling disturbance which carries energy from one point to another, but without the translation of mass

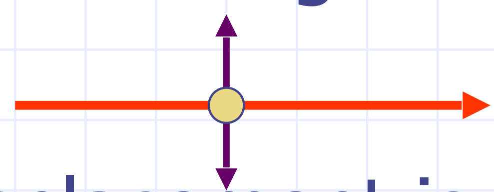


- A wave usually travels through a medium (gas, liquid, solid), except for light (an electromagnetic wave) which can propagate through a vacuum

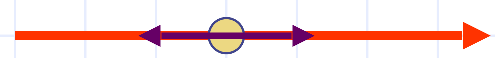
- The particles of the medium which convey the wave, can be said to oscillate about their equilibrium positions (like SHM)

- Waves can be classified according to the direction of the oscillatory displacement

- <u>Transverse waves</u> – the displacement (of the particle conveying the wave) is perpendicular to the direction of the wave; examples are a guitar string and light



- <u>Longitudinal waves</u> – the displacement is along the direction of the wave; examples are sound, the spring, and seismic waves

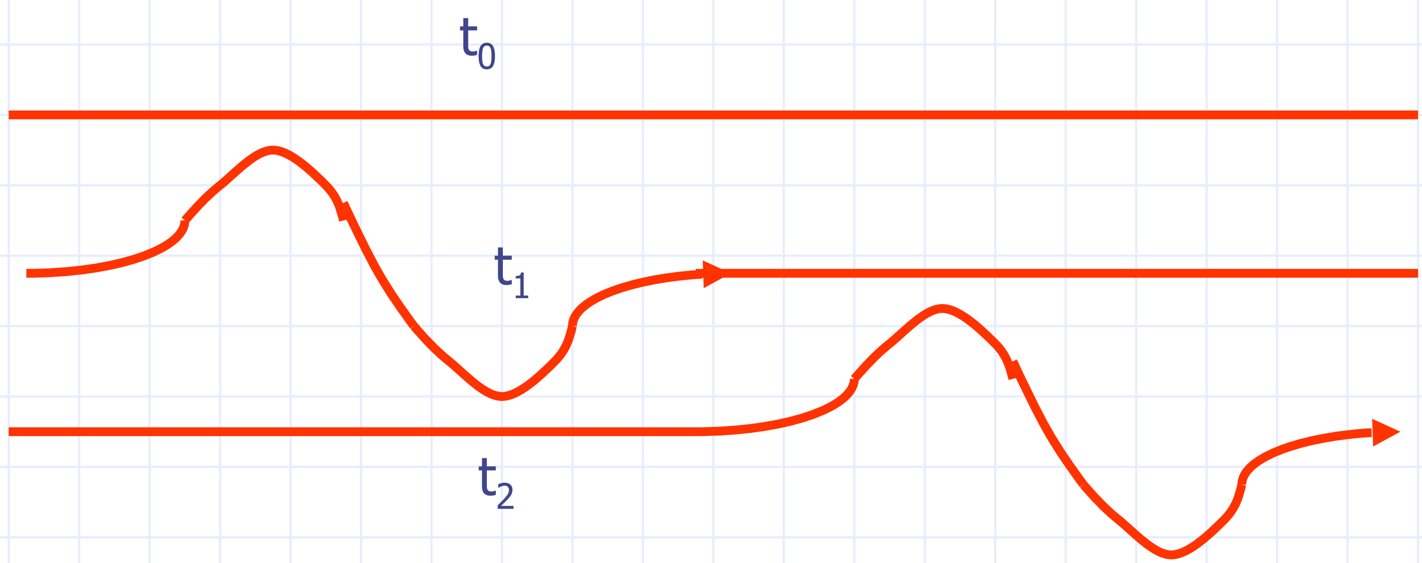


- Some waves can be a mixture of longitudinal and transverse modes – surface water wave

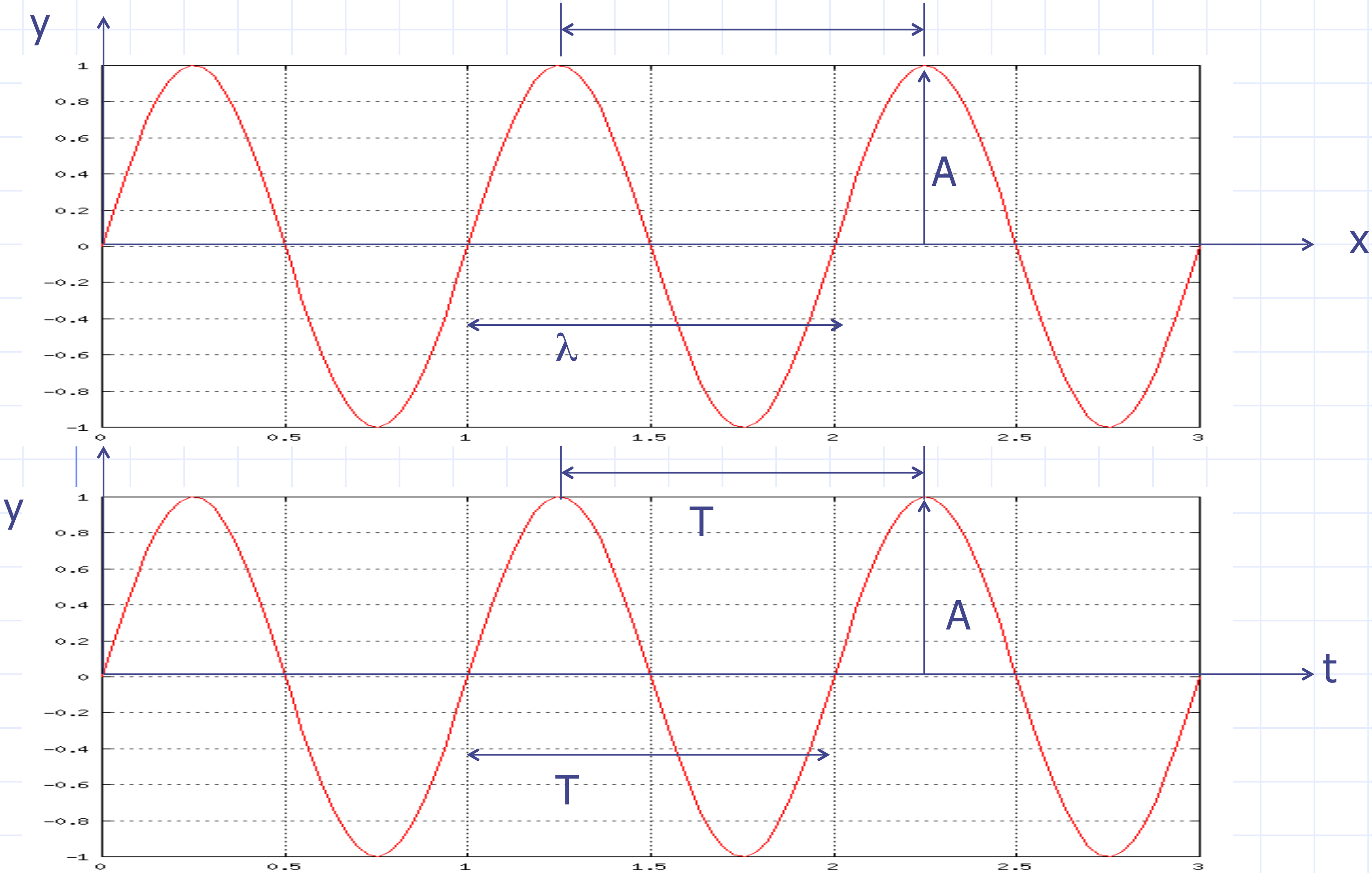
- Can also classify waves as to whether they are
  - a Pulse – a single disturbance
  - Periodic – a repeating pattern of cycles

- In both cases, each particle of the medium experiences SHM, but for the pulse they start from rest, go through one cycle, then return to rest, while for a periodic wave they experience many, many cycles

Pulse on a string



Periodic wave



Can have wave motion in both time (same as SHM) and space (x-direction for example)

- λ = wavelength, the length of one complete wave cycle; units of m

- Analogous to the Period T of a wave (for motion in time). In fact they are related

$$\boxed{\lambda = vT = \frac{v}{f}}$$

- Where v is the velocity of the wave. It is the velocity of a point on the wave (crest, trough, etc), not of a particle

- Since wave motion can be in space and time, we would like to have an equation for the displacement (y) as a function of space and time

- The displacement for a particle as a function of t and x (without proof):
- $$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x \mp \frac{2\pi}{T}t + \phi_o\right)$$

(-) positive x-direction wave motion
(+) negative x-direction wave motion
k=2π/λ, the angular wave number

- The quantity in parentheses is dimensionless (radians) and is called the phase angle (φ) of a wave

$$y = A\sin\varphi = A\cos\left(\varphi - \tfrac{\pi}{2}\right)$$

- The correspondence with Simple Harmonic Motion should be apparent

Example Problem

- The speed of a transverse wave on a string is 450 m/s, while the wavelength is 0.18 m. The amplitude of the wave is 2.0 mm. How much time is required for a particle of the string to move through a distance of 1.0 km?

Solution:

Given: $v = 450$ m/s, $\lambda = 0.18$ m, $A = 2.0$ mm, $D = 1.0$ km = travel distance of particle back and forward in the y-direction

Find: time for particle to cover distance of 1.0 km

Convert to SI units: A=2.0x10^{-3} m, D=1.0x10^{3} m

A is the distance covered in 1/4-cycle. Therefore distance covered in 1 cycle is 8.0x10^{-3} m. The period of each cycle is $\lambda = \mathrm{v}T$

$$T = \frac{\lambda}{\mathrm{v}} = \frac{0.18\ \mathrm{m}}{450\ \mathrm{m/s}} = 4.0x10^{-4}\ \mathrm{s}$$

The number of cycles to cover 1.0 km is

$$\#\text{cycles} = \frac{\text{total distance}}{\text{distance per cycle}} = \frac{1.0\mathrm{x}10^{3}\ \mathrm{m}}{8.0\mathrm{x}10^{-3}\ \mathrm{m}} = 1.25\mathrm{x}10^{5}$$

$$t = (\#\text{cycles})T = (1.25\mathrm{x}10^{5})(4.0\mathrm{x}10^{-4}\ \mathrm{s}) = \boxed{5.0\mathrm{x}10^{1}\ \mathrm{s}}$$

Sound Waves

- Sound is a longitudinal wave
- It requires a medium to convey it, e.g. a gas, liquid, or solid
- In a gas, the amplitude of the sound wave is air pressure – a series of slightly enhanced (crest) and reduced (trough) pressure (or air density) regions
- The speed that these pressure variations move (the wave speed) is the speed of sound

- A sound wave is longitudinal since, for example, the air molecules' positions oscillate in the direction that the wave travels – they oscillate from condensed regions (crest) to underdense regions (trough)
- Lists are available for the sound speeds for various gases, liquids, and solids
- The sound speed in solids > liquids > gases
- Given some physical properties of the medium, it is possible to calculate the sound speed
- For ideal gases (low density gases for which the gas atoms or molecules do not interact -discussed in Chap. 12), the speed of sound is:

$$v_{gas} = \sqrt{\frac{\gamma k_b T}{m}}$$

m = mass of a gas atom or molecule (kg)

T = temperature of the gas (Kelvin, K)

For temperature, we must use the absolute scale of Kelvin: T(K)=T(°C) + 273.15

k_b = Boltzmann's constant = 1.38×10^{-23} J/K

Think of k_b as a conversion factor between temperature and energy

γ = adiabatic index of a gas, a unitless constant which depends on the gas, usually between 1.3-1.7. It is 1.4 for air

- Notice that the speed of sound increases with temperature

- It is also possible to calculate the speed of sound in liquids and solids. We will not consider those expressions. Just be aware of the trends, e.g. v_{air}=343 m/s, v_{water}=1482 m/s, v_{steel}=5960 m/s

Example Problem

The wavelength of a sound wave in air is 2.74 m at 20 °C. What is the wavelength of this sound wave in fresh water at 20 °C? (Hint: the frequency is the same).

Solution: Given λ_{air} = 2.74 m, $f_{air}=f_{water}$

$$\lambda = \mathrm{v}T = \frac{\mathrm{v}}{f} \Rightarrow f = \frac{\mathrm{v}}{\lambda}$$

$$f_{air} = f_{water}$$

$$\frac{\mathrm{v}_{air}}{\lambda_{air}} = \frac{\mathrm{v}_{water}}{\lambda_{water}} \Rightarrow \lambda_{water} = \frac{\mathrm{v}_{water}}{\mathrm{v}_{air}} \lambda_{air}$$

$$\lambda_{water} = \frac{1482 \text{ m/s}}{343 \text{ m/s}} (2.74 \text{ m}) = \boxed{11.8 \text{ m}}$$

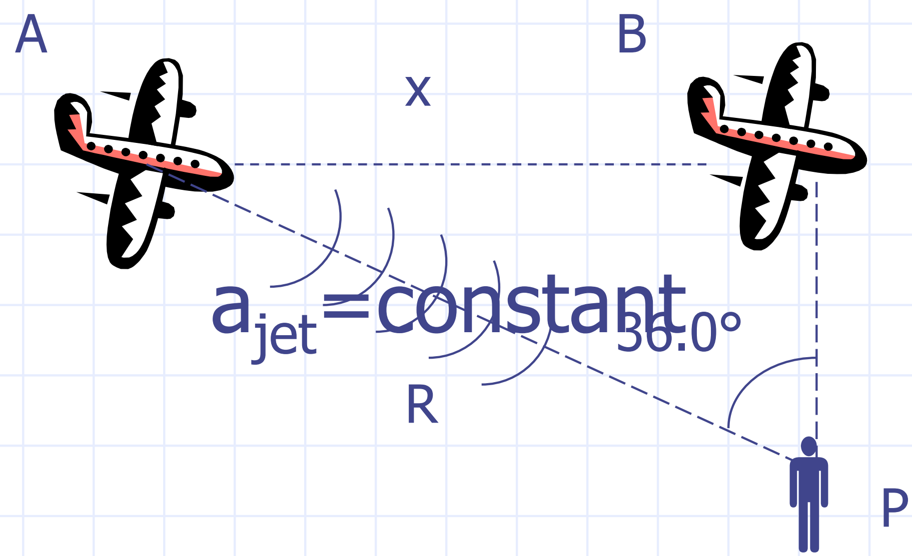
How about for the sound wave in steel?

$$\lambda_{steel} = \frac{5960 \text{ m/s}}{343 \text{ m/s}} (2.74 \text{ m}) = 47.6 \text{ m}$$

- As a sound wave passes from one medium to another, its speed and wavelength changes, but not its frequency

Example Problem

A jet is flying horizontally as shown in the drawing. When the jet is directly overhead at B, a person on the ground hears the sound coming from A. The air temperature is 20 °C. If the speed of the jet is 164 m/s at A, what is its speed at B, assuming it has a constant acceleration?



Solution:

Given: $v_{A,\ jet}$=164 m/s, a_{jet}=constant

v_{air}=343 m/s

Find: $v_{B,jet}$

Let x be the distance between A and B, and R the distance between A and the person (P)

$$x = R\sin\theta$$

The time for the jet to travel from A to B is the same as the time for the sound wave to travel from A to P

$$t_{jet} = t_{sound} = t$$

From 1D kinematics

$$R = \mathrm{v}_{sound} t_{sound} \Rightarrow t_{sound} = \frac{R}{\mathrm{v}_{sound}} = \boxed{\frac{x}{\mathrm{v}_{sound}\sin\theta}} = t$$

$$x = \tfrac{1}{2}(\mathrm{v}_{\mathrm{A}} + \mathrm{v}_{\mathrm{B}})t_{jet} = \tfrac{1}{2}(\mathrm{v}_{\mathrm{A}} + \mathrm{v}_{\mathrm{B}})\frac{x}{\mathrm{v}_{\mathrm{sound}}\sin\theta}$$

$$x(\mathrm{v}_{\mathrm{sound}}\sin\theta) = \tfrac{1}{2}(\mathrm{v}_{\mathrm{A}} + \mathrm{v}_{\mathrm{B}})x$$

$$2v_{sound}\sin\theta = v_A + v_B$$
$$v_B = 2v_{sound}\sin\theta - v_A$$
$$v_B = 2(343\text{ m/s})\sin(36.0^\circ) - 164\text{ m/s}$$
$$v_B = \boxed{239\text{ m/s}}$$

The Doppler Effect of a Sound Wave

- When a car passes you (at rest) holding its horn, the horn sound appears to have a higher pitch (larger f) as the car approaches and a lower pitch (smaller f) as the car recedes – this is the Doppler Effect (named for an Austrian physicist)

- The effect occurs because the number of sound wave condensations (crest) changes from when the car is approaching to when the car is receding (and is different if the car and you are both at rest)
- The frequency of the car horn, we call the source frequency, f_s. Also called the rest frequency since it is the sound frequency you would hear if the car and you (observer) each had zero velocity.

$$f_s = v_{sound} / \lambda = v_{sound} T$$

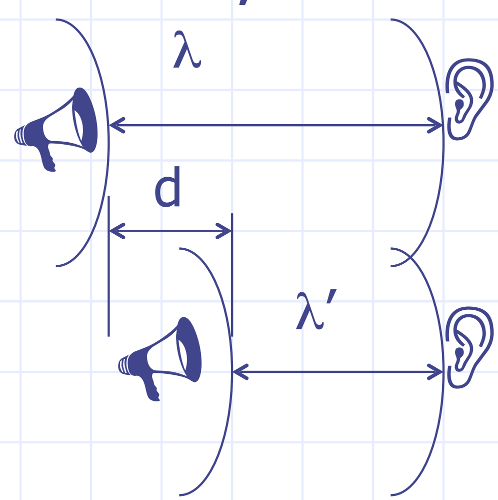
- When both the source and observer are at rest, a condensation (wave crest) passes the observer every T with the distance between each crest equal to the wavelength λ

- The frequency heard by the observer $f_o=f_s$

- Now consider two different cases: 1) the source moving with velocity v_s and the observer at rest and 2) the source at rest and the observer moving with velocity v_o

Moving Source

- The car is moving toward you with v_s. It emits a wave. A time T later it emits another wave, but the car has traveled a distance $d=v_sT$



- The wavelength between each wave is reduced. Therefore the frequency heard by the observer must increase

The reduced wavelength is

$$\lambda' = \lambda - d = \lambda - v_s T$$

The frequency heard by the observer is

$$f_o = \frac{v_{sound}}{\lambda'} = \frac{v}{\lambda - v_s T}$$

Let $v_{sound} = v$

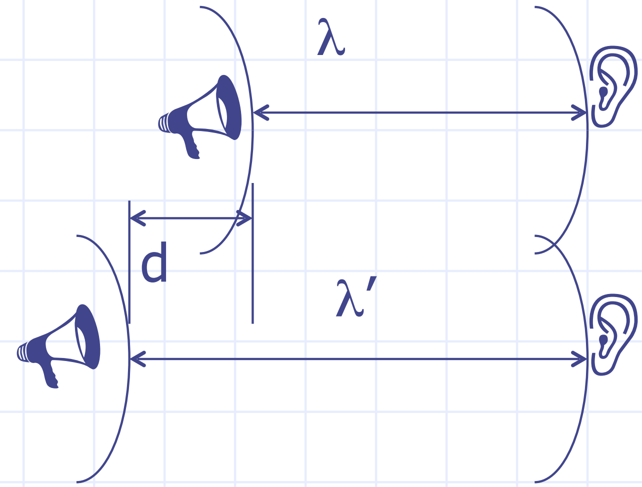
$$f_o = \frac{v}{\frac{v}{f_s} - \frac{v_s}{f_s}} = \frac{v}{\frac{1}{f_s}(v - v_s)} = f_s\left(\frac{v}{v - v_s}\right)$$

$$\boxed{f_o = f_s\left(\frac{1}{1 - v_s/v}\right)}$$

For source moving towards observer, $f_o > f_s$

- For source moving away from observer, wavelength increases

$$\lambda' = \lambda + v_s T$$



- Following the same procedures gives →

$$\boxed{f_o = f_s\left(\frac{1}{1 + v_s / v}\right)}$$

- For source moving away, $f_o < f_s$. Observer hears lower pitch.

Moving Observer

- If the observer moves toward the source (which is at rest) with speed v_o, the emitted wavelength λ remains constant

- But the observer can "run" into more cycles (wave crests) than if she remained at rest. The number of additional cycles encountered is

$$\mathrm{v_o}t/\lambda$$

- Or the additional number of cycles/second, which is a frequency, is $\mathrm{v_o}/\lambda = f'$

- Therefore, the frequency heard by the observer is

$$f_o = f_s + f' = f_s + \frac{\mathrm{v_o}}{\lambda} = f_s\left(1 + \frac{\mathrm{v_o}}{f_s\lambda}\right)$$

$$\boxed{f_o = f_s\left(1 + \frac{\mathrm{v_o}}{\mathrm{v}}\right)} \quad \text{since } \lambda = \mathrm{v}/f_s$$

- Therefore, if the observer is moving towards the source, the frequency heard by the observer is increased, $f_o > f_s$

- Now, for the observer moving away from the source, she will encounter $v_o t/\lambda$ fewer wave crests than if she remained stationary. The observed frequency will be:

$$f_o = f_s - f' = f_s - \frac{v_o}{\lambda} = f_s\left(1 - \frac{v_o}{f_s \lambda}\right)$$

$$\boxed{f_o = f_s\left(1 - \frac{v_o}{v}\right)}$$

In this case $f_0 < f_s$

- To summarize: 1) Moving source

$$f_o = f_s\left(\frac{1}{1 \mp v_s / v}\right)$$

(-) moving together

(+) moving apart

- 2) Moving observer:

$$f_o = f_s\left(1 \pm \frac{v_o}{v}\right)$$

(+) moving together

(-) moving apart

- Note that equations look similar, but mechanisms for frequency shifts ($\Delta f = f_o - f_s$) are different

• Finally, both observer and source can be moving

$$\boxed{f_o = f_s\left(\frac{1 \pm v_o / v}{1 \mp v_s / v}\right)}$$

Example Problem

Suppose you are stopped for a traffic light and an ambulance approaches from behind with a speed of 18 m/s. The siren on the ambulance produces sound with a frequency of 955 Hz. The air sound speed is 343 m/s. What is the wavelength of the sound reaching your ears?

<u>Solution:</u>

Given: v_o=0,v_s=18 m/s, v=343 m/s, f_s=955 Hz

Method: find f_o then $\lambda_{o,}$ use moving source equation

$$f_o = f_s\left(\frac{1}{1 \mp v_s / v}\right)$$

Use (-) since source is approaching observer, $f_o > f_s$

$$f_o = f_s\left(\frac{1}{1 - v_s / v}\right) = (955\ \text{Hz})\left(\frac{1}{1 - (18/343)}\right)$$

$$f_o = 1008\ \text{Hz} = v/\lambda_o$$

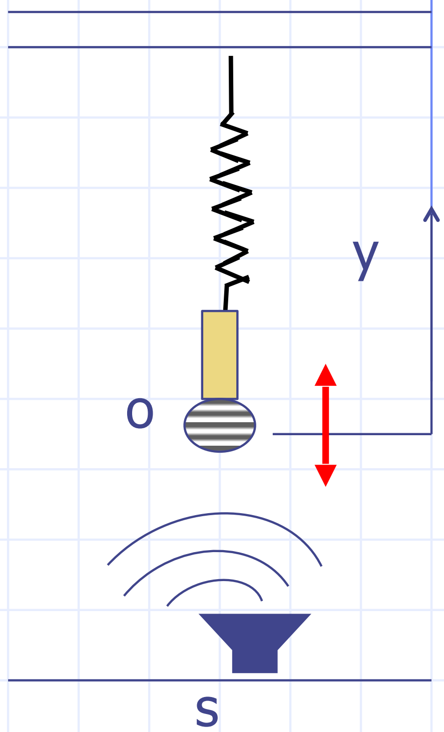
$$\lambda_o = v/f_o = (343\ \text{m/s})/(1008\ \text{Hz}) = \boxed{0.340\ \text{m}}$$

Compare to source wavelength

$$\lambda_s = v/f_s = (343\ \text{m/s})/(955\ \text{Hz}) = 0.359\ \text{m}$$

Example Problem

A microphone is attached to a spring that is suspended from a ceiling. Directly below on the floor is a stationary 440-Hz source of sound. The microphone vibrates up and down in SHM with a period of 2.0 s. The difference between the the maximum and minimum sound frequencies detected by the microphone is 2.1 Hz. Ignoring any sound reflections in the room, determine the amplitude of the SHM of the microphone.



<u>Solution:</u>

Given: $v_s=0$, $f_s=440$ Hz, $f_{o,max} - f_{o,\,min} = 2.1$ Hz $= \Delta f$, $T_{microphone}=T_m=2.0$ s (SHM), assume $v_{sound}=343$ m/s

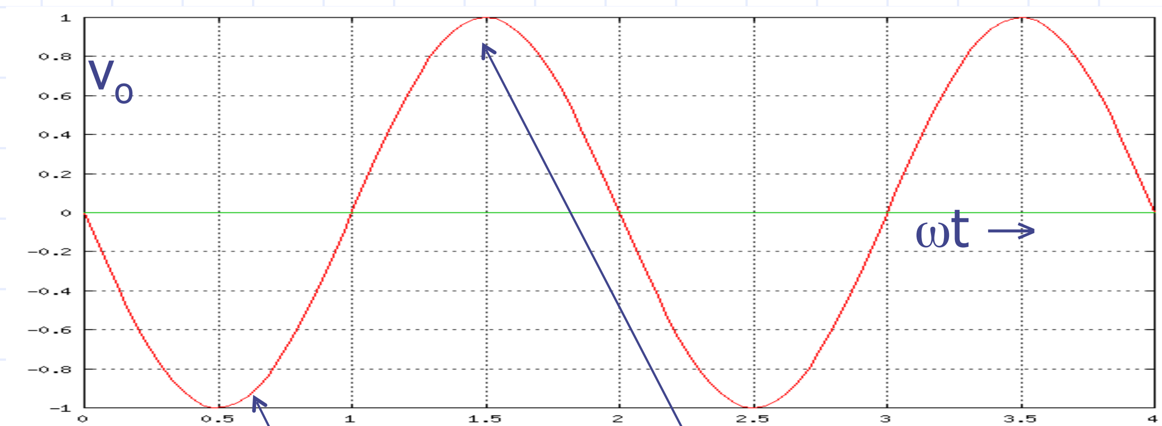
Observer is moving: $$f_o = f_s\left(1 \pm \frac{v_o}{v}\right)$$

Frequency of observer (microphone) is maximum when microphone has maximum velocity approaching the source, and minimum when microphone has maximum velocity receding from source. Since microphone is moving with SHM, its velocity is

$$\mathrm{v}_{\mathrm{o}} = -A\omega \sin(\omega\ \mathrm{t})$$

$$|\,\mathrm{v}_{\mathrm{o,\,max}}\,| = A\omega$$

$$= A(2\pi / T_m)$$



Microphone moving towards source

Microphone moving away from source

A, the SHM amplitude, is what we want to find.

$$f_{o,\max} = f_s\left(1 + \frac{\mathrm{v}_{o,\max}}{\mathrm{v}}\right) = f_s\left(1 + \frac{2\pi A}{\mathrm{v}T_m}\right)$$

$$f_{o,\min} = f_s\left(1 - \frac{\mathrm{v}_{o,\max}}{\mathrm{v}}\right) = f_s\left(1 - \frac{2\pi A}{\mathrm{v}T_m}\right)$$

$$\Delta f = f_{o,\max} - f_{o,\min} = 2.1\ \mathrm{Hz}$$

Measured by microphone

$$= f_s\left(1 + \frac{2\pi A}{\mathrm{v}T_m}\right) - f_s\left(1 - \frac{2\pi A}{\mathrm{v}T_m}\right) = f_s\left(1 + \frac{2\pi A}{\mathrm{v}T_m} - 1 + \frac{2\pi A}{\mathrm{v}T_m}\right)$$

$$= f_s\left(\frac{4\pi A}{\mathrm{v}T_m}\right) \Rightarrow A = \frac{\mathrm{v}T_m\Delta f}{4\pi f_s}$$

$$A = \frac{(343\ \mathrm{m/s})(2.0\ \mathrm{s})(2.1\ \mathrm{Hz})}{4\pi\,(440\ \mathrm{Hz})} = \boxed{0.26\ \mathrm{m}}$$

Microphone is oscillating with this amplitude