Chapter 21: Gauss's Law

- Finding the electric field for a continuous distribution of charge with can be tedious

$$\vec{E} = k_e \int \frac{dq}{r^2} \hat{r}$$

- If the charge distribution has a high degree of symmetry, there is a better way

- First we need to define and understand *Electric Flux*

Flux

- Consider water flowing through a hose with uniform velocity $\vec{v}$

- It passes out the end through a cross sectional area A
- Define the flux as the volume flow rate or volume of water per unit time or

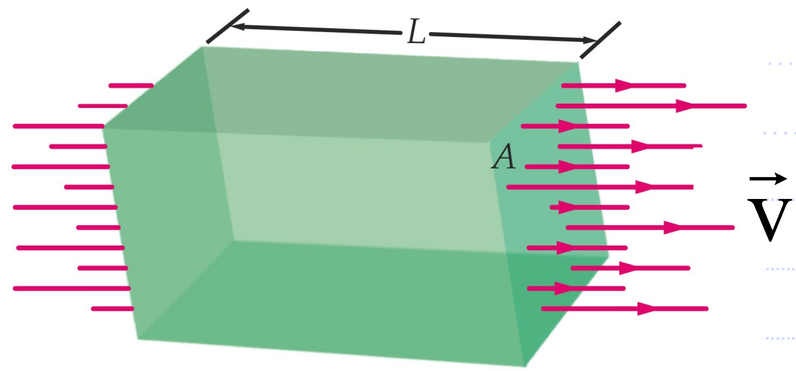
$$\Phi = \mathrm{v}A \qquad [\mathrm{m}^3/\mathrm{s}]$$

- If we define an area vector which has a direction perpendicular to the surface, but magnitude equal to the area then $\Phi = \vec{\mathrm{v}} \cdot \vec{A}$
- At the end of the hose, the area and velocity vectors are parallel, but we imagine we could rotate the area vector by θ, then by the definition of a scalar product

$$\Phi = \mathrm{v}A\cos\theta$$

$\Phi = \vec{v} \cdot \vec{A}$ is a scalar called the volume flux and we can understand it as the volume of water through an area per unit time

- Now abstractly assign a velocity vector to each water droplet in the hose
- The composite of all of the velocity vectors is a velocity field (a vector field)
- Therefore, Φ is the flux of the velocity field through the hose – rather than the actual flow of water



The Electric Flux

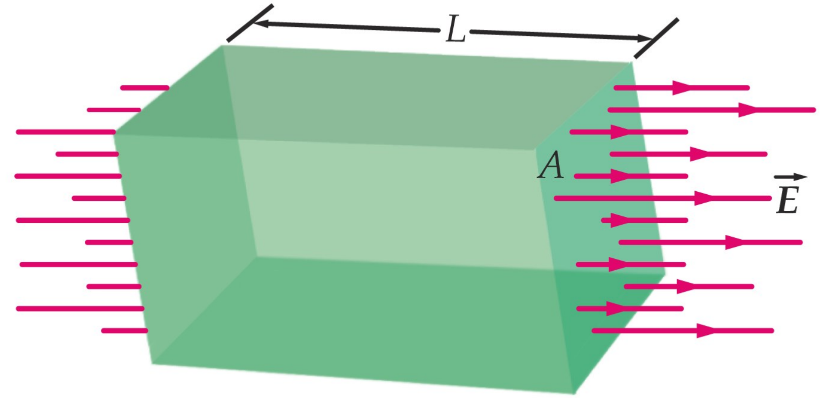
- Consider a uniform electric field passing through a loop of area A
- Since E is a vector field, we can define an Electric Flux

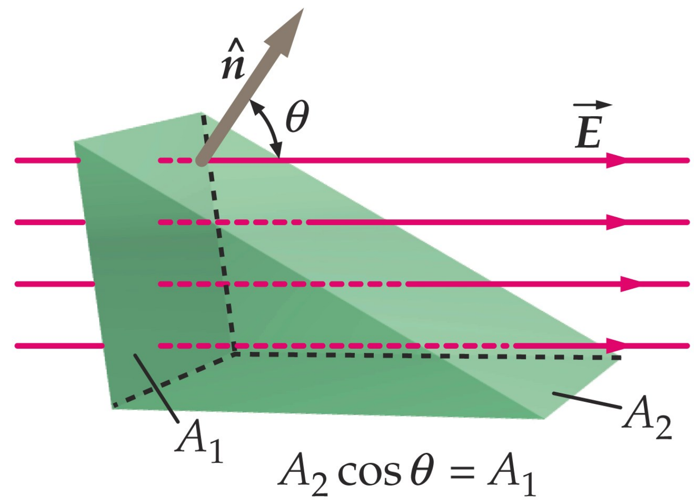
$$\Phi_E = EA \quad [\mathrm{Nm^2/C}]$$

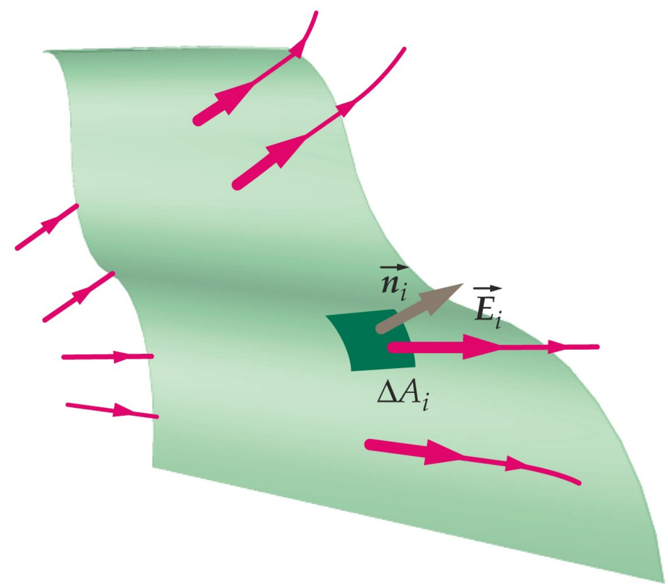
- Or since both E and A are vectors

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

- More often, E is not uniform, or the surface is not flat





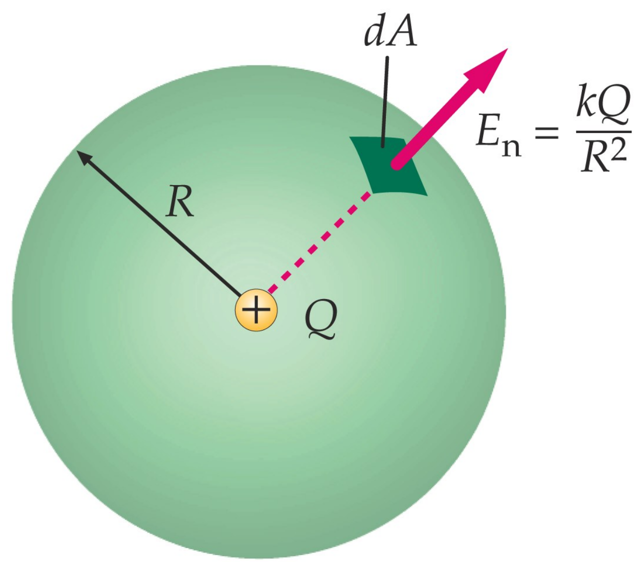


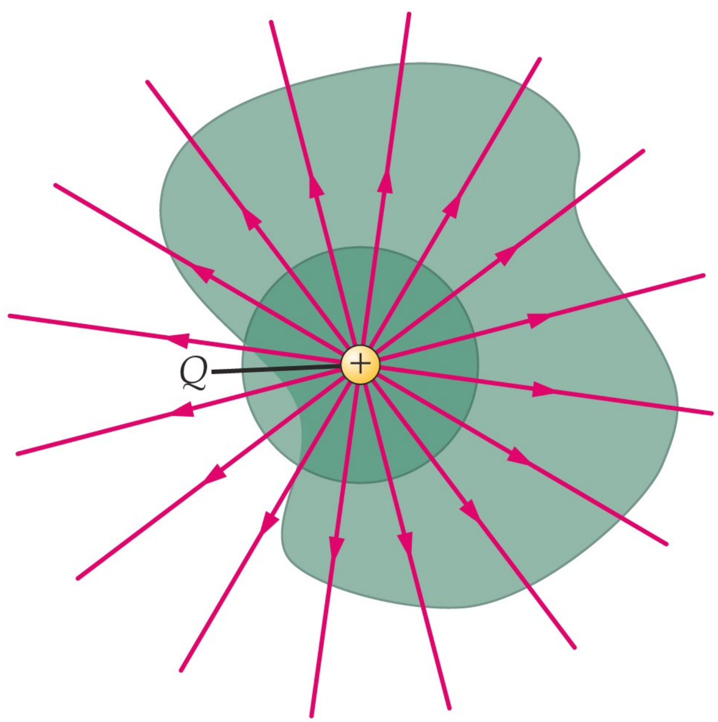
Example Problem

- A pyramid with horizontal square base, 6.0 m on each size, and a height of 4.00 , is placed in a vertical electric field of 52.0 N/C. Calculate the total electric flux through the pyramid’s four slanted surfaces.

Gauss's Law

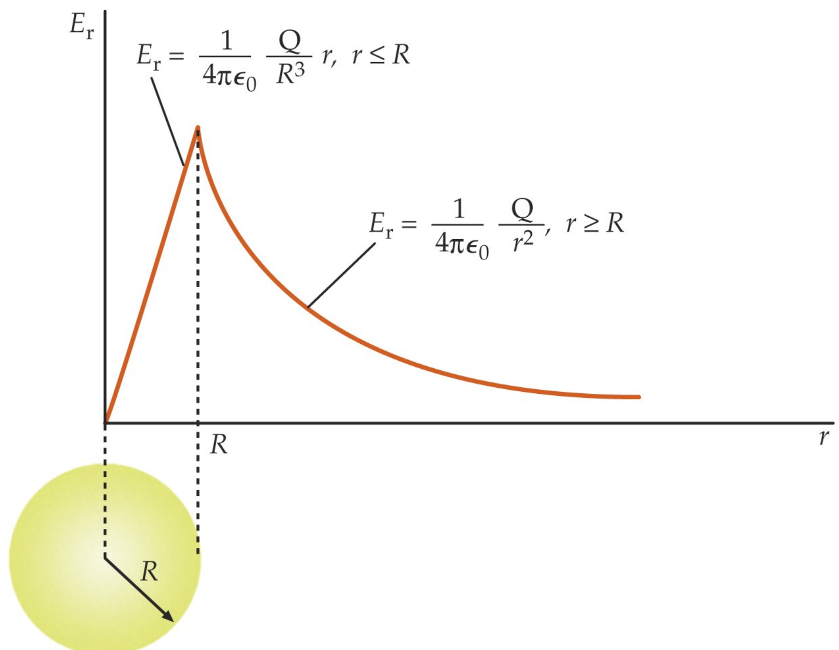
- Consider a point charge

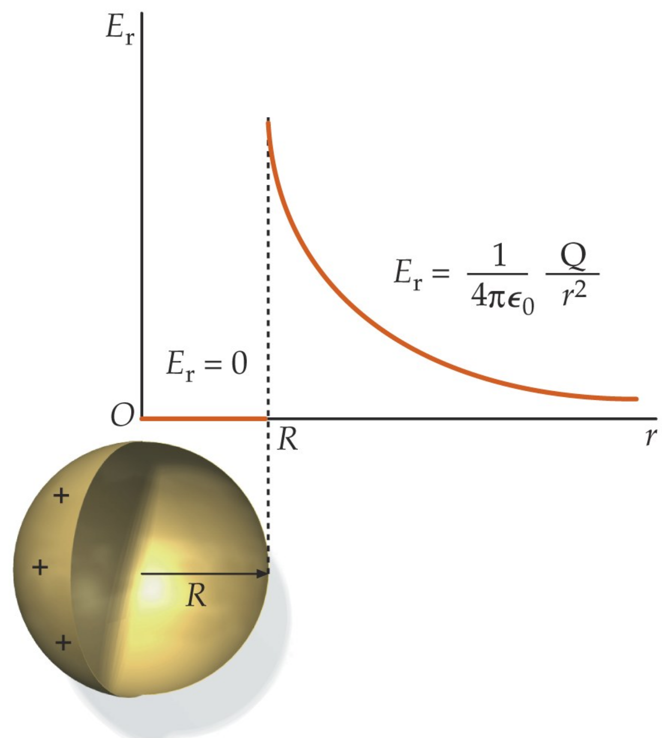




Applications of Gauss's Law

- Spherical distributions





Other distributions