KEY

PHYS 1312 Fall 2017 Test 3
Nov. 7, 2017

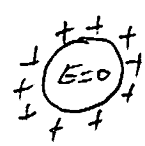
Name ____________ **Student ID** ____________ **Score** ______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

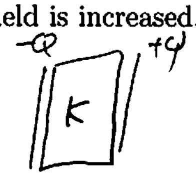
(a) The electric potential inside a conductor in steady-state is a constant.

True



(b) If an insulator (dielectric) is placed between the plates of a parallel-plate capacitor, its original electric field is increased.

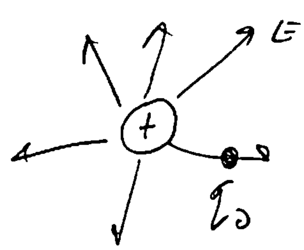
False



$$E_{dielectric} = \frac{E_{vacuum}}{K} \qquad K > 1$$

(c) A proton in a universe by itself will accelerate due to the action of its electric field.

False



$\vec{F} = q_0 \vec{E}$

↑ other charge

Problem 2. An proton is located at $< 0.8, 0.8, -0.7 >$ m. Your task is to find the electric field at location $< 0.6, 1.0, -0.6 >$ m due to the proton. (a) What is the source location? (b) What is the observation location? (c) What are $\vec{r}$, $|\vec{r}|$, and $\hat{r}$, (d) what is the magnitude of the electric field $k_e * q/r^2$, and (e) what is the electric field vector at the observation location? (30 points total)

a) $\vec{r}_{source} = \langle 0.8, 0.8, -0.7 \rangle$ m

b) $\vec{r}_{obs} = \langle 0.6, 1.0, -0.6 \rangle$ m

c) $\vec{r} = \vec{r}_{obs} - \vec{r}_{source} = \langle -0.2, 0.2, 0.1 \rangle$ m

$|\vec{r}| = \sqrt{x^2+y^2+z^2} = \sqrt{0.2^2+0.2^2+0.1^2} = \boxed{0.3 \text{ m}}$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle -0.2, 0.2, 0.1 \rangle \text{ m}}{0.3 \text{ m}} = \langle -0.667, 0.667, 0.333 \rangle$

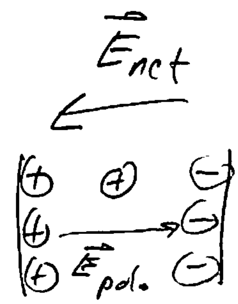
d) $|\vec{E}_e| = \frac{k_e e}{r^2} = \frac{(8.99 \times 10^9)(1.602 \times 10^{-19} \text{C})}{(0.3 \text{ m})^2}$

$= \boxed{1.600 \times 10^{-8} \text{ N/C}}$

e) $\vec{E}_e = |\vec{E}_e| \hat{r} = \boxed{\langle -1.07, 1.07, 0.534 \rangle \times 10^{-8} \frac{N}{C}}$

Problem 3. An electric field given by $\vec{E}_{\text{applied}} =< -5, 0, 0 >$ N/C is applied to an ionic solution. The ions immediately start to move with a drift speed of 1.0×10^{-3} m/s. (a) What is the mobility of the ions in the solution? (b) After a significant time has passed, say 1 sec, what is the electric field inside the solution and what is the electric field due to the induced dipole created by the ions? (c) If initially in part (a), the ions experience a force due to the applied electric field, explain why they move with a constant drift speed and are not accelerated. (30 points total)

a) $\bar{V} = u|\vec{E}| \rightarrow U = \frac{\bar{V}}{|\vec{E}|} = \frac{1.0\times10^{-3}\,m/s}{|-5\,N/C|} = \boxed{2\times10^{-4}\,m/s/(N/C)}$



b) $\vec{E}_{net} = \vec{E}_{applied} + \vec{E}_{polar} = \boxed{0 = \vec{E}_{net}}$

$\vec{E}_{polar} = -\vec{E}_{applied} = \boxed{\langle 5,0,0\rangle \frac{N}{C}}$

c) Initial Force on ions is $\vec{F} = q\vec{E}_{applied} = m\vec{a}$ by Newton's 2nd Law. But the solution has a high density. So ions immatate collide with a solution particle, then collide again, and again. So, on average the ions drift at a constant speed depending on the net Electric Field.

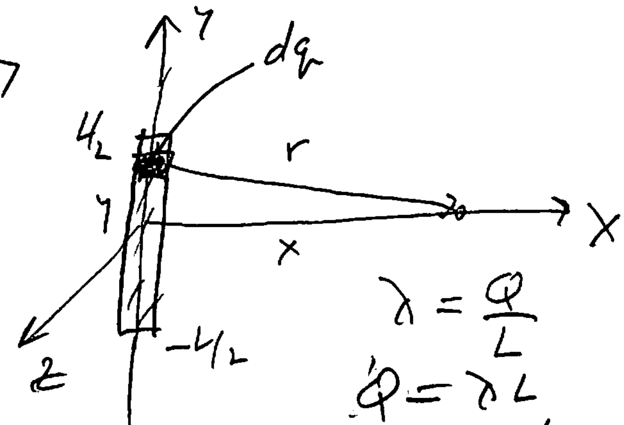
Problem 4. (a) Starting with the integral relation for the electric field due to a continuous charge distribution,

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r} \tag{1}$$

derive the electric field E_x for a rod of length L with uniformly distributed charge Q. Place the rod on the y-axis with its center at the origin. (b) For the same rod in part (a), determine its electric potential on the x-axis using the integral relation

$$V = k_e \int \frac{dq}{|\vec{r}|}. \tag{2}$$

(c) Given the result in part (b), find the electric field by taking the gradient of the electric potential. (30 points total)



a) $\vec{r}_{source} = \langle 0, y, 0\rangle$, $\vec{r}_{obs} = \langle X, 0, 0\rangle$

$\vec{r} = \vec{r}_{obs} - \vec{r}_{source} = \langle X, -y, 0\rangle$

$|\vec{r}| = \sqrt{x^2+y^2}$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle X, -y, 0\rangle}{(x^2+y^2)^{1/2}}$

$\lambda = \frac{Q}{L}$

$Q = \lambda L$

or $dq = \lambda\, dy$

$\vec{E} = k_e \int \frac{dq}{|\vec{r}|^2}\hat{r} = k_e \int \frac{\lambda\, dy}{(x^2+y^2)} \frac{\langle x, -y, 0\rangle}{(x^2+y^2)^{1/2}} = k_e \lambda \int \frac{dy\, \langle x, -y, 0\rangle}{(x^2+y^2)^{3/2}}$

$\vec{E}_y = 0$ by symmetry, $E_z = 0$

$E_x = +k_e \lambda x \int_{-L/2}^{L/2} \frac{dy}{(x^2+y^2)^{3/2}} = +\frac{k_e Q x}{L} \left[\frac{+y}{x^2(x^2+y^2)^{1/2}}\right]\Bigg|_{-L/2}^{L/2}$

$= \frac{k_e Q}{L\, x} \frac{1}{\left(x^2+\left(\frac{L}{2}\right)^2\right)^{1/2}} \left[\frac{L}{2} - \left(-\frac{L}{2}\right)\right] = \boxed{\frac{k_e Q}{x\sqrt{x^2+L^2/4}}}$

Prob 4b

$$V = k_e \int \frac{dq}{|\vec{r}|} = k_e \int_{-L/2}^{L/2} \frac{\lambda\, dy}{\sqrt{x^2+y^2}} = \frac{k_e Q}{L} \int_{-L/2}^{L/2} \frac{dy}{\sqrt{x^2+y^2}}$$

$$= \frac{k_e Q}{L} \Big[\ln\left(y + \sqrt{x^2+y^2}\right) \Big]_{-L/2}^{L/2}$$

$$= \frac{k_e Q}{L} \left[\ln\left(\sqrt{x^2+L^2/4} + \frac{L}{2}\right) - \ln\left(\sqrt{x^2+\frac{L^2}{4}} - \frac{L}{2}\right) \right]$$

$$\boxed{V = \frac{k_e Q}{L} \ln\left[\frac{\sqrt{x^2+L^2/4} + \frac{L}{2}}{\sqrt{x^2+L^2/4} - \frac{L}{2}}\right]}$$

c) $$E_x = -\frac{\partial V}{\partial x} = -\frac{k_e Q}{L}\left[\frac{1}{\sqrt{x^2+\frac{L^2}{4}}+\frac{L}{2}} \frac{d}{dx}\left(\sqrt{x^2+L^2/4}+\frac{L}{2}\right)\right.$$

$$\left. - \frac{1}{\sqrt{x^2+\frac{L^2}{4}}-\frac{L}{2}} \frac{d}{dx}\left(\sqrt{x^2+L^2/4}-\frac{L}{2}\right)\right] =$$

$$= -\frac{k_e Q}{L}\left[\frac{1}{\sqrt{x^2+\frac{L^2}{4}}+\frac{L}{2}} \cdot \frac{(1/2)\,2x}{\sqrt{x^2+\frac{L^2}{4}}} - \frac{1}{\sqrt{x^2+L^2/2}-\frac{L}{2}} \cdot \frac{(1/2)(2x)}{\sqrt{x^2+\frac{L^2}{4}}}\right]$$

$$= -\frac{k_e Q x}{L\sqrt{x^2+L^2/4}}\left[\frac{1}{\sqrt{x^2+L^2/4}+\frac{L}{2}} - \frac{1}{\sqrt{x^2+L^2/4}-\frac{L}{2}}\right] =$$

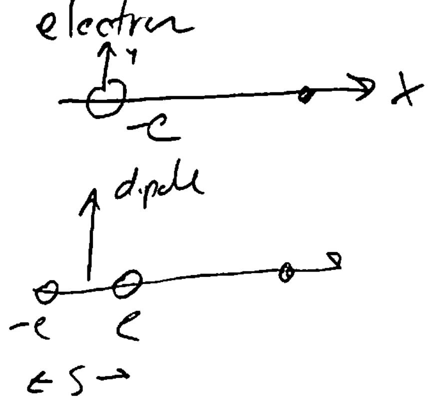
$$= \frac{-k_e Q x}{L\sqrt{x^2+L^2/4}}\left[\frac{\sqrt{x^2+L^2/4}-L/2-\left(\sqrt{x^2+L^2/4}+L/2\right)}{(x^2+L^2/4)-L^2/4}\right] = \frac{-k_e Q x}{L\sqrt{x^2+L^2/4}}\left[\frac{-L}{x^2}\right]$$

$$= \boxed{\frac{k_e Q}{x\sqrt{x^2+L^2/4}}}$$

agrees w/ part a

Bonus Problem. The electric dipole moment of the electron. (a) Obtain a relation for the ratio of the hypothetical electric field due to the dipole moment of an electron E_{dipole} to that of the electric field of the electron treated as a point charge $E_{\text{point charge}}$. Take the charge for both electric fields to be $q = -e$, the dipole length to be s, the dipole to be aligned on the x-axis and centered at the origin, the electron to be at the origin, and compute all electric fields at a distance $x >> s$. (b) Obtain a numerical value of the ratio of $s = 1 \times 10^{-40}$ m and the observation point $x = 0.5 \times 10^{-10}$ m. (5 points total)

a)



$$E_{x,\ \text{point charge}} = \frac{-k_e e}{x^2}$$

$$E_{x,\ \text{dipole}} = \frac{2k_e(-e)s}{x^3}$$

$$\frac{E_{\text{dipole}}}{E_{\text{point charge}}} = \frac{+2\cancel{k_e e}\,s/x^3}{+\cancel{k_e e}/x^2} = \boxed{\frac{2s}{x}} = \text{Ratio}$$

b) $$\text{Ratio} = \frac{2(1\times10^{-40}\,m)}{0.5\times10^{-10}\,m} = \boxed{4\times10^{-30}}$$

Very small