KEY

PHYS 1312 Fall 2017 Test 2
Sept. 28, 2017

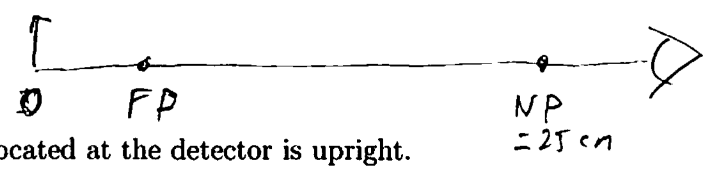
Name ____________________ **Student ID** ____________________ **Score** ______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

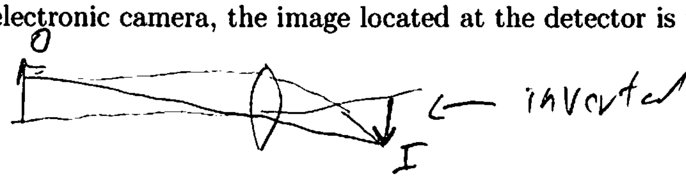
(a) A person who is far-sighted has a near-point which is more than 25 cm from their eye.

True



(b) In an electronic camera, the image located at the detector is upright.

False



(c) A Michelson interferometer which uses a laser of wavelength λ has a bright fringe at the center of its interference pattern. If one of the mirrors is moved a distance $\lambda/4$, a dark fringe will appear at the center.

$d = \frac{\lambda}{4}$ = distance mirror move, $\delta = 2d = \frac{\lambda}{2}$ = path length difference

→ bright → dark

True

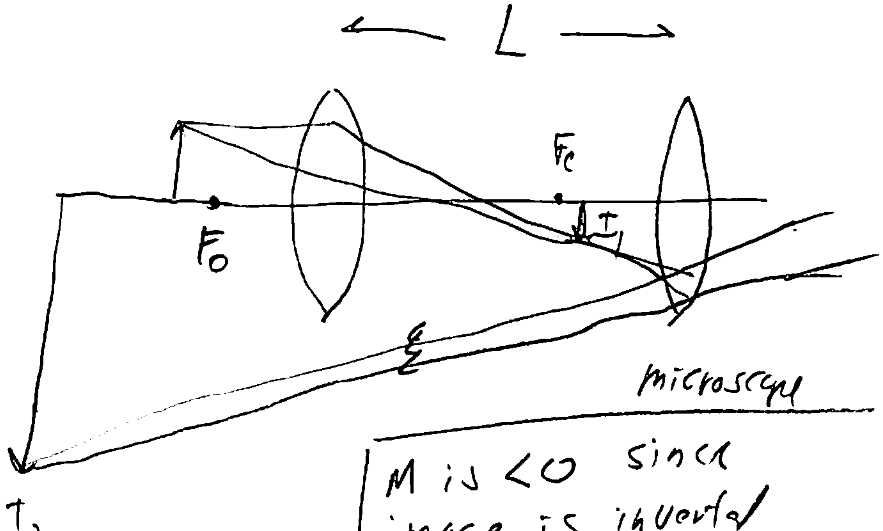
Problem 2. (a) The distance between the eyepiece and the objective lens of a compound microscope is 20.0 cm. If the focal length of the eyepiece is 3.00 cm, what focal length is needed for the objective lens to give a magnification of -500? Why is the magnification negative. (b). If we use the same eyepiece on a telescope, but use an objective lens with focal length 1.00 m, what is the resulting magnification? (30 points total)

a) $L = 20.0\,cm$

$f_e = 3.0\,cm$

$M = -500$

$f_o = ?$



$M = M_o m_e$

$= -\frac{L}{f_o}\frac{25\,cm}{f_e}$

solve for $f_o \Rightarrow f_o = \frac{-L}{M}\left(\frac{25\,cm}{f_e}\right) = -\frac{20.0}{(-500)}\left(\frac{25}{3}\right)$

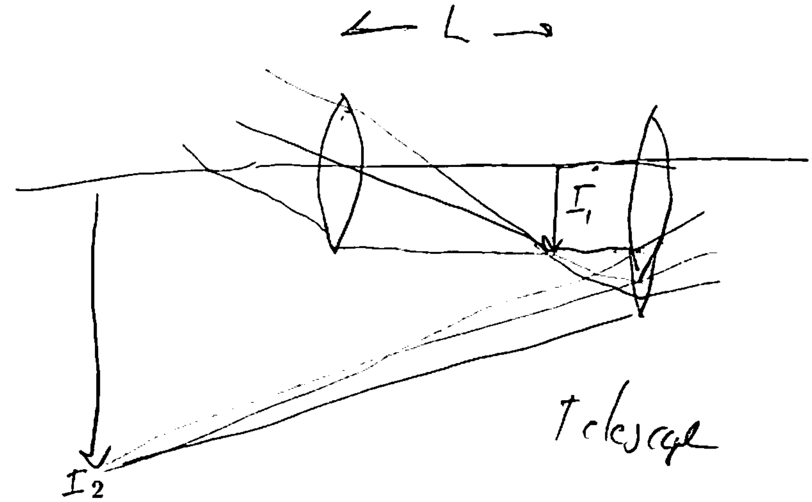
$= \boxed{0.333\,cm}$

M is < 0 since image is inverted

b) $m = -\frac{f_o}{f_e}$

$= -\frac{100\,cm}{3.0\,cm}$

$\Rightarrow \boxed{-33.3}$



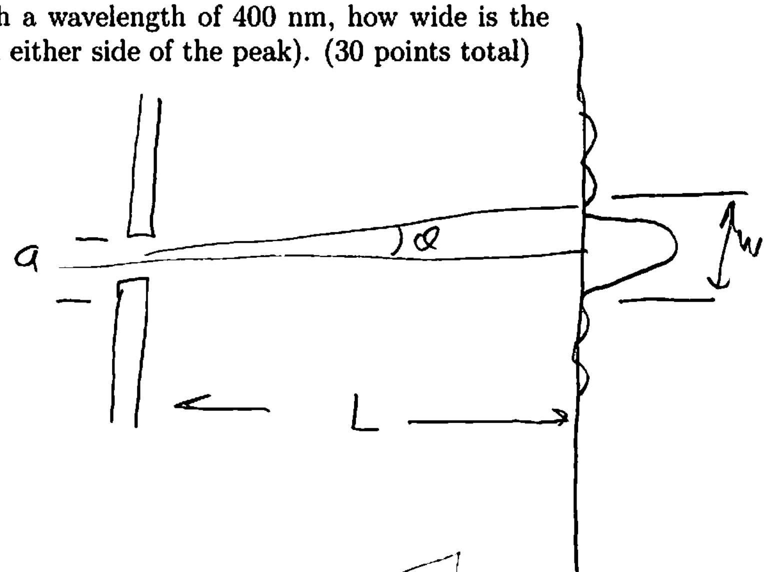
Problem 3. A single slit has a width of 1.00×10^4 nm and is located 1.00 m away from a screen. If the ~~slight~~ slit is illuminated by light with a wavelength of 400 nm, how wide is the central peak? (i.e., distance between minima on either side of the peak). (30 points total)

$a = 10{,}000$ nm

$L = 1.00$ m

$\lambda = 400$ nm

Find y, then $W = 2y$



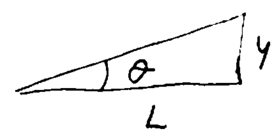
$a \sin\theta_{dark} = m\lambda$, $m = 1$

$\tan\theta = \frac{y}{L}$

$\theta_{dark} = \sin^{-1}\left(\frac{\lambda}{a}\right)$

$= \sin^{-1}\left(\frac{400}{10{,}000}\right)$

$= 4.001\times10^{-2}$ rad $= 2.29°$



$y = L\tan\theta = (1\text{ m})\tan(4.00\times10^{-2}\text{ rad}) = 4.003\times10^{-2}$ m

$W = 8.006\times10^{-2}$ m $=$ **8.0 cm**

Problem 4. A light source consists of two different wavelengths (549 and 551 nm). We want to select a diffraction grating that can resolve the two wavelenghts. (a) What is the required resolving power? (b) If we detect the second-order diffraction peaks, how many slits must the image fall on? (c) What are the angles θ of the diffraction peaks, in second-order, for the two wavelengths? Assume in (c) that all of the slits from part (b) are contained in 1.00 cm. (30 points total)

$\lambda_1 = 549\,nm, \quad \lambda_2 = 551\,nm \qquad \lambda = \frac{\lambda_1 + \lambda_2}{2} = 550\,nm$

$\Delta\lambda = 2\,nm$

a) $R = \frac{\lambda}{\Delta\lambda} = \frac{550}{2} = \boxed{275}$

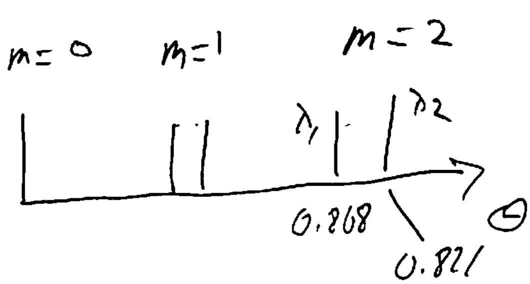
b) $R = Nm \Rightarrow N = \frac{R}{m} = \frac{275}{2} = 137.5 = \boxed{138 \text{ slits}}$

c) $d\sin\theta = m\lambda \qquad m = 2, \quad d = \frac{1\,cm}{N} = 7.246\times10^{-2}\,cm = 7.246\times10^{-5}\,m$

$\sin\theta = \frac{2\lambda}{d}$

or $\theta_1 = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) = \sin^{-1}\left(\frac{2\cdot 549\times10^{-9}}{7.246\times10^{-5}}\right) = \boxed{0.868^\circ}$

$\theta_2 = \sin^{-1}\left(\frac{2\cdot 551\times10^{-9}}{7.246\times10^{-5}}\right) = \boxed{0.871^\circ}$



Bonus Problem. Starting with the relation for the intensity of the single slit

$$I = I_{\max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \tag{1}$$

where

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \tag{2}$$

show that the first maximum (beyond the central maximum at $\theta = 0$) occurs for $\beta \sim 2.86\pi$. Why does it not occur at 3π? (Hint: take a derivative with respect to β). (5 points total)

$$\frac{d(I/I_{max})}{d\beta} = \frac{d}{d\beta}\left[\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2\right] = 2\frac{d}{d\beta}\left[\frac{\sin(\beta/2)}{\beta/2}\right] \cdot \left(\frac{\sin(\beta/2)}{\beta/2}\right)$$

$$= \left[\frac{2}{(\beta/2)}\frac{d}{d\beta}\left(\sin(\beta/2)\right) - \frac{2\sin(\beta/2)}{\beta^2/4}\frac{d}{d\beta}\left(\beta/2\right)\right]\frac{\sin(\beta/2)}{\beta/2}$$

$$= \left[\frac{4}{\beta}\frac{1}{2}\cos(\beta/2) - \frac{4\sin(\beta/2)}{\beta^2}\right]\frac{\sin\beta/2}{\beta/2} = 0$$

First bracket $= 0$ gives location of maxima; $\frac{\sin\beta/2}{\beta/2} = 0$ gives location of minima for $m = \pm 1, \pm 2, \ldots$

or $\boxed{\beta\cos(\beta/2) - 2\sin(\beta/2) = 0}$ ← if second term was not present, solution would result for $\beta = \pi, 3\pi, 5\pi$ instead,

solve by iteration, or substitute guess

$2.86\pi\cos(2.86\pi/2) - 2\sin(2.86\pi/2)$

$= -0.00082$ ← close → So, $\beta = 2.86\pi$, also $\beta = 4.918\pi$