KEY

PHYS 1312 Fall 2017 Test 1
Sept. 5, 2017

Name ____________ **Student ID** ____________ **Score** ______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) To apply the law of reflection for a ray of light, one does not need to know the index of refraction of the material the wave is propagating through.

$\theta_1 = \theta_1'$

$\boxed{\text{True}}$

(b) While you are stopped at an intersection, the frequency of the siren that you hear from a fire truck speeding away from you is higher than the rest frequency of the siren.

$\boxed{\text{False}}$

$f_o < f_s$

$f_o = \dfrac{f_s (1)}{(1 + v_s/v)}$

(c) It is possible for a reference frame to be moving away from another reference frame at a speed ~~getting~~ greater than that of light in a vacuum.

$\boxed{\text{True}}$

A reference frame has neither mass nor energy

Problem 2. A 3.97 m long pole stands vertically in a lake having a depth of 1.81 m. When the sun is 39.5° above the horizontal, determine the length of the pole's shadow on the bottom of the lake. Take the indices of refraction of air and water to be 1.00 and 1.33, respectively. (30 points total)

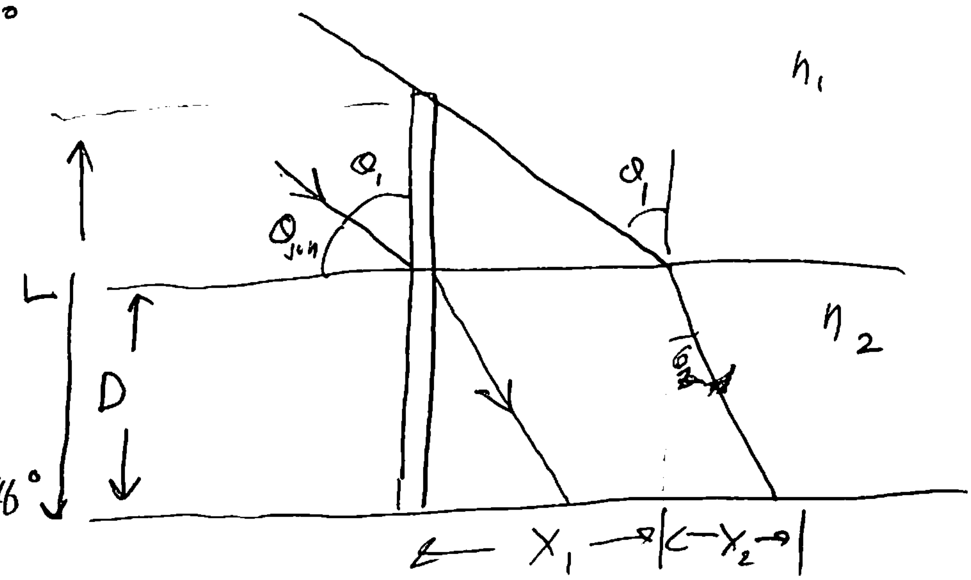
$\theta_1 = 90° - \theta_{sun} = 50.5°$

From Snell's Law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$

$= \sin^{-1}\left(\frac{1}{1.33}\sin 50.5°\right) = 35.46°$



From diagram

$\frac{X_2}{D} = \tan\theta_2 \Rightarrow X_2 = D\tan\theta_2 = 1.81\tan 35.46°$

$= 1.289\ m$

$X_1 = (L-D)\tan\theta_1 = (3.97-1.81)\tan 50.5°$

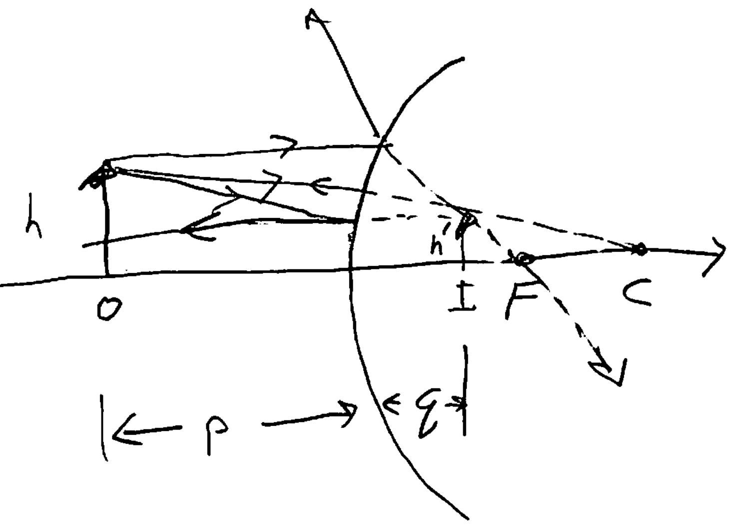
$= 2.62\ m$

$X = X_1 + X_2 = \boxed{3.91\,m}$

h p

Problem 3. An object of height 2.00 cm is 30.0 cm in front of a convex mirror. (a) Draw a ray diagram to locate the image. (b) Determine the location of the image q. (c) Determine the magnification of the image M. (d) Is the image real or virtual, upright or inverted? (30 points total)

a)



b) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}$

$$\boxed{q=\frac{1}{\frac{1}{f}-\frac{1}{30}}}$$

need f, but q will be negative since f must be negative

c) $M=-\frac{q}{p}=-\frac{1}{\left(\frac{1}{f}-\frac{1}{p}\right)p}=-\frac{1}{\frac{p}{f}-1}$

$=\frac{f}{f-p}=\boxed{\frac{f}{f-30}}=M>0$

d) Virtual and upright image

Problem 4. Two waves of equal amplitude A and frequency f propagate on a string of length L, but in opposite directions. Starting with the general wave equation, derive a relation for the supposition of the two waves (a standing wave) as a function of x and t. Then prove that nodes (or zeros) occur on the string for $x = n\lambda/2$, where $n = 0, 1, 2, \ldots$ (30 points total)

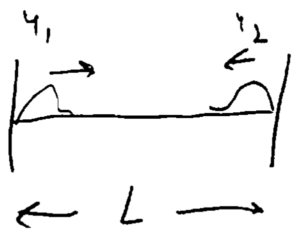
Start with general wave equations

$$y_1(x_1, t) = A_1 \sin(k_1 x \mp \omega_1 t + \phi_1) = A \sin(kx - \omega t)$$

$$y_2(x, t) = A_2 \sin(k_2 x \mp \omega_2 t + \phi_2) = A \sin(kx + \omega t)$$

$A_1 = A_2$, $f_1 = f_2 = f$, $\omega_1 = \omega_2 = \omega$, $k_1 = k_2 = k$

$\phi_1 = \phi_2 = 0 \equiv$ no phase shift difference



the sum is

$$y = y_1 + y_2 = A[\sin(\varphi_1) + \sin(\varphi_2)]$$

$$\varphi_1 - \varphi_2 = -2\omega t$$

$$\varphi_1 + \varphi_2 = 2kx$$

using the identity

$$\sin A + \sin B = 2\cos\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$

to give $\boxed{y(x,t) = 2A\cos(\omega t)\sin(kx)}$ since $\cos(-\omega t) = \cos\omega t$

We want to find the location of nodes (when $y = 0$) at any time t. So, we just consider the sine term.

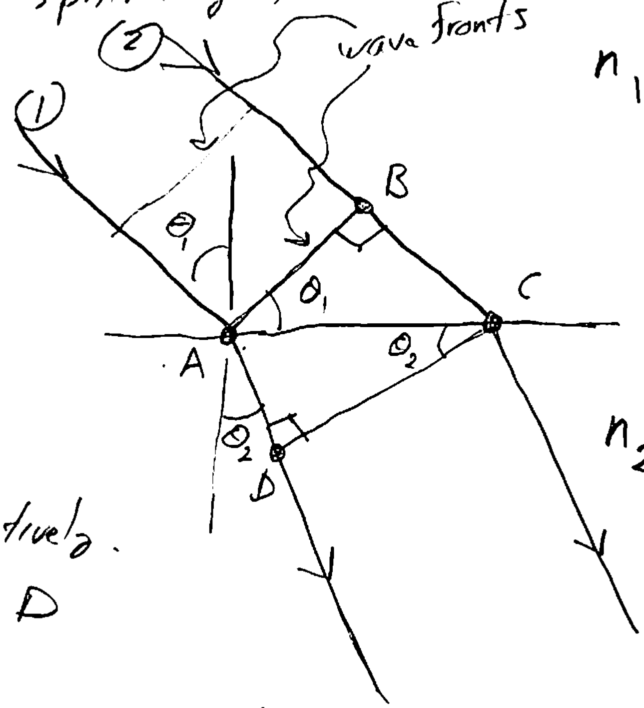
$y = 0 = \sin(kx)$ occurs when $kx = n\pi$

where $n = 0, 1, 2, 3, \ldots$ then $k = \frac{2\pi}{\lambda}$

$\frac{2\cancel{\pi}}{\lambda} x = n\cancel{\pi}$ or $\boxed{x = \frac{n\lambda}{2}}$ for $n = 0, 1, 2, 3, \ldots$

Bonus Problem. Using Huygens principle derive Snell's Law. (5 points total)

Huygen's principle sayings that we treat a wave front as composed of a large number of point sources which radiate spherically in all directions. So consider two rays propagating from medium with n_1 into a medium with n_2. Points A and B are points on the wave front. Rays 1 and 2 propagate from A and B, respectively for a time Δt and travel to D and D.



$\overline{BC} = V_1 \Delta t$ $\overline{AD} = V_2 \Delta t$

also $n_1 = \frac{c}{V_1} \Rightarrow V_1 = \frac{c}{n_1}$

$n_2 = \frac{c}{V_2} \Rightarrow V_2 = \frac{c}{n_2}$

$$\sin\theta_1 = \frac{\overline{BC}}{\overline{AC}} = \frac{c\Delta t/n_1}{\overline{AC}} \Rightarrow \overline{AC} = \frac{c\Delta t}{n_1 \sin\theta_1}$$

equal

$$\sin\theta_2 = \frac{\overline{AD}}{\overline{AC}} = \frac{c\Delta t/n_2}{\overline{AC}} \Rightarrow \overline{AC} = \frac{c\Delta t}{n_2 \sin\theta_2}$$

$$\therefore \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$