KEY

PHYS 1312 Fall 2016 Test 3

Nov. 15, 2016

Name ______________________________ **Student ID** ______________________________ **Score** ________

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) The electric field between the plates of a particular parallel-plate capacitor depends on the distance to the observation point.

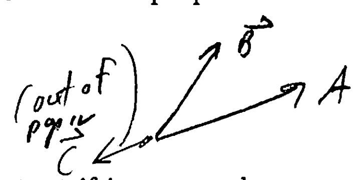
False. $E = \frac{Q/A}{\epsilon_0}$

(b) In steady-state, the electric field inside of an insulator may be non-zero.

True (zero inside of a conductor)

(c) The cross product $\vec{A} \times \vec{B}$ results in a vector $\vec{C}$ which is perpendicular to the plane defined by $\vec{A}$ and $\vec{B}$.

True



(d) The electric potential of a charge remains constant if it moves along a equipotential curve.

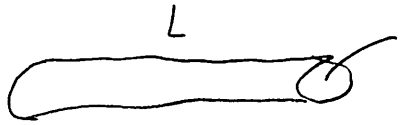
True

means does not change!

Problem 2. In a particular cube-shaped metal, the mobility of the mobile electrons is 0.0077 (m/s)/(N/C). At a particular moment, the net electric field everywhere inside the metal cube is $\vec{E} =< 0.053, 0, 0 >$ N/C. (a) What is the average drift speed of the mobile electrons at this moment in time? (b). If we wait a sufficiently long time (a few seconds, say), what would be the new drift speed and electric field instead the cube? Explain. (c) If we now use the same material to make a long wire of radius 1.0 mm, place it on the x-axis and apply the same electric field, what is the current I in the wire? Take the density of mobile electrons to be 9×10^{28} m^{-3}. (30 points total)

a) $\bar{v} = uE = \left(0.0077 \frac{m/s}{N/C}\right)\left(0.053\ N/c\right) = \boxed{4.1 \times 10^{-4}\ m/s}$

b) After a short time, the system reaches steady-state. For a conductor $\boxed{E = 0 \text{ and } \bar{v} = 0}$

c) 

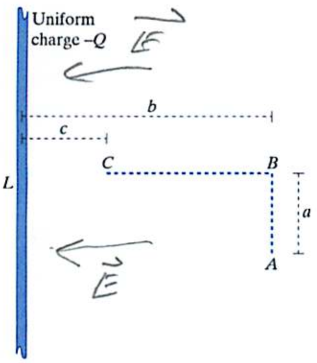
$A = \pi r^2$

$r = 1 \times 10^{-3}\ m$

$n = 9 \times 10^{28}\ m^{-3}$

$$I = |q| n A \bar{v} = (1.602 \times 10^{-19}\ C)(9 \times 10^{28}\ m^{-3})\,\pi\,(1 \times 10^{-3}\ m)^2 \times \left(4.1 \times 10^{-4}\ \frac{m}{s}\right) = \boxed{18.6\ A}$$

Problem 3. The long rod shown in the figure has a length L and carries a uniform charge $-Q$. Calculate the potential difference $V_A - V_C$ in terms of Q, c, and b. All of the distances are small compared to L. Explain your work. (30 points total)



$$\Delta V_{AC} = \Delta V_{AB} + \Delta V_{BC}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

The electric field for a rod of length L is

$$\vec{E}_{rod} = \frac{k_e Q}{r\sqrt{r^2 + (L/2)^2}} \hat{r}$$

points A and B are both r from the rod and therefore have the same potential

$\Rightarrow \Delta V_{AB} = 0$

since $L \gg c, b$
then $L \gg r$

$$\vec{E}_{rod} \approx \frac{k_e Q}{r L/2} = \frac{2 k_e Q/L}{r}$$

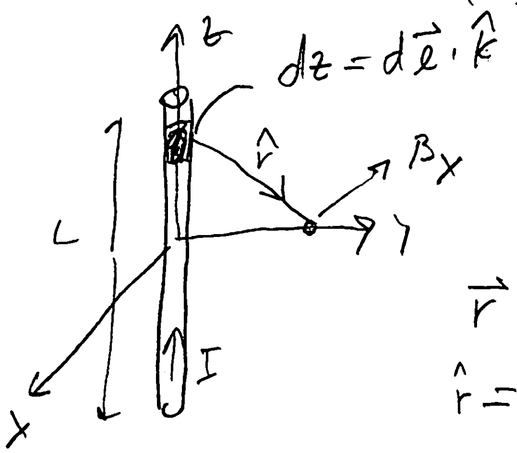
$$\Delta V_{BC} = -\int_c^b \vec{E}_{rod} \cdot d\vec{r} = -\int_c^b \frac{2k_e Q/L}{r} dr =$$

$$= -2k_e Q/L \int_c^b \frac{dr}{r} = \frac{-2k_e Q}{L} \ln \Big|_c^b = \boxed{-\frac{2k_e Q}{L} \ln(b/c)}$$

Problem 4. Consider a straight conducting wire of length L placed along the z-axis with its center at the origin. If conventional current I flows in the positive z direction, determine the magnetic field $\vec{B}$ (magnitude and direction) at $r =< 0, y, 0 >$. Start with the Biot-Savart law for a current in a segment of wire given by

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{\ell}\times\hat{r}}{r^2} \tag{1}$$

and show all work. (30 points total)



$\vec{r}_{obs} = \langle 0, y, 0\rangle$

$\vec{r}_{source} = \langle 0, 0, z\rangle$

$\vec{r} = \langle 0, y, -z\rangle$, $|\vec{r}| = \sqrt{y^2+z^2}$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle 0, y, -z\rangle}{\sqrt{y^2+z^2}}$, $\Delta\vec{\ell} = \Delta z\langle 0,0,1\rangle$

$\langle 0,0,1\rangle \times \langle 0, y, -z\rangle$
$= \langle -y, 0, 0\rangle$

$$\Delta\vec{B} = \frac{\mu_0}{4\pi} I \frac{\Delta z\langle 0,0,1\rangle \times}{y^2+z^2}\frac{\langle 0, y, -z\rangle}{\sqrt{y^2+z^2}}$$

$$= \frac{\mu_0}{4\pi}\frac{I\Delta z}{(y^2+z^2)^{3/2}}\langle -y, 0, 0\rangle$$

or

$$\int d\vec{B} = \frac{\mu_0}{4\pi} I\langle -y, 0, 0\rangle \int_{-L/2}^{L/2}\frac{dz}{(y^2+z^2)^{3/2}}$$

or

$$B_x = -\frac{\mu_0}{4\pi} I y\left[\frac{z}{y^2\sqrt{y^2+z^2}}\right]_{-L/2}^{L/2}$$

$$B_x = -\frac{\mu_0}{4\pi}\frac{I}{y}\frac{1}{\sqrt{y^2+(L/2)^2}}\left[\frac{L}{2} - -\frac{L}{2}\right]$$

$$\boxed{B_x = -\frac{\mu_0}{4\pi}\frac{LI}{y\sqrt{y^2+L^2/4}}}$$

$B_y = 0$, $B_z = 0$

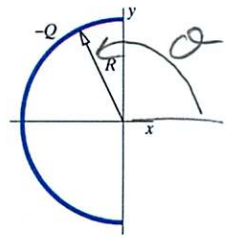
$$\vec{P}_f = \left\langle \frac{2k_e Q}{\pi R^2}, 0, 0\right\rangle(-2e)\Delta t$$

or $\boxed{P_x = +\frac{4k_e e|Q|\Delta t}{\pi R^2}}$ to positive x since $Q<1$.

Note for small Δt since for large ΔX, E_x will change.

Bonus Problem. Consider a thin plastic rod bent into a semicircular arc of radius R with center at the origin as shown in the figure. The rod carries a uniform charge of $-Q$. An ion with charge $-2e$ and mass M is placed at rest at the origin. After a very short time Δt the ion has moved a very short distance, but acquired some momentum $\vec{p}$. Calculate $\vec{p}$. (5 points total)

$$\vec{P}_i = 0, \quad \vec{P}_f = \vec{P}_i + \vec{F}_{net}\Delta t = 0 + \vec{E}(-2e)\Delta t$$



Find $\vec{E}$ at the origin, start with electric field for continuous charge distribution $\vec{E} = k_e\int \frac{dq}{r^2}\hat{r}$

$\vec{r}_{obs} = \langle 0, 0, 0\rangle$, $\vec{r}_{source} = \langle R\cos\theta, R\sin\theta, 0\rangle$

$\vec{r} = \langle -R\cos\theta, -R\sin\theta, 0\rangle$

$|\vec{r}| = R$, $\hat{r} = \langle -\cos\theta, -\sin\theta, 0\rangle$

$\vec{E}$

$-2e$ $\vec{F}_{net} = \vec{E}q = -\vec{E}2e$

$\lambda \equiv$ linear charge density $= \frac{Q}{\ell}$ or

$dq = \lambda\, d\ell = \lambda R \Delta\theta$

$d\ell$ is the path length (arclength) → from $\theta = \pi/2$ to $3\pi/2$

$\ell = R\pi$

$\lambda = \frac{Q}{\pi R}$

Then
$$\vec{E} = k_e\int_{\pi/2}^{3\pi/2} \frac{\lambda R\, d\theta}{R^2}\langle -\cos\theta, -\sin\theta, 0\rangle$$

$$E_x = -\frac{k_e\lambda}{R}\int_{\pi/2}^{3\pi/2}\cos\theta\, d\theta$$
$$= -\frac{k_e\lambda}{R}\sin\theta\Big|_{\pi/2}^{3\pi/2} = -\frac{k_e\lambda}{R}[-1-1]$$
$$= \frac{2k_e\lambda}{R} = \frac{2k_e Q/\pi R}{R} = \boxed{\frac{2k_e Q}{\pi R^2}}$$

$$E_y = -\frac{k_e\lambda}{R}\int_{\pi/2}^{3\pi/2}\sin\theta\, d\theta$$
$$= \frac{k_e\lambda}{R}\cos\theta\Big|_{\pi/2}^{3\pi/2} = \frac{k_e\lambda}{R}[0]$$
$$= 0$$

[to the top]

y-component cancel but check