KEY

PHYS 1312 Fall 2016 Test 2
Oct. 6, 2016

Name ____________________ **Student ID** ____________________ **Score** ______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) For an electric dipole of length s, the magnitude of the resulting electric field is proportional to r^{-3} when $r >> s$.

True $\quad |\vec{E}_{\parallel}| = \frac{2k_e p}{r^3}$

(b) The electron has a radius of 0.529×10^{-10} m. ← radius of electron orbit in Hydrogen

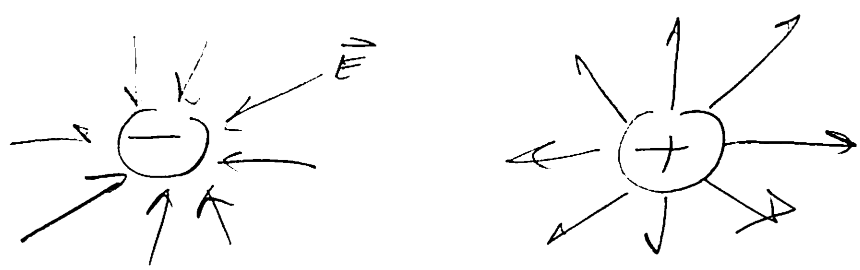
False, electron is a "true point" particle

(c) Since information is massless, it can be transferred at speeds greater than that of light.

False, nothing can travel faster than c

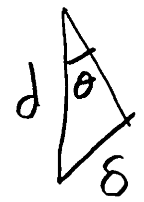
(d) Electric field lines point toward a negative charge.

True



Problem 2. If monochromatic light of wavelength λ illuminates a double slit with slit separation d, derive a relation for the distance y of the interference minima from the central maximum if the interference pattern is displayed on a screen a horizontal distance L away from the slits. (a) Assume the small angle approximation. (b) Repeat, but do not assume the small angle approximation. (c) Make a sketch of the arrangement and interference pattern marking the locations of minima for a few values of m. (30 points total)

a)



$\delta = r_2 - r_1 \equiv$ pathlength difference

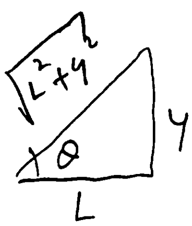
$= d\sin\theta$

For destructive interference

$\delta = \left(m+\frac{1}{2}\right)\lambda \;, m=0, \pm 1, \pm 2$

$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda \simeq d\theta$ ← small θ

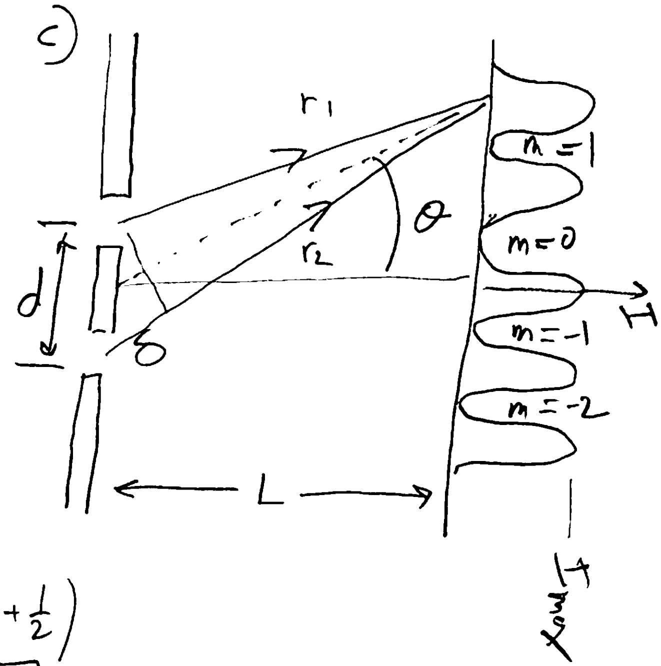
or $\theta = \left(m+\frac{1}{2}\right)\frac{\lambda}{d}$



$y = L\tan\theta \simeq L\theta = \frac{L\lambda}{d}\left(m+\frac{1}{2}\right)$

or $\boxed{y = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right), m = 0, \pm 1, \pm 2, .}$

c)



b) $\sin\theta = \left(m+\frac{1}{2}\right)\frac{\lambda}{d} = \frac{y}{\sqrt{L^2+y^2}}$

$y = \sqrt{L^2+y^2}\,\frac{\lambda}{d}\left(m+\frac{1}{2}\right)$

$y^2 = (L^2+y^2)\left[\frac{\lambda}{d}\left(m+\frac{1}{2}\right)\right]^2$

$y^2\left\{1-\left[\frac{\lambda}{d}\left(m+\frac{1}{2}\right)\right]^2\right\} = L^2\left[\frac{\lambda}{d}\left(m+\frac{1}{2}\right)\right]^2$

$y = \frac{\frac{L\lambda}{d}\left(m+\frac{1}{2}\right)}{\sqrt{1-\left[\frac{\lambda}{d}\left(m+\frac{1}{2}\right)\right]^2}}$

$\boxed{y = \frac{\lambda L\left(m+\frac{1}{2}\right)}{\sqrt{d^2-\lambda^2\left(m+\frac{1}{2}\right)^2}}}$

λ θ $m=1$

Problem 3. For an optics project you need to design a diffraction grating that will disperse the visible spectrum (400-700 nm) over 30.0° in first order. (a) How many lines per millimeter does your grating need? (b) What is the first-order diffraction angle of light from a sodium lamp (λ = 589 nm). (c) What is the maximum order (m) that can be detected with your grating for λ = 400 nm and 700 nm? (d) What is the chromatic resolving power of your grating in second-order if the sodium lamp illuminates a 2.0-mm long region of the grating? (30 points total)

a) $d \sin\theta_{bright} = m\lambda \Rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{(1)(400\times10^{-9}\,m)}{\sin 30^\circ}$

$= 8\times10^{-7}\,m = 8\times10^{-4}\,mm$

$\frac{\#\,slits}{mm} = \frac{1}{d} = \frac{1}{8\times10^{-4}\,mm} = 1260\,slits/mm$

for $\lambda = 700\,nm$

$d = 1.4\times10^{-3}\,mm$

$\Rightarrow \boxed{714\,slits/mm}$

b) $\theta_{bright} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(1)(589\times10^{-9}\,m)}{1.4\times10^{-6}\,m}\right)$

$= \boxed{24.8^\circ}$

c) $\sin\theta = \frac{m\lambda}{d} \le 1$

$m \le \frac{d}{\lambda}$

400nm $\rightarrow m_{max} = \frac{1.4\times10^{-6}\,m}{400\times10^{-9}\,m} = \boxed{3.5}$

700nm $\rightarrow m_{max} = \frac{1.4\times10^{-6}\,m}{700\times10^{-9}\,m} = \boxed{2}$

d) $R = Nm \equiv$ chromatic resolving power, $m=2$

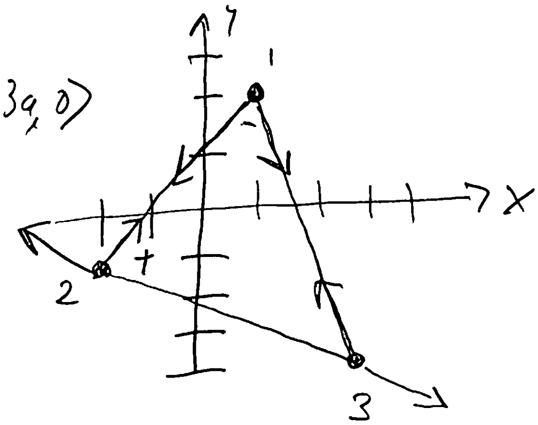
$\lambda = 589\,nm$

$N =$ number of illuminated slits $= (714\,slits/mm)(2\,mm) = 1428$ slits

$R = (1428)(2) = \boxed{2856}$

Problem 4. Three point charges are located in space at $\vec{r}_1 =< a, b, 0 >$, $\vec{r}_2 =< -b, -a, 0 >$ and $\vec{r}_3 =< c, -d, 0 >$ with charges $q_1 = -q$, $q_2 = 2q$, and $q_3 = 3q$. Take $a = 1\mu$m, $b = 2a$, $c = 3a$, $d = 4a$, and $q = 1 \times 10^{-10}$ C. (a) If $m_1 = 2m_2 = 3m_3 = 1 \times 10^{-6}$ kg, determine the acceleration of each charge. (b) Make a sketch of the electric field lines due this initial configuration of the charges. (30 points total)

a)



$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = \langle a, 2a, 0\rangle - \langle -2a, -a, 0\rangle = \langle 3a, 3a, 0\rangle$$

$$|\vec{r}_{12}| = \sqrt{x^2 + y^2} = \sqrt{9a^2 + 9a^2} = 3a\sqrt{2}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{\langle 3a, 3a, 0\rangle}{3a\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\rangle$$

$$\vec{r}_{13} = \vec{r}_1 - \vec{r}_3 = \langle a, 2a, 0\rangle - \langle 3a, -4a, 0\rangle = \langle -2a, 6a, 0\rangle$$

$$|\vec{r}_{13}| = \sqrt{4a^2 + 36a^2} = a\sqrt{4\cdot 5\cdot 2} = 2a\sqrt{10}$$

$$\hat{r}_{13} = \frac{\vec{r}_{13}}{|\vec{r}_{13}|} = \frac{\langle -2a, 6a, 0\rangle}{2a\sqrt{10}} = \left\langle -\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, 0\right\rangle$$

$$\vec{r}_{23} = \vec{r}_2 - \vec{r}_3 = \langle -2a, -a, 0\rangle - \langle 3a, -4a, 0\rangle = \langle -5a, 3a, 0\rangle$$

$$|\vec{r}_{23}| = \sqrt{25a^2 + 9a^2} = a\sqrt{34}, \quad \hat{r}_{23} = \frac{\vec{r}_{23}}{|\vec{r}_{23}|} = \frac{\langle -5a, 3a, 0\rangle}{a\sqrt{34}} = \left\langle -\frac{5}{\sqrt{34}}, \frac{3}{\sqrt{34}}, 0\right\rangle$$

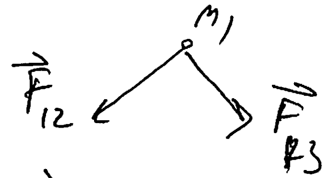
Force magnitudes

$$|\vec{F}_{12}| = k_e \frac{q_1 q_2}{|r_{12}^2|} = \frac{(8.99\times10^{9})(+1\times10^{-10}\,\mathrm{C})(2\times10^{-10}\,\mathrm{C})}{18(1\times10^{-6}\,\mathrm{m})^2} = 9.99\ \mathrm{N}$$

$$|\vec{F}_{13}| = k_e \frac{q_1 q_3}{|r_{13}^2|} = \frac{(8.99\times10^{9})(1\times10^{-10}\,\mathrm{C})(3\times10^{-10}\,\mathrm{C})}{40(1\times10^{-6}\,\mathrm{m})^2} = 6.743\ \mathrm{N}$$

Problem 4 (con't'd)

$$|\vec{F}_{23}| = \frac{k_c q_2 q_3}{|r_{23}^2|} = \frac{(8.99\times10^{9})(2\times10^{-10}c)(3\times10^{-10}c)}{34\,(1\times10^{-6}m)^2}$$



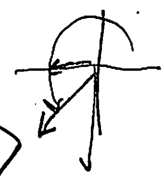
$$= 15.86\ N$$

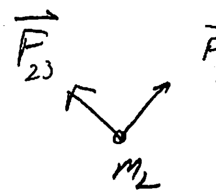
mass m_1 $\quad \Sigma\vec{F}_1 = m_1\vec{a}_1 \Rightarrow \vec{a}_1 = \frac{\Sigma\vec{F}_1}{m_1} = \frac{|\vec{F}_{12}|(-\hat{r}_{12}) + |\vec{F}_{13}|(-\hat{r}_{13})}{m_1}$

$$\vec{a}_1 = \left[\frac{9.99N}{1\times10^{-6}kg}\langle -\tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}, 0\rangle + \frac{6.743N}{1\times10^{-6}kg}\langle +\tfrac{1}{\sqrt{10}}, -\tfrac{3}{\sqrt{10}}, 0\rangle\right]$$

$$= \langle -7.064\times10^{6}N, -7.064\times10^{6}N, 0\rangle + \langle 2.132\times10^{6}, -6.397\times10^{6}, 0\rangle$$

$$\boxed{\vec{a}_1 = \langle -4.932\times10^{6}, -1.346\times10^{7}\rangle \frac{m}{s^2}}$$ or $14.3\times10^{6}\frac{m}{s^2}$ @ $250°$





$$\vec{a}_2 = \frac{|\vec{F}_{21}|}{m_2}\langle \tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\rangle + \frac{|\vec{F}_{23}|}{m_2}\langle -\tfrac{5}{\sqrt{34}}, \tfrac{3}{\sqrt{34}}, 0\rangle$$

$$= \frac{9.99}{2\times10^{-6}}\langle \tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\rangle + \frac{15.86}{2\times10^{-6}}\langle -\tfrac{5}{\sqrt{34}}, \tfrac{3}{\sqrt{34}}, 0\rangle$$

$$\boxed{\vec{a}_2 = \langle -3.268\times10^{6}, 7.610\times10^{6}, 0\rangle \frac{m}{s^2}}$$

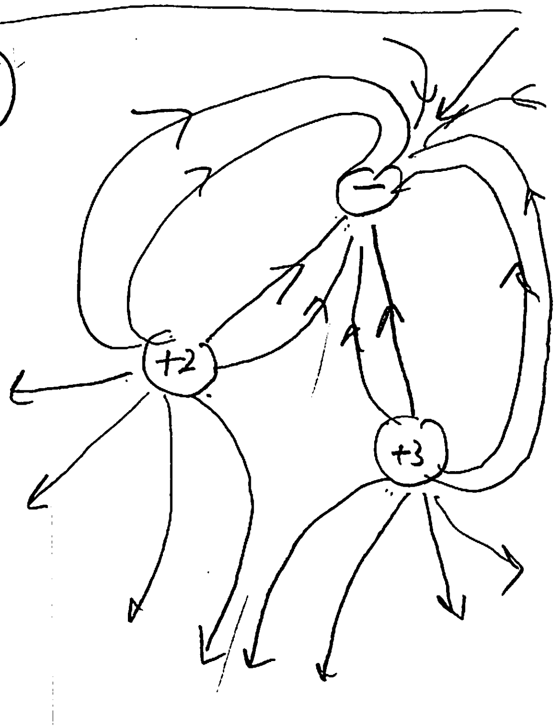
or $\vec{a}_2 = 8.30\times10^{6}\frac{m}{s^2}$ @ $113°$

and

$$\boxed{\vec{a}_3 = \langle 3.822\times10^{6}, -0.588\times10^{6}, 0\rangle \frac{m}{s^2}}$$

or $\vec{a}_3 = 3.87\times10^{6}\frac{m}{s}$ @ $-8.75°$

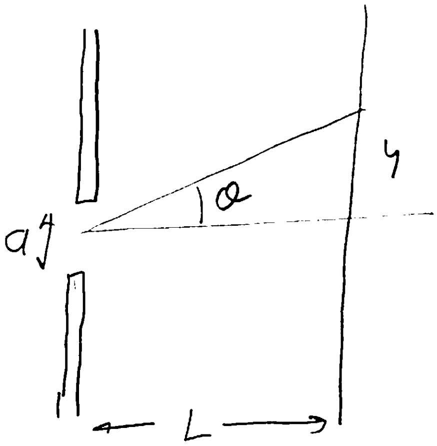
b)



Bonus Problem. For a single slit, locate the position (angle θ and height y) of the first diffraction maximum (not the central maximum) if the slit has a width a and the interference pattern is displayed on a screen a distance L away from the slit. Use the relation for the intensity given by

$$I = I_{\text{max}} \frac{\sin^2(\beta/2)}{(\beta/2)^2} \tag{1}$$

where $\beta = (2\pi a/\lambda)\sin\theta$. (5 points total)



To find maxima (and minima) take a derivative of I w.r.t β

$$\frac{dI}{d\beta} = I_{max}\frac{d}{d\beta}\left[\frac{\sin\beta/2}{\beta/2}\right]^2$$

$$= 2I_{max}\left[\frac{\sin\beta/2}{\beta/2}\right]\frac{d}{d\beta}\left[\frac{\sin\beta/2}{\beta/2}\right]$$

$$= 2I_{max}\left[\frac{\sin\beta/2}{\beta/2}\right]\left[\frac{1}{\beta/2}\frac{d}{d\beta}(\sin\beta/2) - \frac{\sin\beta/2}{\beta^2/4}\frac{d(\beta/2)}{d\beta}\right]$$

$$= 2I_{max}\underbrace{\left[\frac{\sin\beta/2}{\beta/2}\right]}_{=0}\underbrace{\left[\frac{1/2}{\beta/2}\cos(\beta/2) - \frac{1}{2}\frac{\sin\beta/2}{\beta^2/4}\right]}_{=0} = 0$$

Find roots of this equation

or $\boxed{\beta\cos(\beta/2) - 2\sin(\beta/2) = 0}$ $\Rightarrow$ expect $\frac{\beta}{2} \simeq (m+\frac{1}{2})\pi$

m=1

β	y
3π	-2
2π	6.28
$\frac{5\pi}{2}$	4.14
2.86π	0.0082 ← 1st root

$$\beta = \frac{2\pi a}{\lambda}\sin\theta = 2.86\pi$$

$$\boxed{\theta = \sin^{-1}\left(\frac{2.86\lambda}{2a}\right)}$$

$$\tan\theta = \frac{y}{L} \qquad \cancel{y = L\tan\theta}$$

$$\boxed{y = L\tan\left(\sin^{-1}\left(\frac{2.86\lambda}{2a}\right)\right)}$$