KEY

PHYS 1312 Fall 2016 Test 1

Sept. 8, 2016

Name ____________________ **Student ID** ____________________ **Score** ______

Note: This test consists of one set of conceptual questions, three problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

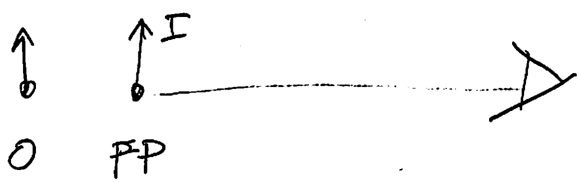
(a) If a police car siren has a frequency of f_s and the car travels toward you at speed v_s, the wavelength of the sound wave you hear is smaller than that emitted by the siren.

Since source and observer are traveling toward each other

$f_0 > f_s$, but $\lambda = \frac{v}{f}$, ∴ $\lambda_0 < \lambda_s$

True

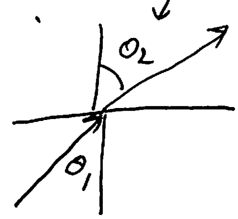
(b) For a person who is nearsighted, the role of a corrective lens is to create an image at her far point of an object located at infinity.



True

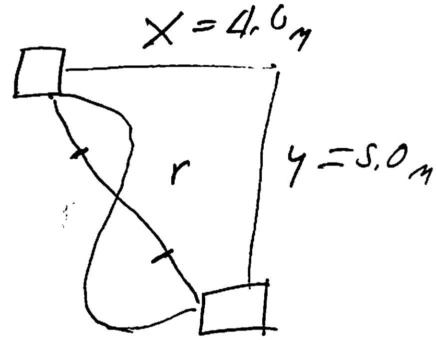
(c) For total internal reflection, the angle of refraction $\theta_2 = 45°$. $\theta_2 = 90°$

False



Problem 2. Two in-phase loudspeakers, which emit sound in all directions, are sitting side by side. One of them is moved sideways by 4.0 m, then forward by 5.0 m. Afterward, constructive interference is observed $\frac{1}{4}$ and $\frac{3}{4}$ of the distance between the speakers along the line that joins them. (a) What is the maximum possible wavelength of the sound waves? (b) What is the frequency of this wave if the speed of sound in air is 343 m/s? (30 points total)

a) The general phase difference equation is



$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0$$

In-phase $\rightarrow$ $\Delta\phi_0 = 0$

$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 5^2} = 6.403\,m$$

From diagram, one wavelength can fit exactly between speakers. $\therefore$ $\lambda = r = \boxed{6.403\,m}$

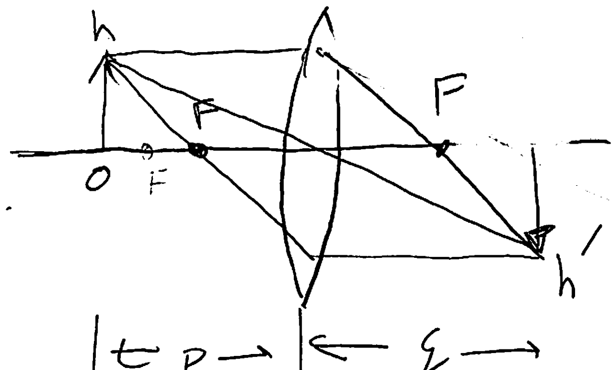
(also $\Delta\varphi = m2\pi$)

b) $V = F\lambda$

$$f = \frac{V}{\lambda} = \frac{343\,m/s}{6.403\,m} = \boxed{53.6\,Hz}$$

Problem 3. A 2.0-cm-tall stub of a pencil is placed in front of a thin converging lens that has a focal length of 20 cm. Sketch a ray diagram including the pencil, lens, image, and three rays. Calculate the position of the image, the image height, the magnification, and indicate whether the image is real or virtual for (a) the pencil 25 cm from the lens. (b) Repeat with the pencil is located 15 cm from the lens. (30 points total)

a)



$f = 20\,cm,\quad p = 25\,cm,\quad h = 2\,cm$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

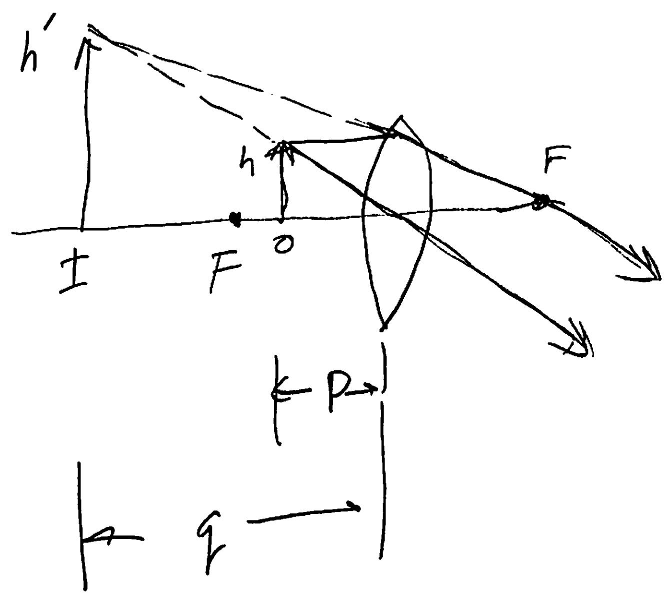
or $q = \dfrac{1}{\frac{1}{f} - \frac{1}{p}} = \dfrac{1}{\frac{1}{20} - \frac{1}{25}}$

$q = \boxed{100\,cm}$

$M = -\dfrac{q}{p} = \dfrac{-100}{25} = \boxed{-4} = \dfrac{h'}{h} \Rightarrow h' = hM = (2)(-4) = \boxed{-8}\,cm$

real, inverted

b)



$p = 15\,cm,\quad f = 20\,cm,\quad h = 2\,cm$

$q = \dfrac{1}{\frac{1}{f} - \frac{1}{p}} = \dfrac{1}{\frac{1}{20} - \frac{1}{15}}$

$= \boxed{-60\,cm}$

$M = -\dfrac{q}{p} = \dfrac{-60}{15} = \boxed{4}$

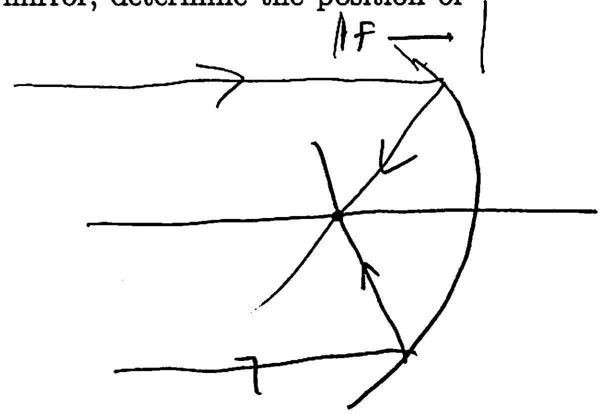
$h' = Mh = (4)(2) = \boxed{8\,cm}$

Virtual, upright

Problem 4. For a concave mirror with radius of curvature R, start with the mirror equation of the form

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}. \qquad (1)$$

(a) Prove that the focal length $f = R/2$ (think "distant galaxy"). (b) If $R = 20$ cm and an object is located at 5.0 cm from the concave side of the mirror, determine the position of the image and its magnification. (30 points total)



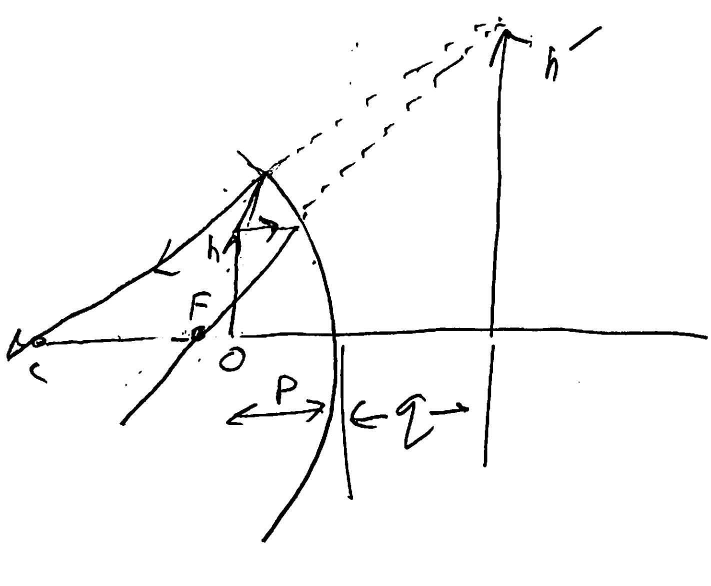
a) Let the object go to a large distance ($p \to \infty$), all rays are parallel to the principle axis and go through the focal point

$$\frac{1}{\infty} + \frac{1}{q} = 0 + \frac{1}{f} = \frac{2}{R} \Rightarrow \boxed{f = \frac{R}{2}}$$

Then we get the mirror equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

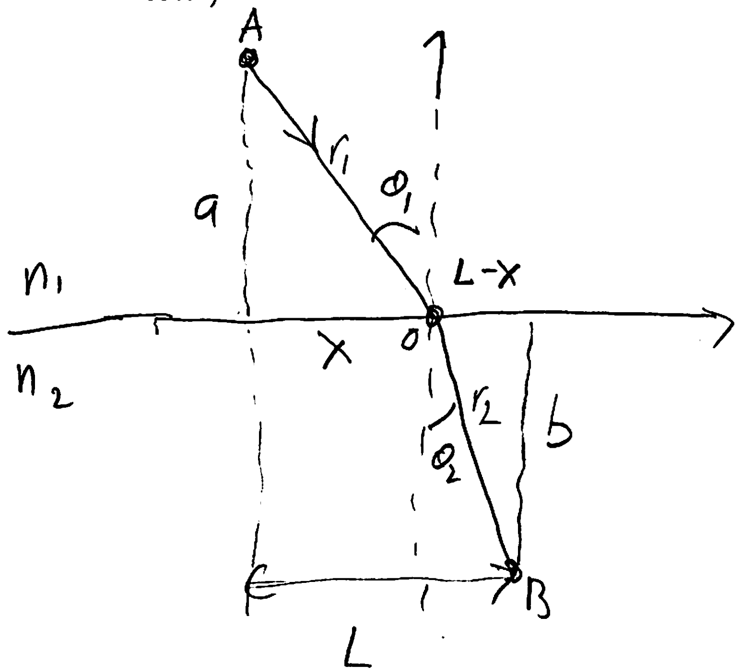
b) $R = 20\text{cm}, \; f = 10\text{cm}, \; p = 5\text{cm}$

$$q = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = \boxed{-10\text{cm}}$$



$$M = -\frac{q}{p} = \frac{-(-10)}{5} = \boxed{2}$$

Bonus Problem. Derive Snell's Law of Refraction using Fermat's Principle (i.e., the path of least time). Make a diagram showing appropriate rays, angles, and lengths. (5 points total)



$n_1 = \frac{c}{V_1}$, $n_2 = \frac{c}{V_2}$

total time for ray to travel from point A to B on path A-O-B is

$$t = \frac{r_1}{V_1} + \frac{r_2}{V_2}$$

$$t = \frac{\sqrt{a^2+x^2}}{c/n_1} + \frac{\sqrt{b^2+(L-x)^2}}{c/n_2}$$

$r_1^2 = a^2 + x^2$

$r_2^2 = b^2 + (L-x)^2$

applying Fermat's principle to find the least time $\frac{dt}{dx} = 0$

$$\frac{dt}{dx} = \frac{n_1 2}{c}\left(\frac{1}{2}\right)(a^2+x^2)^{-1/2} x + \frac{n_2 2}{c}\left(\frac{1}{2}\right)(L-x)(b^2+(L-x)^2)^{-1/2} = 0$$

or $$\frac{x\, n_1}{\sqrt{a^2+x^2}} - \frac{(L-x) n_2}{\sqrt{b^2+(L-x)^2}} = n_1 \sin\theta_1 - n_2 \sin\theta_2 = 0$$

or $$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$