KEY

PHYS 1311 Spring 2023 Test 2
March 2, 2023, 9:35 - 10:50 am

Name _____ **Student ID** _____ **Score** _____

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last pages.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) Kinetic friction is one of the four fundamental forces.

False

(b) One of the postulates of the theory of special relativity is that the speed of light is the same in all reference frames.

True

(c) The two forces involved in Newton's third law must originate from the same physical mechanism.

True

(d) The strong nuclear force can be used as a good model to describe the tension force.

False, the spring force

Problem 2. A kayaker needs to paddle north across a 100-m-wide harbor. The tide is going out creating a tidal current that flows to the east at a speed of 2.0 m/s. The kayaker can paddle with a speed of 3.0 m/s. (a) In which direction should she paddle in order to travel straight across the harbor? (b) How long will it take her to cross? (Hint: draw a velocity vector diagram). (15 points total)

Use galilean transformation

$\vec{V}' = \vec{V} - \vec{V}_0$

$|\vec{V}'| = 3.0\ m/s, \quad \vec{V}_0 = 2.0\ m/s\ \hat{i}$

$\vec{V} = |\vec{V}|\hat{j}, \quad y = 100\ m$

a) Find θ', consider X-direction

$-V'\cos\theta' = 0 - V_0$

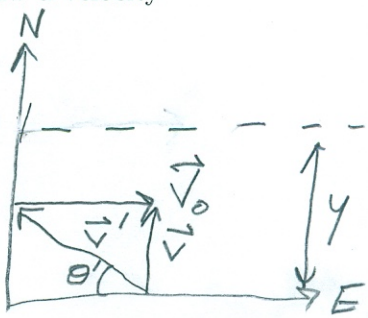
$\cos\theta' = \frac{V_0}{V'}$

$\theta' = \cos^{-1}\left(\frac{2}{3}\right) = \boxed{48.1^\circ\ N\ of\ W} = \boxed{131.8^\circ}$

b) consider y-direction

$V'\sin\theta' = V = y/t$

$t = \frac{y}{V'\sin\theta'}$



$= \frac{100}{3\sin 48.1} = \boxed{44.8\ s}$

Problem 3. Measurements show that Jupiter's gravitation force on a mass of 1 kg near Jupiter's "surface" is 24.9 N. If the radius of Jupiter is 71,500 km, what is the mass of Jupiter? (15 points total)

$= m$

$= R_J$

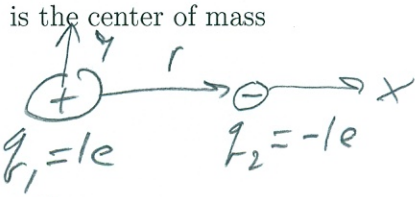
mg_J

$\Sigma F_y = ma_y$

$\frac{GM_J m}{R_J^2} = mg_J$

$M_J = \frac{mg_J R_J^2}{mG} = \frac{(24.9)(71{,}500 \times 10^3)^2}{(1)(6.673 \times 10^{-11})} = \boxed{1.91 \times 10^{27}\ kg}$

Problem 4. A proton and an electron are separated by a distance of 0.529×10^{-10} m. (a) Calculate the force on the electron due to the proton. (b) Compute the center of mass of the proton-electron system. If the radius of a proton is 0.84×10^{-15} m, is the center of mass outside the proton radius? (15 points total)



a) $F_e = \frac{k_e q_1 q_2}{r^2} = \frac{k_e e(-e)}{r^2} = \frac{-k_e e^2}{r^2}$

$= \frac{-8.99\times10^{9}(1.60\times10^{-19})^2}{(.529\times10^{-10})^2} = \boxed{-8.2\times10^{-8}\ \text{N}}$ or $\langle -8.2, 0, 0 \rangle \times 10^{-8}$ N

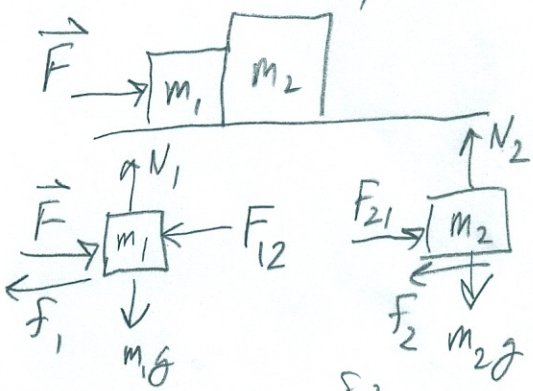
b) $x_{cm} = \frac{m_p x_p + m_e x_e}{m_p + m_e} = \frac{0 + m_e r}{m_p + m_e} = \frac{m_e}{m_p + m_e} r$ $\boxed{> r_p}$ yes

$= \left(\frac{9.109\times10^{-31}}{9.101\times10^{-31} + 1.6726\times10^{-27}}\right)(0.529\times10^{-10}) = \boxed{2.88\times10^{-14}\ \text{m}}$

Problem 5. Two boxes of cereal are sitting next to each other on a flat surface. The left one of mass 1.0 kg is a box of Cheerios, while the right one of mass 3.0 kg is a box of Wheaties. A force of $< 12.0, 0, 0 >$ N is applied to the Cheerios box and both boxes move to the right. If the magnitude of the friction force on the Cheerios and Wheaties boxes are 2.0 N and 4.0 N, respectively, what is the force on the Wheaties box due to the Cheerios box? Draw all necessary free body diagram(s). (15 points total)

f_2

f_1



Consider the Wheaties box

in X-direction

$\pm \rightarrow \sum F_{x_2} = m_2 a_{x_2}$

(1) $F_{21} - f_2 = m_2 a_{x_2}$

Cheerios box

$\rightarrow \sum F_{x_1} = m_1 a_{x_2}$

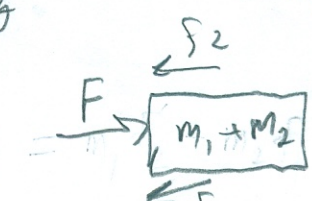
$F - F_{12} - f_1 = m_1 a_{x_1}$ (2)

but $a_{x_1} = a_{x_2} = a$ and $F_{12} = -F_{21}$

(3) For both masses

$F - f_1 - f_2 = (m_1 + m_2) a$

$a = \frac{F - f_1 - f_2}{m_1 + m_2} = \frac{12 - 2 - 4}{4} = \frac{3}{2}\ \text{m/s}^2$



From (1)

$F_{21} = m_2 a_x + f_2$

$= (3)\left(\frac{3}{2}\right) + 4 = \boxed{8.5\ \text{N}}$

Problem 6. (a) Beginning with Newton's 2nd Law, derive the impulse-momentum theorem for the case where the force is a function of time, $F(t)$. (b) State the necessary condition(s) for and derive conservation of linear momentum. (c) Using part (b) consider a 1000 kg cart rolling to the right on a horizontal surface at 5.0 m/s. A 70 kg man is standing on the right end of the card. What is the speed of the cart if the man suddenly starts running to the left with a speed of 10 m/s relative to the cart? (30 points total)

a) $\Sigma \vec{F} = \vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$

or $\vec{F}_{net}(t) = \frac{d\vec{p}}{dt}$

or $\int_{t_i}^{t_f} \vec{F}_{net}(t)\,dt = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p}$

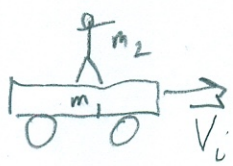
or $\vec{J} = \vec{p}\Big|_{\vec{p}_i}^{\vec{p}_f} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$

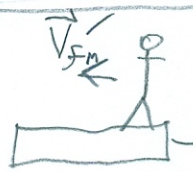
or $\boxed{\vec{J} = \Delta\vec{p}}$ where $\vec{I} = \int \vec{F}_{net}(t)\,dt$

b) For the conservation of linear momentum to hold, the net external force $\vec{F}_{net}$ acting on a system of particles of total momentum $\vec{P} = \sum_{j=1}^{N} \vec{p}_j$ must be zero, $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$ or

$$\boxed{\vec{P}_f = \vec{P}_i}$$

c)





$m_1 = 1000$ kg

$m_2 = 70$ kg

$\vec{V}_{i1} = \vec{V}_{i2} = 5.0\ \text{m/s}\ \hat{i}$

$\vec{V}_{fm} = -10.0\ \text{m/s}\ \hat{i}$

Use conservation of linear momentum, and the galilean transformation → solve for V_{f1}

$P_{i1} + P_{i2} = P_{f1} + P_{f2}$

$m_1 V_{i1} + m_2 V_{i2} = m_1 V_{f1} + m_2 V_{f2}$

$(m_1 + m_2)V_i = m_1 V_{f1} + m_2 V_{f2}$

$\vec{V}_{f2} = \vec{V}'_{f2} + \vec{V}_0$, $V_0 = V_{i1}$

$= \vec{V}_{f2} + \vec{V}_{i1}$

$V_{f2} = -10 + 5$

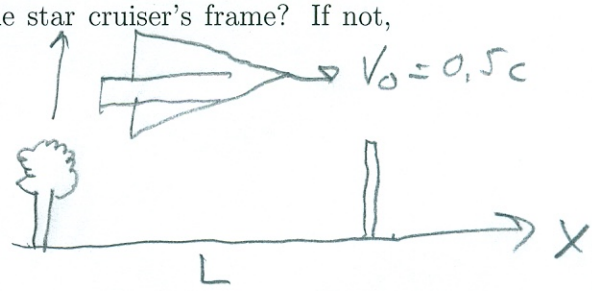
$= -5\ \text{m/s}$

$V_{f1} = \left(\frac{m_1 + m_2}{m_1}\right)V_i - \frac{m_2}{m_1}V_{f2}$

$= \left(\frac{1000 + 70}{1000}\right)5 - \frac{70}{1000}(-5)$

$= \frac{1140}{1000}(5) = \boxed{5.7\ \text{m/s}}$

Bonus Problem. In Earth's reference frame, a tree is at the origin and a pole is at $x = 30$ km. Lightning strikes both the tree and the pole at the same instant at a time of $t = 10\ \mu$s. The lighting strikes are also observed by a SpaceY luxury star cruiser traveling at 0.5c in the x-direction. (a) What are the horizontal coordinates and times of the two events as viewed by the star cruiser? (b) Are the events simultaneous in the star cruiser's frame? If not, which occurs first? (5 points)



$L = 30\text{ km} = X_p$, $X_{tree} = 0$

$t = 10\ \mu s = 10^{-5}\ s$

Use Lorentz transformation

a) $x' = \gamma(x - V_0 t)$, $V_0 = 0.5c$, $\gamma = \dfrac{1}{\sqrt{1 - \left(\frac{V}{c}\right)^2}}$

$t' = \gamma\left(t - \frac{V_0}{c^2}x\right)$

$= \dfrac{1}{\sqrt{1 - \left(\frac{.5c}{c}\right)^2}}$

$= 1.1547$

tree | $x' = 1.1547(0 - 0.5c\,t)$

$= 1.1547\left(0 - \frac{3\times10^8}{2}(10^{-5})\right)$

$= \boxed{-1732\text{ m}}$

$t' = 1.1547\left(10^{-5} - \frac{1}{2(3\times10^8)}0\right) = \boxed{1.1547\times10^{-5}\ s}$

pole | $x' = 1.1547\left(30{,}000 - \frac{3\times10^8}{2}(10^{-5})\right)$

$= \boxed{32909\text{ m}}$

$t' = 1.1547\left(10^{-5} - \frac{30{,}000}{2(3\times10^8)}\right) = \boxed{4.62\times10^{-5}\ s}$

b) pole before tree