KEY

**PHYS 1311 Spring 2023 Test 3**

April 11, 2023

**Name** ____________________ **Student ID** ____________________ **Score** ______

**Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.**

**Problem 1. Conceptual questions**. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The slope of a potential energy function evaluated at a spatial point tells us the magnitude of the conservative force at that point.

True

(b) MeV/c$^2$ is a unit of mass.

True

(c) The orbital velocity of Jupiter is much smaller than that for Mercury.

True

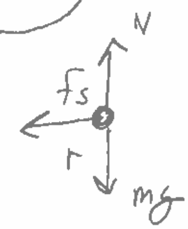
(d) A massless particle, e.g. the photon, must have zero momentum.

False

**Problem 2.** A 30 kg child rides on a playground merry-go-round and sits 1.4 m from the center. The merry-go-round makes one complete revolution every 5.0 s. How large is the net force on the child? What type of force keeps the child on the merry-go-round and in what direction does it point? (15 points total)

$m = 30\ kg,\ r = 1.4 m$

$T = 5.0 s$


a) $+\sum F_r = m a_r$

$$f_s = m\frac{V_t^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 \quad V_t = r\omega = r\frac{2\pi}{T}$$

$$= \frac{m}{r}\frac{4\pi^2 r^2}{T^2} = \frac{4\pi^2 r m}{T^2}$$

$$= \frac{4\pi^2(1.4)(30)}{(5)^2} = \boxed{66.3 N}$$

b) friction, static, points to the center

**Problem 3.** Given the relativistic total energy and momentum relations, show that $E^2 - (pc)^2$ is a constant. What is that constant? (15 points total)

$$E = \gamma mc^2, \quad \vec{p} = \gamma m\vec{v}$$

$$E^2 - p^2c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 = \gamma^2 m^2 c^4\left(1 - \frac{v^2}{c^2}\right)$$

but $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ or $\gamma^2 = \dfrac{1}{1-\frac{v^2}{c^2}}$

therefore

$$E^2 - p^2c^2 = \gamma^2 m^2 c^4 \frac{1}{\gamma^2} = m^2c^4$$

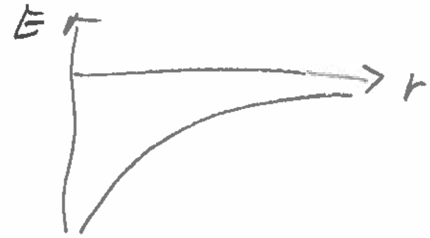
or $\boxed{E^2 - (pc)^2 = (mc^2)^2}$

The constant is the square of the rest energy $mc^2$

$R_M = 1.75 \times 10^6$ m

$m_M$

**Problem 4.** The radius of the Moon is 1750 km, and its mass is $7 \times 10^{22}$ kg. What would be the escape speed for a rocket to leave the surface of the Moon. Take the system to include only the rocket and the Moon. (15 points total)




By conservation of energy

$E = K + U = 0$ For $r \to \infty$, $E \to 0$, $U \to 0$

$$= \frac{1}{2}\cancel{m}v^2 - \frac{GM_m \cancel{m}}{R_M} = 0$$

$$v = \sqrt{\frac{2GM_m}{R_M}} = \sqrt{\frac{2(6.673 \times 10^{-11})(7 \times 10^{22})}{1.75 \times 10^6}} = 2310 \frac{km}{s}$$

$\vec{r}_i$

$V_i$

**Problem 5.** Outside the space station, you and a friend pull on two ropes to dock a satellite whose mass in 700.0 kg. The satellite is initially at a position $< 3.5, -1.0, 2.4 >$ m and has a speed of 4.00 m/s. You exert a constant force of $< -400, 310, -250 >$ N. When the satellite reaches the position $< 7.1, 3.2, 1.2 >$ m, its speed is 4.01 m/s. How much work did your friend do? (15 points total)

$\vec{r}_f$ $V_f$ $\vec{F}_1$

$$W_{total} = \Delta K = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_i^2 = \frac{1}{2}m(V_f^2 - V_i^2)$$

$$= \frac{1}{2}(700)\left[(4.01)^2 - 4^2\right] = 28.035 \text{ J}$$

Work due to Forces

$W = W_1 + W_2 \Rightarrow W_2 = W - W_1$

$$W_1 = \vec{F} \cdot \Delta\vec{r} = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

$$= (-400)[7.1 - 3.5] + (310)[3.2 - (-1)] + (-250)[1.2 - 2.4]$$

$$= -1440 + 1302 + 300 = \boxed{162 \text{ J}}$$

$W_2 = 28.035 - 162 = \boxed{-134 \text{ J}}$ ← friend

**Problem 6.** A mass of 0.12 kg is attached to a horizontal spring with the mass sitting on a frictionless surface and the spring attached at the other end to a vertical wall (on the left). With the spring unstretched, you push the mass to the right. The speed of the mass just after leaving your hand is < 3.40, 0, 0 > m/s. If the spring stretches to a maximum displacement of 0.07 m, determine (a) the phase constant $\phi_0$, (b), the angular frequency of vibration $\omega$, (c) the spring constant $k$, and (d) the amount of work done by spring on the mass as the mass ~~most~~ moves from $x$ = 0 to $x$ =0.07 m and (e) from 0.07 m to the origin. (f) Determine the total energy for the system (neglect the rest energy) and (g) draw a potential energy diagram of the system labeling the turning points with their values in "energy-position" space. (30 points total)

$m = 0.12\ kg$ , $\vec{v}(t=0) = \langle 3.40, 0, 0 \rangle\ m/s$

or $V_{ox} = 3.40\ m/s$


$X(t=0) = 0$

$A = 0.07\,m$

a) Start with simple harmonic motion equations

$x(t) = A\cos(\omega t + \phi_0)$

$v(t) = -A\omega \sin(\omega t + \phi_0)$

$t=0$ $X(t=0) = 0 = A\cos\phi_0$

or $\cos\phi_0 = 0 \Rightarrow \phi_0 = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots \rightarrow \phi_0 = \frac{\pi}{2}$

b) $V_{ox} = -A\omega\sin\phi_0 = -A\omega$

$|\omega| = +\left|\frac{V_{ox}}{A}\right| = \frac{3.4}{0.07} = \boxed{48.57\ rad/s}$

c) $\omega = \sqrt{\frac{K}{m}} \Rightarrow K = \omega^2 m$

or $K = (48.57\ \frac{rad}{s})^2 (0.12\ kg)$

$= \boxed{283.1\ N/m}$

from b) $-\frac{V_{ox}}{A\omega} = \sin\phi_0 \rightarrow$

$\boxed{\phi_0 = -\frac{\pi}{2}}$

d) $U = \frac{1}{2}kx^2$

$W = -\Delta U = -(U_f - U_i)$

$= U_i - U_f = \frac{1}{2}k(x_i^2 - x_f^2)$

$= \frac{1}{2}(283.1)[0^2 - 0.07^2]$

$= \boxed{-0.694\ J}$

e) $W = -W_d = \boxed{0.694\ J}$

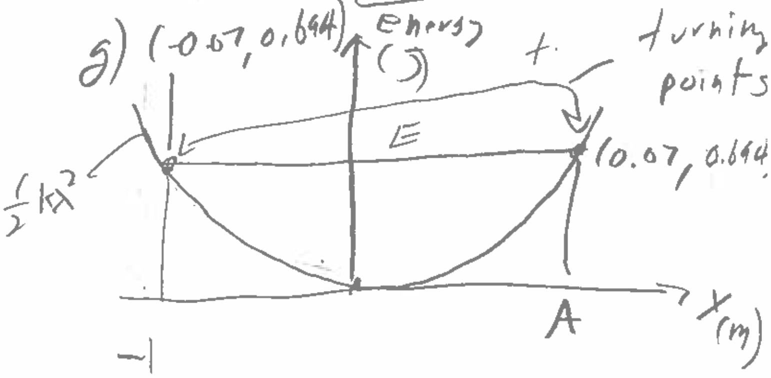
f) $E = K + U$

$= \frac{1}{2}kA^2$ when $v = 0$

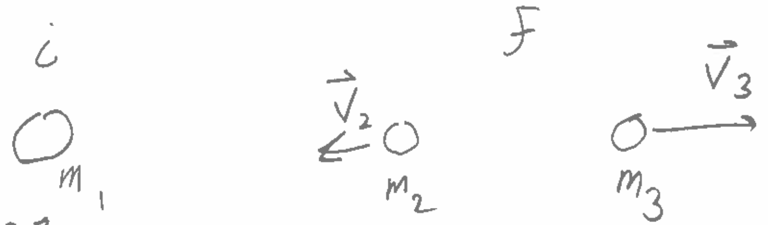
$= \frac{1}{2}(283.1)(0.07)^2$

$= \boxed{0.694\ J}$

g)



**Bonus Problem.** A nucleus whose mass is 239.3837 u undergoes spontaneous alpha decay. In this process, the original nucleus is replaced by an alpha particle (helium nucleus with mass 4.001506 u) and another nucleus of mass 232.0376 u. (a) What is the total kinetic energy of the new particle and the alpha particle in eV when they are infinitely far apart? (b) Determine the individual kinetic energies of the two particles. (5 points)




$m_1 = 239.3837\,u$

$m_2 = 232.0376\,u$

$m_3 = 4.001506\,u = m_{He}$

a) Apply conservation of energy

$E_1 = E_2 + E_2$

$\gamma_1 m_1 c^2 = \gamma_2 m_2 c^2 + \gamma_3 m_3 c^2$

$\gamma_1 = 1$, $\gamma_2$ & $\gamma_3$ are unknown

but $E = E_{rest} + K$

so

$m_1 c^2 = m_2 c^2 + K_1 + m_3 c^2 + K_2 + U_{12}$

for $r \to \infty$, $U_{12} \to 0$

so

$K_3 + K_2 = (m_1 - m_2 - m_3)c^2$

$= (239.3837 - 232.0376 - 4.001506)\,c^2$

$= (3.344594\,u)\,c^2$

$= 3.344594\,u \times \left(\dfrac{1.6726\times10^{-27}\,kg}{1\,u}\right)(3\times10^8\,m/s)^2$

$= 5.02804\times10^{-10}\,J \times \dfrac{1\,eV}{1.602\times10^{-19}\,J} = \boxed{3.139\,GeV}$ 5

b) Apply conservation of momentum

$\vec{P}_1 = \vec{P}_2 + \vec{P}_3$

$0 = \gamma_2 m_2 \vec{V}_2 + \gamma_3 m_2 \vec{V}_3$