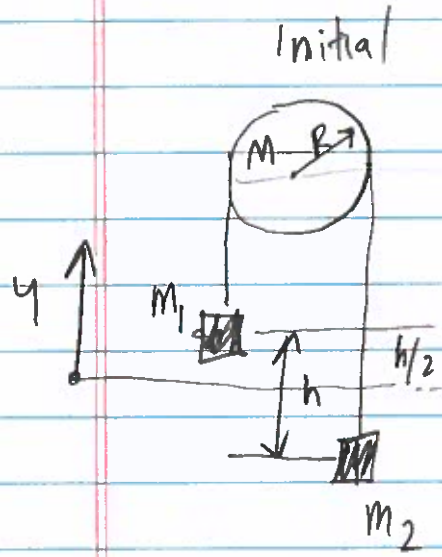
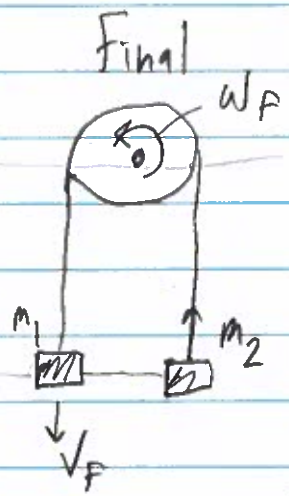
Chapter 12 example 2

3-30-21

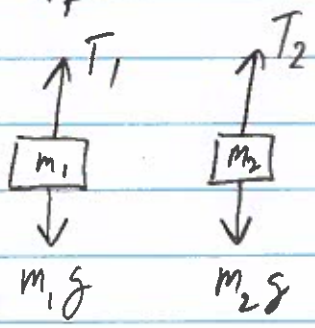


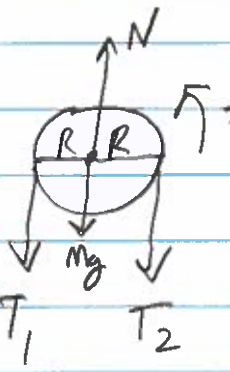


$m_1 = 15$ kg, $m_2 = 10$ kg
$M = 3$ kg, $R = 0.1$ m
$h = 3$ m
$v_i = 0$, $v_f = ?$
$\omega_i = 0$, $\omega_f = ?$

$I_{disk} = \frac{1}{2}MR^2$

Three FBDs

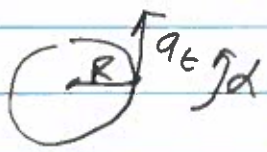




$+\uparrow \Sigma F_y = m_1 a_1$ $\quad +\uparrow \Sigma F_y = m_2 a$ $\quad +\circlearrowleft \Sigma \tau = I\alpha$

$T_1 - m_1 g = m_1 a_1$ $\quad T_2 - m_2 g = m_2 a_2$ $\quad T_1 R - T_2 R = I\alpha$

$T_1 = m_1(g + a_1)$ $\quad T_2 = m_2(g + a_2)$



$a_2 = -a_1 = a$

$a_t = r\alpha = a$

$\rightarrow T_1 = m_1(g - a)$ $\quad T_2 = m_2(g + a)$

$T_1 R - T_2 R = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

$T_1 - T_2 = \frac{1}{2}Ma$ $\quad$ now algebra

$m_1(g - a) - m_2(g + a) = \frac{1}{2}Ma$

Example 2 cont'd 3-30-21

$$m_1 g - m_1 a - m_2 g - m_2 a = \tfrac{1}{2} M a$$

$$a\left(\tfrac{1}{2}M + m_1 + m_2\right) = (m_1 - m_2) g$$

$$a = \left(\frac{m_1 - m_2}{\frac{1}{2}M + m_1 + m_2}\right) g = \left(\frac{15-10}{\frac{3}{2}+15+0}\right) 9.8 = 1.85\ m/s^2$$

Now kinematics for m_2

$$v_f^2 = v_i^2 + 2a(y_f - y_i) \qquad y_f - y_i = \frac{h}{2}$$

$$v_f^2 = 2a\frac{h}{2}$$

$$v_f = \sqrt{ah} = \sqrt{\left(\frac{m_1 - m_2}{\frac{1}{2}M + m_1 + m_2}\right) g h}$$

$$= \sqrt{(1.85)(3)} = \boxed{2.36\ m/s}$$

Now, easy to do with conservation of energy !! Total energy of system is

$$E = \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2} I \omega^2 + m_1 g y + m_2 g y$$

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For masses and pulley $V_i = 0$, $\omega_i = 0$

$$\Rightarrow E_i = m_1 g y_1 + m_2 g y_2 = mg\frac{h}{2} - m_2 g\frac{h}{2}$$

$y_f = 0$, no potential energy

$$E_f = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2} I \omega_f^2$$

$$K_{Rot} = \frac{1}{2} I \omega_f^2 = \frac{1}{2}\left(\frac{1}{2} M R^2\right)\left(\frac{V_t}{R}\right)^2$$

$V_t = \omega R$

$V_t = V_f$

$$= \frac{1}{4} M V_f^2$$

V_f = speed of blocks and tangential on rim of pulley

$$\Rightarrow E_i = E_f$$

$$(m_1 - m_2) g\frac{h}{2} = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{4} M V_f^2$$

$$= \frac{1}{2} V_f^2 \left(m_1 + m_2 + \frac{1}{2} M\right)$$

solve for V_f

$$V_f = \sqrt{\frac{m_1 - m_2}{\left(\frac{1}{2}M + m_1 + m_2\right)} g h}$$

same as above