Chapter 8 Energy Quantization

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- We know that the energy (total) for a multiple particle system is

$$E = E_{rest} + K + U + E_{int}$$

- $E_{rest} = mc^2$ which are "discrete" values for fundamental particles: $E^{e}_{rest} = 511$ keV
  muon $E^{\mu}_{rest} = 106$ MeV
  proton $E^{p}_{rest} = 938$ MeV

  of course macroscopic objects can have any mass

- Also, K, U, E_{int} can take on any real number

- However for atomic systems (and smaller, even larger), only certain values of $K+U$ are possible

- (also in quantum system $K+U \equiv E_{int}$)

- Quantized energy was first introduced by Planck as an approach to describe light

- In the late 1800s, it was well-known that light was a wave (Chapter 23, S3)

- Planck deduced that light came in small

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packets (wave packet) which behaved like particles — photon

- "Light" has both wave-like and particle-like properties
  → wave-particle duality

- Likewise, QM says that particles of matter have wave-like properties.

- Now we are only interested in the total energy of the photon (particle)

$$E^2-(pc)^2=(mc^2)^2=0 \Rightarrow \boxed{E=pc}=E_{ph}$$

- Next semester we will discuss its wave properties but

$$E_{ph}=\hbar\omega=hf=\frac{hc}{\lambda}$$

$h \equiv$ planck's constant $= 6.6\times10^{-34}$ J.s

$\hbar = h/2\pi$

- Discrete spectra from atomic gases (molecules) was observed — mystery lead to development of quantum mechanics)

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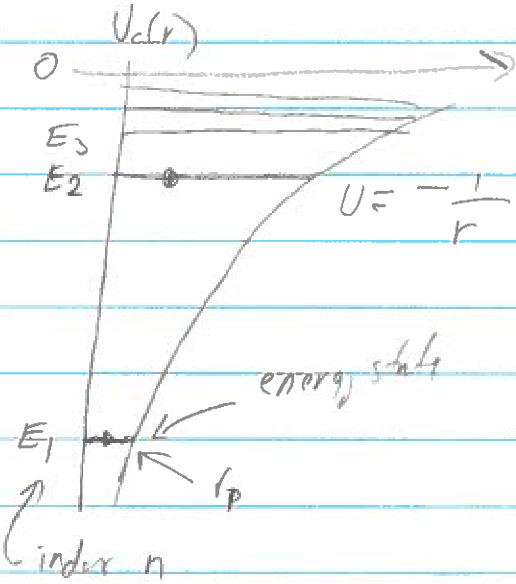
- Given a potential energy function, we can solve the SE and get solutions for the "allowed" energies

Coulomb potent (e⁻–p system)



for H

$$E_n = \frac{-13.6\,eV}{n^2}$$

$n = 1, 2, 3, 4, \ldots$ integer

$n = 1$ ground state

$n > 1$ excited states

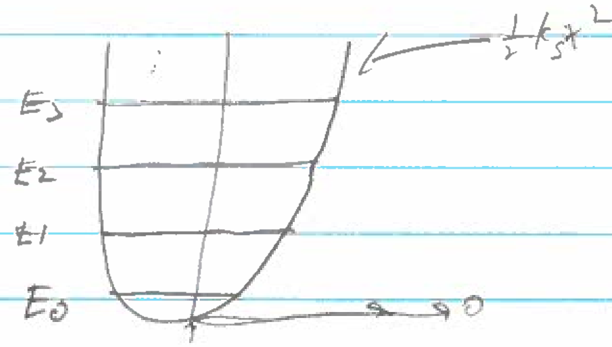
$n = 1 \rightarrow \infty$

- picture electron moving (ellipse) from $r_1 \rightarrow$ to $r = \infty$

- Electronic states or electronic energy levels

- Relation not valid for 2 or more electrons, but all atoms and molecules have similar energy level

- Harmonic oscillator



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- $E_n = n\hbar\omega_0 + E_0 \qquad n = 0, 1, 2, 3,$

  $\omega_0 = \sqrt{\frac{k_s}{m}}$ = property of oscillator

- $\Delta E = E_n - E_{n-1} = \hbar\omega_0$ = evenly spaced

- If k_s is increased, like squeeze U, levels get pushed up

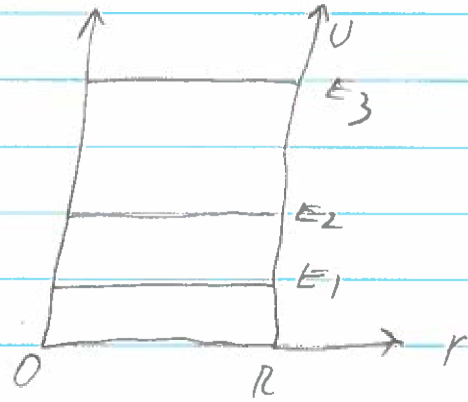
- If m is increased, widen U, levels drop down

- $E_0 = \frac{1}{2}\hbar\omega_0 \Rightarrow$ a quantum effect due to Heisenberg uncertainty principle

- good model for a vibrating distance

- Real molecule

- Strong Force → Infinite square well



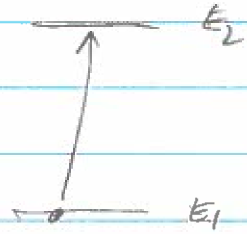
$U = 0, \quad 0 < r < R$

$U = \infty, \quad r > \infty, \quad r < 0$

$E_n = \left(\frac{h^2}{8mR^2}\right)n^2, \quad n = 1, 2,$

Transitions between quantum states 4-14-2016

- Energy level diagram

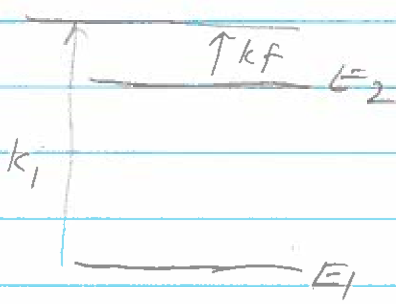


by photon

- Photons must have exact energy $\Delta E = E_2 - E_1$ or it is not absorbed

- Collisions by particles with $K \geq \Delta E$

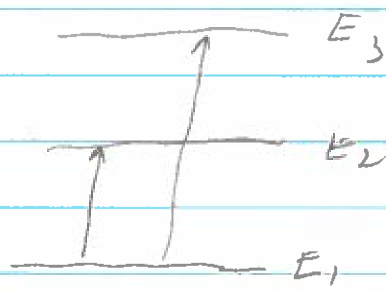
$$H(E_1) + e^-(K_i) \rightarrow H(E_2) + e^-(K_f)$$



$$K_i + E_1 = K_f + E_2$$

$$K_f = K_i - (E_2 - E_1) = K_i - \Delta E$$

- Multiple energy levels



For cold temperature most atoms in the ground state
So transition originate from E_1

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- for photons shining on sample only two transitions are possible — 2 photon energies are absorbed

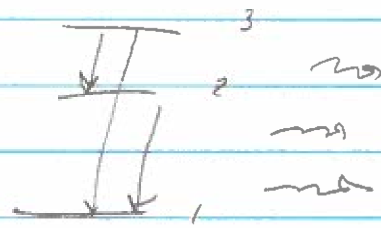
$$E_{ph_1} = E_2 - E_1, \quad E_{ph_2} = E_3 - E_1, \quad E_3 - E_2$$

- However, if gas is hit with a beam of particles with $K_i > E_3$, then levels 1, 2, 3 can be populated and one observes three photons being emitted

$E_{ph_1} = E_2 - E_1$

$E_{ph_2} = E_3 - E_1$

$E_{ph_3} = E_3 - E_2$



- Excited levels can also be populated if gas is hot

fraction of atoms in level n is (1.0)

$$\frac{N_n}{N_{total}} = e^{-\frac{E_n}{k_B T}} = e^{-\frac{1.602\times10^{-19}\,J}{(1.38\times10^{-23}\,\frac{J}{K})(300\,K)}} = 1.6\times10^{-17}$$

$E_1 = 1\,eV = 1.602\times10^{-19}\,J$

$T = 300\,K$

↳ no emissions seen

if $T = 10{,}000\,K$, $\frac{N_1}{N_{total}} = 0.31 \rightarrow 31\%$

↳ emission seen

$E_2 = 2\,eV$, $\frac{N_2}{N_{tot}} = 0.098$

$E_3 = 3\,eV$, $\frac{N_3}{N_{tot}} = 0.03$