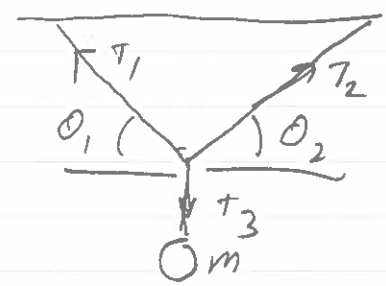
EOC HW #6, Chapter 5, PHYS 1311

Problems P11, P21, P23, P25, P27, P31, P39, P47, P49, and P51 — 3-15-2016

Problem P11 $\theta_1 = 60^\circ$, $\theta_2 = 40^\circ$

$m = 800\,kg$

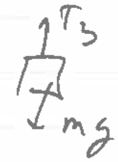
a) Find T_1, T_2, T_3



knot $\Sigma F_x = 0$ $\qquad$ $\Sigma F_y = 0$

$T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \qquad T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0$

mass $\quad \Sigma F_y = 0$



$T_3 - mg = 0$

$$T_3 = mg = (800\,kg)(9.8\,m/s^2) = \boxed{7840\,N}$$

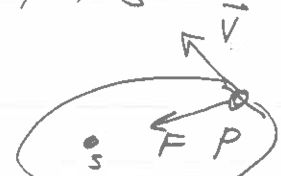
$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = (6098\,N)\frac{\cos 60^\circ}{\cos 40^\circ} = \boxed{3980\,N}$$

$$T_1 \sin\theta_1 + T_1 \frac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 = T_3$$

$$T_1 \left[\sin\theta_1 + \cos\theta_1 \tan\theta_2\right] = T_3$$

$$T_1 = \frac{T_3}{\sin\theta_1 + \cos\theta_1 \tan\theta_2} = \frac{7840\,N}{\sin 60^\circ + \cos 60^\circ \tan 40^\circ} = \boxed{6098\,N}$$

Problem P21 $\vec{V} = \langle 4.5\times10^4, -1.7\times10^4, 0\rangle\,\frac{m}{s}$



$$\vec{F}_{PS} = \langle 1.5\times10^{22}, 1.9\times10^{23}, 0\rangle\,N$$

$$\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}, \quad \vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel}$$

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Problem P21 (cont'd)

$$\vec{F}_{\parallel} = (\vec{F}\cdot\hat{v})\,\hat{v} \qquad \hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

$$|\vec{v}| = \sqrt{4.5^2 + 1.7^2} \times 10^4 = 4.8104 \times 10^4 \text{ m/s}$$

$$\hat{v} = \frac{\langle 4.5, -1.7, 0\rangle}{4.8104} = \langle 0.9354, -0.3534, 0\rangle$$

$$\vec{F}\cdot\hat{v} = (1.5\times10^{22})(0.9354) + (+1.9\times10^{23})(-0.3534) + 0$$
$$= 1.4031\times10^{22} + -6.7146\times10^{22} = -5.3115\times10^{22}\text{ N}$$

$$\vec{F}_{\parallel} = -5.3115\times10^{22}\langle 0.9354, -0.3534, 0\rangle$$

$$\boxed{\vec{F}_{\parallel} = \langle -4.9684, 1.877, 0\rangle \times 10^{22}\text{ N}}$$

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel} = [\langle 1.5, 19, 0\rangle - \langle -4.9684, 1.877, 0\rangle] \times 10^{22}\text{ N}$$

$$\boxed{\vec{F}_{\perp} = \langle 6.47, 17.1, 0\rangle \times 10^{22}\text{ N}}$$

Problem P23 $\quad T = 12\text{ s},\ r = 7\text{ m}$

a) speed of atom on outer rim

$$V_t = r\omega, \quad T = 2\pi/\omega, \quad \omega = \frac{2\pi}{T}$$

$$= \frac{r\,2\pi}{T} = \frac{2\pi(7)}{12} = \boxed{3.665\text{ m/s}}$$

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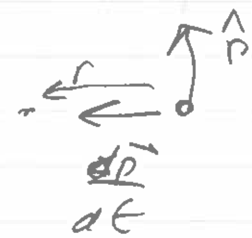
Problem P23 (cont'd)

b) direction of momentum, $\hat{p} = \frac{\vec{p}}{|\vec{p}|} = \frac{m\vec{V}}{m|\vec{V}|}$ $\frac{\vec{V}}{|\vec{V}|} = \frac{\vec{V}_t}{|\vec{V}|}$

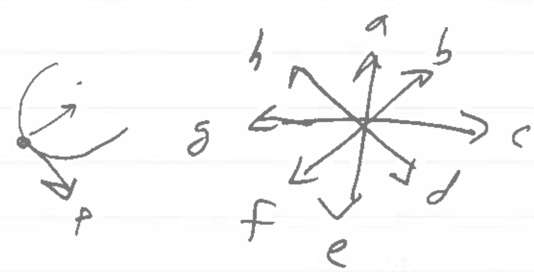
or tangent to circle (parallel) to V_t

c) $\frac{d\vec{p}}{dt} = m\vec{a} = m\vec{a}_r = m\vec{a}_\perp$

or perpendicular to motion, inward toward the center



Problem P25



a) Proton's momentum $\rightarrow d$

b) $\frac{d\vec{p}}{dt} \rightarrow b$

c) $\frac{d\vec{p}}{dt} = m\vec{a}$

$$\left|\frac{d\vec{p}}{dt}\right| = \frac{mV_t^2}{r} = \frac{(1.7\times10^{-27}\,kg)(6.0\times10^{5}\,m/s)}{0.07m}$$

$$= \boxed{8.74\times10^{-15}N}$$

Problem P27

$m = 40\,kg$, $r = 5\,m$

$T = 90\,s$

$\frac{d|\vec{p}|}{dt}\ \hat{p}$

$V_t = \frac{2\pi r}{T}$

$P_t = \frac{2\pi r m}{T}$

$$\left|\frac{d\vec{p}}{dt}\right| = ma_r = \frac{mV_t^2}{r} = \frac{m4\pi^2r^2}{T^2 r} = \left(\frac{2\pi}{T}\right)^2 m r = \left(\frac{2\pi}{90}\right)^2(5)(40)$$

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Problem P27 (cont'd)

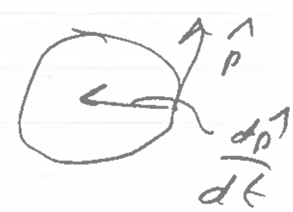
$$\left|\frac{d\vec{p}}{dt}\right| = \boxed{0.975\ N} = \vec{F}_\perp$$ force acting on child

$$\boxed{\frac{d|\vec{p}|}{dt}\hat{p} = \vec{F}_\parallel = 0}$$ since in uniform circular motion

$$|\vec{p}| = p_t = mv_t = \frac{2\pi(5)(40)}{90} = \boxed{13.96\ kg\frac{m}{s^2}}$$

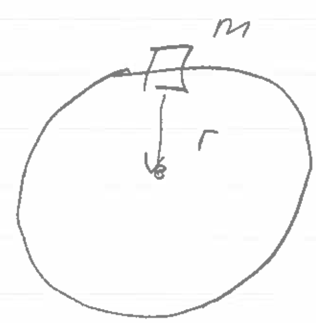
$$\frac{d\hat{p}}{dt} = \hat{F}_\perp$$ = radially toward center

$$|\vec{p}|\left(\frac{d\hat{p}}{dt}\right) = \boxed{13.96\ \frac{km}{s^2}\ \hat{F}_\perp}$$



→ The force is static friction between child and wooden horse

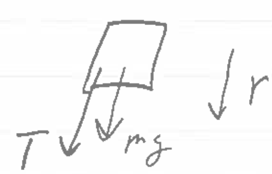
Problem P31



$m = 3.5\ kg$, $r = 1.3\ m$

$v_t = 4\ m/s$

$$\Sigma F_r = \frac{mv^2}{r}$$



$$T + mg = \frac{mv^2}{r}$$

$$T = m\left(\frac{v^2}{r} - g\right) = 3.5\left(\frac{4^2}{1.3} - 9.8\right) = \boxed{8.78\ N}$$

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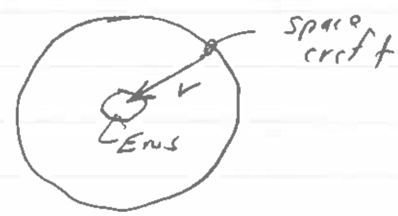
Problem P39

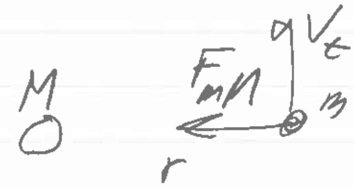
$r = 45 \times 10^3\ m$



$T = 1.04\,\text{days} \times \frac{24\,hr}{1\,dy} \times \frac{60\,min}{1\,hr} \times \frac{60\,s}{1\,min}$

$= 89{,}856\ s$

Apply UCM to orbital motion



$$\frac{GMm}{r^2} = m a_r = m\frac{V_t^2}{r}$$

$$V_t = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T} \qquad \text{solve for } M$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2} \Rightarrow M = \frac{4\pi^2 r^3}{GT^2}$$

$$M = \frac{4\pi^2 (45{,}000)^3}{(6.67\times10^{-11})(89{,}856)^2} = \boxed{6.68\times10^{15}\ kg}$$

Problem P47

$$\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM}}}$$

For orbital speed of Earth around the Sun

$$V_t = \sqrt{\frac{GM_s}{R_{ES}}} = \sqrt{\frac{(6.67\times10^{-11})(2\times10^{30}\,kg)}{1.5\times10^{11}\,m}} = \boxed{29.8\ km/s}$$

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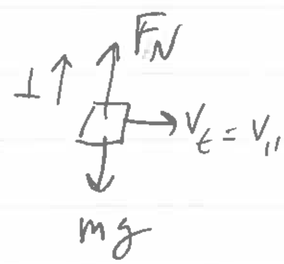
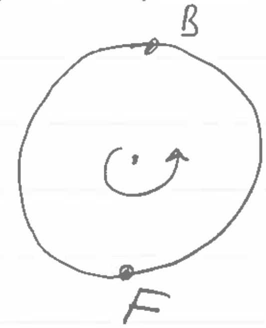
Problem P49 $T = 10.5\,s$, $m = 56\,kg$, $r = 10\,m$

a) Point F





$$\left|\frac{d\vec{p}}{dt}\right| = \Sigma F_r = m a_r$$

$$= m\frac{V_t^2}{r} \qquad V_t = r\omega = \frac{r2\pi}{T}$$

$$= m\left(\frac{2\pi}{T}\right)^2 r$$

$$= 56\left(\frac{2\pi}{10.5}\right)^2(10) = 200\,N \text{ in radial } (\perp) \text{ direction}$$

$$\Rightarrow \boxed{\frac{d\vec{p}}{dt} = \langle 0, 200, 0\rangle\ N}$$

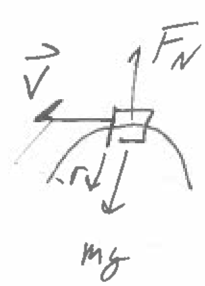
b) $\vec{F}_g = \langle 0, -mg, 0\rangle = \langle 0, -549, 0\rangle N$

c) $F_N - mg = m\frac{V_t^2}{r} = \left|\frac{d\vec{p}}{dt}\right|$

$$F_N = mg + \left|\frac{d\vec{p}}{dt}\right| = 549 + 200 = 749\,N$$

$$\boxed{\vec{F}_N = \langle 0, 749, 0\rangle N}$$

d) Point B $\left|\frac{d\vec{p}}{dt}\right|_B = \left|\frac{d\vec{p}}{dt}\right|_A$



$$\boxed{\left.\frac{d\vec{p}}{dt}\right|_B = \langle 0, -200, 0\rangle N}$$

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e) $|\vec{F}_g|_B = |\vec{F}_g|_F = \boxed{\langle 0, -549, 0 \rangle N}$

f) $mg - F_N = \dfrac{mv_t^2}{r} = \left|\dfrac{d\vec{p}}{dt}\right|$

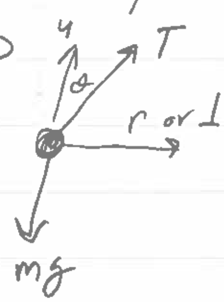
$F_N = mg - \left|\dfrac{d\vec{p}}{dt}\right| = 549 - 200 = 349$

$\boxed{\vec{F}_N = \langle 0, 349, 0 \rangle N}$

Problem PS1

$l = 1.1\,m, \quad \theta = 28°$

FBD



$\Sigma F_y = 0$

$T\cos\theta - mg = 0 \rightarrow T = \dfrac{mg}{\cos\theta}$

$\Sigma F_r = ma_r = \dfrac{mv_t^2}{r}$ | $r = l\sin\theta$

$T\sin\theta = \dfrac{mv_t^2}{r}$

$T = \dfrac{mV_t^2}{(\sin\theta)\, l\sin\theta} = \dfrac{\cancel{m}V_t^2}{l\sin^2\theta} = \dfrac{\cancel{m}g}{\cos\theta}$

$V_t^2 = gl\dfrac{\sin\theta}{\cos\theta}\sin\theta$

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Problem PS1 (Cont'd)

or $V_t = \sqrt{g l \tan\theta \sin\theta}$

$= \sqrt{(9.8)(1.1) \tan 28° \sin 28°} = \boxed{1.64 \text{ m/s}}$

$V_t = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{V_t} = \frac{2\pi l \sin\theta}{V_t} = \frac{2\pi (1.1) \sin 28°}{1.64}$

$\boxed{T = 1.98 \text{ s}}$