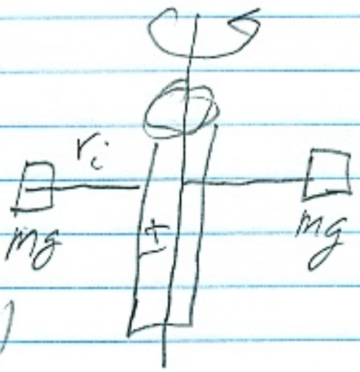
# Example Problem - Chapter 12 3-31-21

- Student sits in rotating chair, she has her arms out horizontally, In each hand she holds a weight of 35.6N
Her arms (From body center) are 0.9m in length.
She has a moment of Inertia about a Vertical axis through her body of $5.4\ kg\ m^2$
She starts with angular speed $\omega_i = \pi$ rad/s.
She pulls her arms in to $r_f = 0.15m$.
What is her new angular speed?

Solution

→ assuming no friction in the rotating chair, No torque act on the student




→ Use conservation of angular momentum.

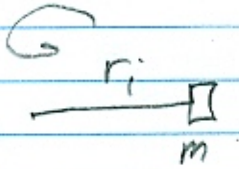
$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$I_i = I_{studen} + I_{weights}$$

Example Problem - (cont'd) 3-31-21

Treat weights as point particle




$$I_{weight} = m r^2$$

$$\Rightarrow I_i = I_{student} + 2 m r_i^2$$

$$I_f = I_{student} + 2 m r_f^2$$

$$\Rightarrow \left(I_{student} + 2 m r_i^2\right)\omega_i = \left(I_{student} + 2 m r_f^2\right)\omega_f$$

solve for $\omega_f$

$$\omega_f = \frac{\left(I_{student} + 2 m r_i^2\right)}{\left(I_{student} + 2 m r_f^2\right)}\,\omega_i$$

$$= \frac{\left[5.4 + 2\left(\frac{35.6}{9.8}\right)(0.9)^2\right]}{\left[5.4 + 2\left(\frac{35.6}{9.8}\right)(0.15)^2\right]}\,\pi \simeq \boxed{2\pi\ \text{rad/s}}$$

$\rightarrow$ Is energy conserved?

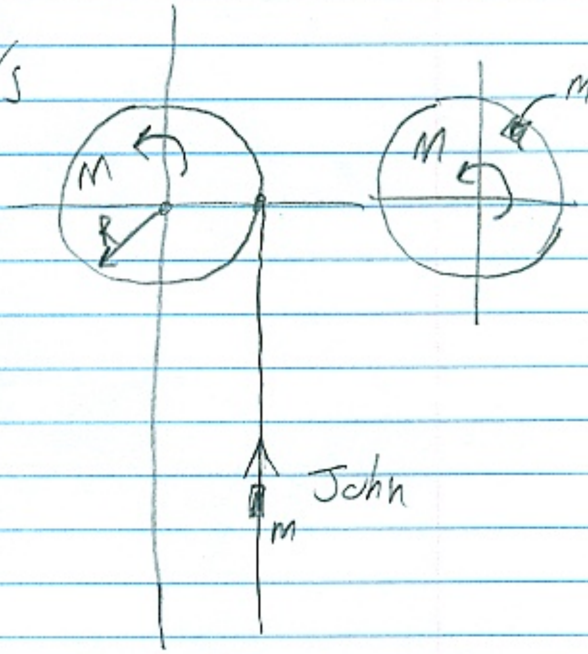
Example Problem 12.81 – Merry-go-round 3-31-4

---

$D = 3\,m$, $M = 250\,kg$

$\omega_i = 20\,rpm$, $V_i = 5.0\,m/s$

$m = 30\,kg$, $\omega_f = ?$




Solution

- Is energy conserved?

  No, inelastic collision

- Is linear momentum conserved?

  No, $P_i = mV_i$, $P_f = 0$

- Is angular momentum conserved?

  Yes, if no friction in axle, No torque,

$$L_i = L_f$$

$$L_i = L_{i,M} + L_{im}$$

– assume merry-go-round is a thin disk

$$= \frac{1}{2}MR^2\omega_i + mR^2\omega_m$$

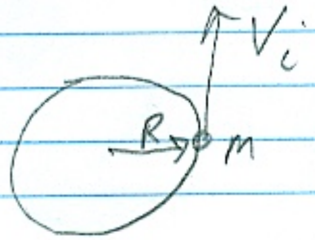
– assume John is a point particle

$$L_f = \frac{1}{2}MR^2\omega_f + mR^2\omega_f$$ ⟵ sits on merry go round

What is $\omega_m$?

Problem 12.81 (Cont'd) 3-31-21

- before John jumps on the merry-go-round, he has a tangential speed




$$V_i = V_t = R\omega_m \leftarrow \text{with respect to axis}$$

or $\omega_m = \dfrac{V_i}{R}$

Substitute in for $\omega_m$, solve for $\omega_f$

$$\frac{1}{2}MR^2\omega_i + mR^2\left(\frac{V_i}{R}\right) = \left(\frac{1}{2}MR^2 + mR^2\right)\omega_f$$

or

$$\omega_f = \frac{\frac{1}{2}MR^2\omega_i + RmV_i}{\left[\frac{1}{2}M + m\right]R^2} = \frac{\frac{1}{2}(250)(1.5)^2(2.09\,\text{rad/s}) + (1.5)(30)(5.0)}{\left[\frac{1}{2}(250) + (30)\right](1.5)^2}$$

$$= 2.33\ \text{rad/s} \times \frac{60\,\text{s}}{1\,\text{min}} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{22\ \text{rpm}}$$

$$\omega_i = 2.0\,\frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{sec}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} = 2.09\,\frac{\text{rad}}{\text{s}}$$

Note $RmV_i = Rp_i = L$

PHYS 1211

# The Vector Product

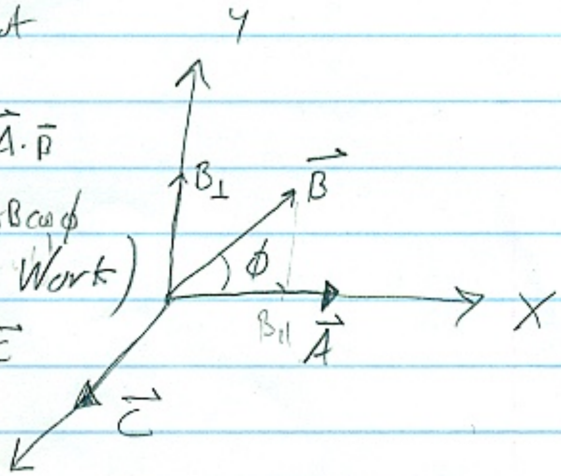
10-23-2007

(cross) — dot

- $\vec{C} = \vec{A} \times \vec{B}$
  $C = AB\sin\phi$
  (e.g. Torque)

  | $D = \vec{A}\cdot\vec{B}$
  | $D = AB\cos\phi$
  | (e.g. Work)

Use RHR to get direction of $\vec{C}$




- Not commutative
  (different from scalar product) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$$\vec{B} \times \vec{A} = -\vec{C} \qquad \text{or}$$
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- if $\vec{A} \parallel \vec{B}$, $\phi = 0, 180°$, $\vec{C} = 0$

  so
  $$\vec{A} \times \vec{A} = 0$$

- $\vec{A} \perp \vec{B}$ $\quad \phi = 90°, 270°$ $\quad |C| = |AB|$

- Is distributive $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

- derivative $\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d}{dt}(\vec{B})$

$$= \frac{d\vec{A}}{dt} \times \vec{B} - \frac{d\vec{B}}{dt} \times \vec{A}$$

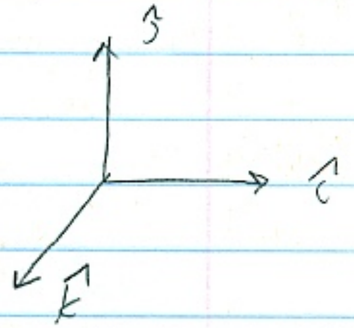
10-23-2002

- Unit vectors

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{i} \times \hat{k} = -\hat{j}$, $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{j} = -\hat{i}$




$\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$\vec{C} = \vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= \cancel{A_x B_x \underbrace{\hat{i} \times \hat{i}}_{0}} + A_x B_y \underbrace{\hat{i} \times \hat{j}}_{\hat{k}} + A_x B_z \underbrace{\hat{i} \times \hat{k}}_{-\hat{j}}$$

$$+ A_y B_x \underbrace{\hat{j} \times \hat{i}}_{-\hat{k}} + \cancel{A_y B_y \underbrace{\hat{j} \times \hat{j}}_{0}} + A_y B_z \underbrace{\hat{j} \times \hat{k}}_{\hat{i}}$$

$$+ A_z B_x \underbrace{\hat{k} \times \hat{i}}_{\hat{j}} + A_z B_y \underbrace{\hat{k} \times \hat{j}}_{-\hat{i}} + \cancel{A_z B_z \underbrace{\hat{k} \times \hat{k}}_{0}}$$

$$\boxed{\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x)\hat{k}}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

in determinant form

## Differential Relations for Rotational Work and Power

3-23-2004

$$dW = F_s\,ds \quad \Rightarrow \quad dW_R = \tau\,d\theta$$

$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v} \quad \Rightarrow \quad P_R = \tau\frac{d\theta}{dt} = \tau\omega$$

## Work-Energy Principle for Rotation

- Begin with Newton's 2nd law for rotation

$$\Sigma\tau = I\alpha = I\frac{d\omega}{dt}$$

- use chain rule

$$I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\frac{d\theta}{dt} = I\omega\frac{d\omega}{d\theta}$$

- multiply both sides by $d\theta$

$$\Sigma\tau\,d\theta = I\omega\,d\omega$$

$$\int \Sigma dW_R = \int I\omega\,d\omega$$

$$\int_0^{W_T} dW_{Total,R} = \int_{\omega_i}^{\omega_f} I\omega\,d\omega \qquad I \neq f(\omega)$$

$$W_{R,total} = I\frac{\omega^2}{2}\Big|_{\omega_i}^{\omega_f} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \Delta K_R$$

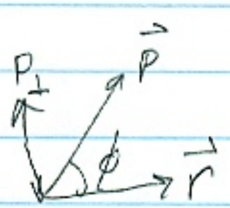
$$\boxed{W_{R,total} = \Delta K_R}$$

# Cross Product relations

10-23-2002

$\tau = rF\sin\phi = rF_{\perp}$

$\vec{\tau} = \vec{r} \times \vec{F}$ (order matters)




$L = I\omega = rp_{\perp} = rp\sin\phi$

$\vec{L} = \vec{r} \times \vec{p}$

$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \cancel{\frac{d\vec{r}}{dt} \times \vec{p}} + \vec{r} \times \frac{d\vec{p}}{dt}$

1st term $\frac{d\vec{r}}{dt} = \vec{v}$, $\vec{p} = m\vec{v}$ $\quad \frac{d\vec{r}}{dt} \times \vec{p} = \vec{v} \times m\vec{v} = 0$

$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}$

Newton's 2nd law $\boxed{\Sigma \vec{F} = \frac{d\vec{p}}{dt}}$

now $\Sigma\vec{\tau} = \vec{r} \times \Sigma\vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}$

$\boxed{\Sigma\vec{\tau} = \frac{d\vec{L}}{dt}}$

$= \underbrace{\Sigma\vec{\tau}_{int}}_{0} + \Sigma\vec{\tau}_{ext}$

$\Sigma\vec{\tau}_{ext} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \quad \vec{L} = \text{constant}$

$\Sigma\vec{\tau}_{ext} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(I\vec{\omega}) = I\frac{d\vec{\omega}}{dt}$

$\boxed{\Sigma\vec{\tau}_{ext} = I\vec{\alpha}}$ 2nd Law

bike demo

atomic electron configuration

$n\ell m$

↑

1s 2p