PHYS 1311 Spring 2022 Test 3
April 7, 2022

Name ______________ **Student ID** ______________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) In uniform circular motion, the tangential acceleration is zero.

True $a_t = r\alpha = 0$

(b) Because of special relativity, the total energy of a particle moving at speed v cannot be greater than mc^2.

False $E = \gamma mc^2$, $\gamma \xrightarrow[\text{can}]{} \infty$

(c) Every point on a rotating disk of radius r has the same tangential speed.

False $v_t = r\omega$ depends on r

(d) Non-conservative forces do no work.

False $W = \int \vec{F} \cdot d\vec{r}$

conservative or non conservative

Problem 2. An astronaut whose mass is 70 kg holds onto the outer rim of a rotating space station whose radius is 14 m and which takes 30 s to make on complete rotation. What is the magnitude of the force the astronaut must exert in order to hold on? (15 points total)

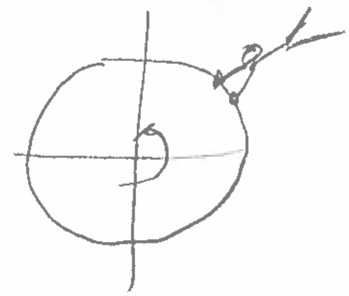
Apply Newton's 2nd Law in radial direction

$r = 14\,m$

$m = 70\,kg$

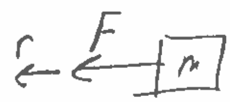
$T = 30\,s$

$V = V_{tangential} = \frac{2\pi r}{T}$



$\sum F_r = m a_r$

$F = m\frac{v^2}{r}$



$= \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{m 4\pi^2 r}{T^2} = \frac{(70) 4\pi^2 (14)}{(30)^2} = \boxed{42.99N}$

Problem 3. Starting with Newton's 2nd Law in the radial direction, derive the period of a satellite of mass m orbiting the Earth of mass M_E at a radial distance r from the center of the Earth ($r > R_E$). Give your answer in terms of r and M_E. Bonus point: What is the name of this law? (15 points total)

$\sum F_r = m a_r$

$\frac{G M_E m}{r^2} = m\frac{v^2}{r}$

$\frac{G M_E}{r} = v^2$

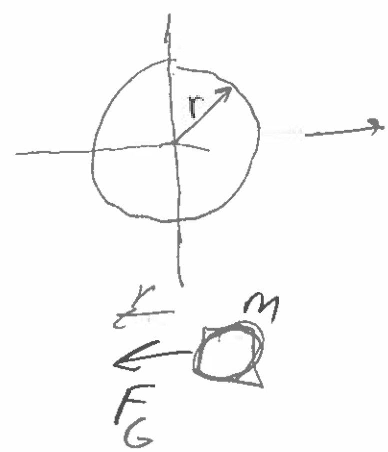
$V = \sqrt{\frac{G M_E}{r}}$

$V = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{V}$

$T = \frac{2\pi r}{\sqrt{\frac{G M_E}{r}}}$

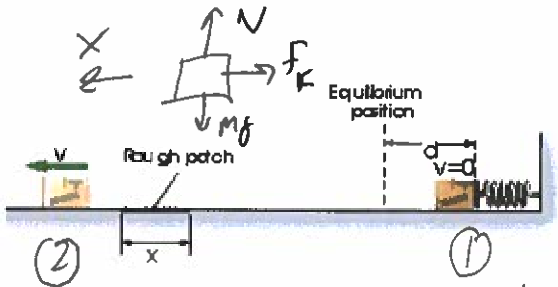
$\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{G M_E}}}$



$\boxed{\text{Kepler's 3rd Law}}$

m

Problem 4. In the figure, a 1.50-kg UPS package is held at rest against a spring with a spring constant k = 737 N/m. Initially, the spring is compressed a distance d. When the package is released from rest it slides to the left across a flat frictionless surface, except for a rough patch of width x = 0.0514 m that has a coefficient of kinetic friction between the surface and the package of 0.419. Calculate d such that the package's speed after crossing the rough patch is 2.49 m/s. (15 points total)



Total energy at any time is

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$E_1 = \frac{1}{2} k d^2, \quad E_2 = \frac{1}{2} m v^2$$

but there is a non conservative force acting so, use

$$E_2 = E_1 + W_{surr} \qquad W_{surr} = f_K x \cos 180° = -f_K x = -\mu_K N x = -\mu_K m g x$$

$$\frac{1}{2} m v^2 = \frac{1}{2} k d^2 - \mu_K m g x$$

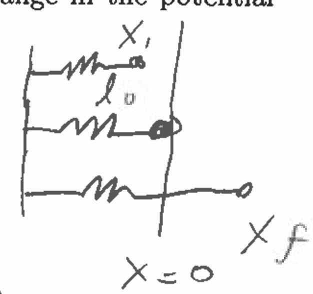
solve for d

$$\frac{1}{2} k d^2 = \frac{1}{2} m v^2 + \mu_K m g x$$

$$d = \sqrt{\frac{m v^2 + 2 \mu_K m g x}{k}} = \sqrt{\frac{1.50(2.49^2 + 2(0.419)(9.8)(0.051))}{737}} = \boxed{0.116 \text{ m}}$$

l_0

Problem 5. A spring whose stiffness is 800 N/m has a relaxed length of 0.66 m. If the length of the spring changes from 0.55 m to 0.96 m, what is the change in the potential energy of the spring? (15 points total)



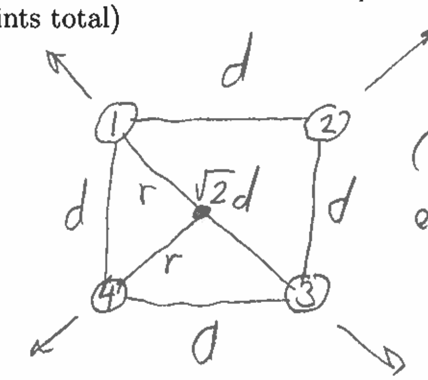
$$\Delta U = U_f - U_i = \frac{1}{2} k (X_f^2 - X_i^2)$$

$$= \frac{1}{2}(800)\left[(0.96 - 0.66)^2 - (0.55 - 0.66)^2\right]$$

$$= \boxed{31.16 \text{ J}}$$

Problem 6. Four protons, each with mass m_p and charge $+e$ are initially held in place at the corners of a square that has sides of length d. What is the speed of each proton when the protons are far apart (assume non-relativistic speeds)? Give your answer in terms of m_p, e, and d. What is the final momentum of the system? (30 points total)

Total energy $E = \sum_{j=1}^{4} K_j + \sum_{j \neq k} U_{jk}$



Coulomb explosion

$E = K_1 + K_2 + K_3 + K_4 + U_{12} + U_{13} + U_{14}$
$+ U_{23} + U_{24} + U_{34}$

$K_j = \frac{1}{2} m_p v^2$ $\quad U_{12} = \frac{k_e e e}{\sqrt{2} r}$, $\quad U_{13} = \frac{k_e e e}{2r} = U_{24}$

$= U_{14} = U_{23} = U_{34}$

or

$E = 2 m_p v^2 + \frac{4 k_e e^2}{\sqrt{2} r} + \frac{2 k_e e^2}{2r}$

$E_i = \frac{4 k_e e^2}{d} + \frac{2 k_e e^2}{\sqrt{2}\, d} = \frac{2 k_e e^2}{d}\left(2 + \frac{1}{\sqrt{2}}\right) = \frac{2 k_e e^2}{d} 2.707$

$r = d/\sqrt{2}$
$V = 0$

$E_f = 2 m_p V_f^2 = E_i = \frac{2 k_e e^2 \, 2.707}{d}$

$r \to \infty$

$\boxed{V_f = \sqrt{\frac{k_e e^2}{d m_p} 2.707}}$

$\boxed{V_f = e \sqrt{\frac{k_e}{d m_p}} 1.645}$

Since all protons start from rest

$\vec{P}_i = 0 = \boxed{\vec{P}_f = 0}$

Bonus Problem Repeat Problem 6, but take the final speed of the protons to be relativistic. First find the final γ_f, then the final speed v_f. Find a relation for p_f. What initial quantity would we have to adjust to ensure the problem is relativistic? Note that the rest energy of the proton is 938 MeV. (5 points)

Similar to problem 6, but $K = (\gamma - 1)mc^2$

$$E = 4K + 4U_{12} + 2U_{13}$$

$$E_i = \frac{2k_e e^2 (2.707)}{d}$$

$$E_f = 4(\gamma_f - 1) m_p c^2$$

$E_i = E_f$

$$4(\gamma_f - 1) m_p c^2 = \frac{2k_e e^2}{d}(2.707)$$

$$\boxed{\gamma_f = \frac{2k_e e^2 (2.707)}{4 m_p c^2 d} + 1}$$

$$\frac{1}{\sqrt{1 - \frac{v_f^2}{c^2}}} = \frac{2.707}{2}\frac{k_e e^2}{m_p c^2 d} + 1$$

$$1 - \frac{v_f^2}{c^2} = \left(\frac{2.707}{2}\frac{k_e e^2}{m_p c^2 d} + 1\right)^2$$

or

$$\boxed{v_f = c\sqrt{1 - \left(\frac{2.707}{2}\frac{k_e c^2}{m_p c^2 d} + 1\right)^2}}$$

$$p_f = \gamma_f m_p v_f$$

complicated!

$$p_f = \left(\frac{2.707 k_e e^2}{2 m_p c^2 d} + 1\right) m_p c \sqrt{1 - \left(\frac{2.702}{2}\frac{k_e e^2}{m_p c^2 d} + 1\right)^2}$$

To ensure the problem is relativistic, make $\underline{d}$ very small, say, $\gamma_f > 1.1$

$$1.1 = \frac{2.707}{2}\frac{k_e e^2}{m_p c^2 d} + 1$$

$$\frac{2.707}{2}\frac{k_e e^2}{m_p c^2 d} = 0.1$$

or $d \lesssim \frac{15 k_e e^2}{m_p c^2}$

Also the final total kinetic energy

$$4(\gamma - 1) m_p c^2 \geq 0.4 m_p c^2$$

$$\gtrsim 375 \text{ MeV}$$