KEY

PHYS 1311 Spring 2022 Test 2
March 1, 2022

Name ______________ **Student ID** ______________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) The linear momentum of a system is conserved if no net external force acts on the system.

True

$$\vec{P}_f = \vec{P}_i + \underbrace{\cancel{\vec{F}_{ext}}}_{0}\Delta t$$

(b) The simple pendulum approximation assumes that $\cos\theta \approx \theta$ as θ becomes small, where θ is the angle of the pendulum measured with respect to the negative y-axis.

False

$$\sin\theta \underset{\theta\to 0}{\approx} \theta \quad / \quad \cos\theta \underset{\theta\to 0}{\approx} 1$$

(c) The spring force is good model of a chemical bond, i.e., the force between two atoms.

True

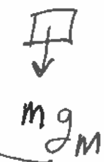
(d) The friction force is a manifestation of the fundamental strong nuclear force.

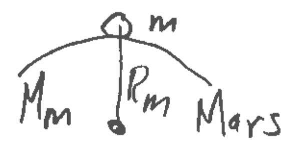
False

electric force

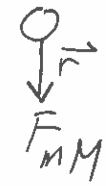
Problem 2. The mass of Mars is 6.4×10^{23} kg and its average radius is 3.4×10^{6} m. What is the value of the acceleration due to gravity g_M on the surface of Mars? At what height above Mars' surface is the value of g_M reduced by half? (15 points total)

$$\Sigma F_r = m g_m = \frac{G M_m m}{R_M{}^2} \Rightarrow g_m = \frac{G M_m}{R_m{}^2}$$





$$g_{Mars} = \frac{(6.673\times10^{-11})(6.4\times10^{23})}{(3.4\times10^{6}\,m)^2} = \boxed{3.69\,m/s^2}$$



$$g_{1/2} = \frac{g_M}{2} = \frac{G M_m}{(R_m+h)^2} \Rightarrow R_m + h = \sqrt{\frac{2GM_m}{g_m}}$$

$$g_{1/2} = \frac{g_M}{2}$$

$$h = \sqrt{\frac{2GM_m}{g_m}} - R_M = \sqrt{\frac{2GM_m R_m^2}{GM_m}} - R_M = \sqrt{2}\,R_m - R_M = R_m(\sqrt{2}-1) = \boxed{1.41\times10^{6}\,m}$$

Problem 3. At a particular instant a proton exerts an electric force of < 0, 5.76 × 10^{-13}, 0 > N on an electron. How far apart are the proton and electron? (15 points total)

$$|\vec{F}_e| = \frac{K_e q_e q_p}{|\vec{r}|^2}$$

$$r = \sqrt{\frac{K_e q_e q_p}{|F_e|}} = \sqrt{\frac{(9\times10^{9})(1)(1)(1.6\times10^{-19}c)^2}{5.76\times10^{-13}}}$$

$$= \boxed{2\times10^{-8}\,m}$$

$r = 25$

Problem 4. At the center of a 50-m diameter circular ice rink, a 75 kg skater traveling north at 2.5 m/s collides with and holds on to a 60 kg skater who had been heading west at 3.5 m/s (i.e., they collide at the center). (a) How long will it take them to glide to the edge of the rink? (b) Where will they reach it? Give your answer as an angle north of west. (15 points total)

a) Conservation of linear momentum

$\vec{P_i} = \vec{P_f}$

x: $P_{2x} = P_{f_x} = (m_1 + m_2) V_{fx}$

$-m_2 V_{2x} = (m_1 + m_2) V_{fx}$

$V_{fx} = -\frac{m_2 V_{2x}}{m_1 + m_2} = -\frac{60}{(75+60)} 3.5$

y: $P_{1y} = P_{fy} \quad = 1.556 \text{ m/s}$

$m_1 V_{1y} = (m_1 + m_2) V_{fy}$

$V_{fy} = \left(\frac{m_1}{m_1 + m_2}\right) V_{1y} = \left(\frac{75}{75+60}\right) 2.5 = 1.389 \text{ m/s}$

$V_f = \sqrt{1.389^2 + 1.556^2} = 2.086 \text{ m/s}$

$r = V_f t \rightarrow t = \frac{25}{2.086} = \boxed{11.99 \text{ s}}$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{1.389}{1.556}\right)$

$= \boxed{41.8° \text{ N of W}} = 138.2°$

Problem 5. (a) If the equation for displacement for simple harmonic motion is $x(t) = A\cos(\omega t + \phi_0)$, derive the relation for velocity $v(t)$ and acceleration $a(t)$. (b) What are the maximum magnitudes of the velocity and acceleration? (c) If the initial conditions at $t = 0$ are $x(0) = 0$ and $v(0) = v_0$, find the phase constant ϕ_0 and a relation for the amplitude A. (15 points total)

a) $x(t) = A\cos(\omega t + \phi_0) \quad A, \omega, \phi_0$ constants

$v(t) = \frac{dx(t)}{dt} = -A\sin(\omega t + \phi_0)\frac{d}{dt}(\omega t + \phi_0) = -A\omega\sin(\omega t + \phi_0)$

$a(t) = \frac{dv(t)}{dt} = -A\omega^2\cos(\omega t + \phi_0)$

b) since $\sin(\omega t), \cos(\omega t)$ go from -1 to 1

$v_{max} = A\omega, \quad a_{max} = A\omega^2$

c) $x(0) = 0 = A\cos(\omega \cdot 0 + \phi_0)$

$0 = A\cos\phi_0 \Rightarrow \boxed{\phi_0 = \frac{\pi}{2}}$

$v(0) = V_0 = -A\omega\sin(\omega \cdot 0 + \phi_0)$

$V_0 = -A\sin\phi_0\,\omega$

$= -A\omega\sin\frac{\pi}{2}$

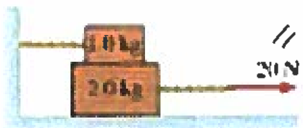
$= -A\omega$

$\boxed{|A| = +\left|\frac{V_0}{\omega}\right|}$

Problem 6. The 1.0 kg block in the figure is tied to the wall with a rope. It sits on top of the 2.0 kg block. The lower block is pulled to the right with a tension of 20 N. The coefficient of kinetic friction at both the lower and upper surfaces of the 2.0 kg block is $\mu_k = 0.40$. (a) Draw free body diagrams. (b) What is the tension in the rope attached to the wall? (c) What is the acceleration of the 2.0 kg block? (30 points total)

m_1

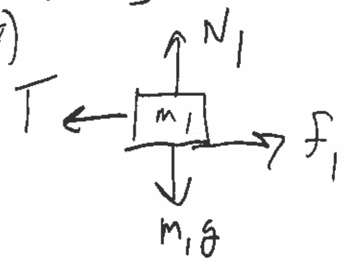
m_2

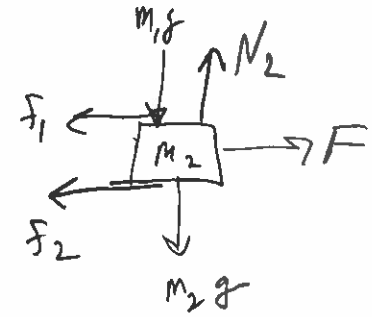


$= F$

a) FBD

$\uparrow y \; \rightarrow x$





b) mass 1

$\Sigma F_x = f_1 - T = 0$

$\mu_k N_1 = T$

$\Sigma F_y = N_1 - m_1 g = 0$

$N_1 = m_1 g$

$T = \mu_k m_1 g$

$= (0.4)(1)(9.8)$

$T = \boxed{3.92\,N}$

mass 2

$\Sigma F_y = 0$

$N_2 - m_1 g - m_1 g = 0 \Rightarrow N_2 = (m_1 + m_2) g$

$\Sigma F_x = m_2 a_x$

$F - f_1 - f_2 = m_2 a_x$

$F - \mu_k m_1 g - \mu_k (m_1 + m_2) g = m_2 a_x$

or

$$a_x = \frac{F - \mu_k g (2m_1 + m_2)}{m_2}$$

$$= \frac{20 - (0.4)(9.8)(2 \cdot 1 + 2)}{2}$$

$$= 10 - (0.8)(9.8) = \boxed{2.16\,m/s^2}$$

Bonus Problem. A block of mass m is at rest at the origin at $t = 0$. It is pushed with constant force F_0 from $x = 0$ to $x = L$ across a horizontal surface whose coefficient of kinetic friction is $\mu_k = \mu_0(1 - x/L)$. That is, the coefficient of friction decreases from μ_0 at $x = 0$ to zero at $x = L$. (a) Prove that

$$a_x = v_x \frac{dv_x}{dx}. \tag{1}$$

(b) Find an expression for the block's speed as it reaches $x = L$. (5 points total)

a) $a_x = \frac{dV_x}{dt} = \frac{dV_x}{dt}\frac{dx}{dx} = \frac{dV_x}{dx}\frac{dx}{dt}$ chainrule

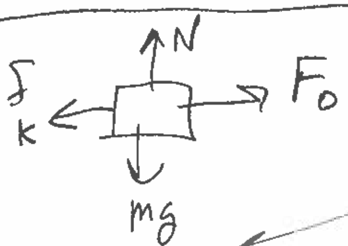
$$\boxed{a_x = V_x \frac{dV_x}{dx}}$$

$\Sigma F_x = F_0 - f_k = m a_x$

$F_0 - \mu_k N = F_0 - \mu_0(1 - \frac{x}{L}) m g$

$\Sigma F_y = 0$

$N = mg$

b) 

or $a_x = \frac{F_0}{m} - \mu_0 g(1 - \frac{x}{L}) = V_x \frac{dV_x}{dx}$

multiply both sides by dx and integrate

$$\int_0^{V_x} V_x\, dV_x = \int_0^L \left[\frac{F_0}{m} - \mu_0 g\left(1 - \frac{x}{L}\right)\right] dx = \int_0^L \left[\frac{F_0}{m} - \mu_0 g\right] dx + \int_0^L \frac{\mu_0 g}{L} x\, dx$$

$$\frac{V_x^2}{2}\Big|_0^{V_x} = \left[\frac{F_0}{m} - \mu_0 g\right] x \Big|_0^L + \frac{\mu_0 g}{L}\frac{x^2}{2}\Big|_0^L$$

$$\frac{V_x^2}{2} = \left[\frac{F_0}{m} - \mu_0 g\right] L + \frac{\mu_0 g}{L}\frac{L^2}{2} = \frac{F_0}{m} - \mu_0 g L + \mu_0 g \frac{L}{2}$$

$$= \frac{F_0}{m} - \mu_0 g \frac{L}{2} \Rightarrow V_x^2 = 2\frac{F_0}{m} - \mu_0 g L$$

$$\boxed{V_x = \sqrt{\frac{2F_0}{m} - \mu_0 g L}}$$