Key

Sun $\bigcirc \rightarrow \vec{F}_{SE}$

PHYS 1311: In Class Problems
Chapter 3
Feb. 10, 2022

$\vec{F}_{ES} \leftarrow \bigcirc$ Earth

Problem 1. Consider the Sun-Earth system with a center-to-center distance of 1.5×10^{11} m. Suppose that at some instance the Sun's momentum is zero and its location is at the origin. Ignoring all effects but that of the Earth, what will the Sun's velocity, momentum, and position be after 1 day. Compute the same quantities for the Earth ignoring the fact the Earth is in a circular orbit (i.e., assume it to initially be at rest). Compute the change in momentum for the Sun. Compute the center-of-mass for the Sun-Earth system at the initial instance and compare that to the radius of the Sun. Treat this as a 1D problem. (Look up appropriate masses)

$$t = 1\,\text{day} \times \frac{24\,\text{hr}}{\text{day}} \times \frac{60\,\text{min}}{1\,\text{hr}} \times \frac{60\,\text{sec}}{1\,\text{min}} = 86{,}400\,\text{s}$$

a) Sun, $F_{SE} = \dfrac{G M_S M_E}{r^2} = M_S a_S$

$$a_S = \frac{G M_E}{r^2} = \frac{(6.674\times10^{-11})(5.97\times10^{24}\,\text{kg})}{(1.5\times10^{11})^2}$$

$a_S = 1.77\times10^{-8}\,\text{m/s}^2 \Rightarrow V_f = V_i + at = 0 + (1.77\times10^{-8}\,\text{m/s}^2)(86{,}400\,\text{s}) = \boxed{1.529\times10^{-3}\,\text{m/s}}$

Now assume F_{SE} & a_S do not change over this time period

Problem 2. Repeat Problem 1, but replace the Sun by a proton, the Earth by an electron, and a starting proton-electron distance of 0.529×10^{-10} m. Use a time interval of 1.0×10^{-18} s. (Look up the appropriate masses)
(on back)

sun at origin

$$r_f = r_i + V_{avg}\,\Delta t = 0 + \left(\frac{1.529\times10^{-3} + 0}{2}\right) 86{,}400\,\text{s}$$
$$= \boxed{66.1\,\text{m}} \leftarrow \text{small compared to } 1.5\times10^{11}\,\text{m}$$

b) Earth $a_E = \dfrac{G M_S}{r^2} = \dfrac{(6.674\times10^{-11})(1.989\times10^{30})}{(1.5\times10^{11})^2} = 5.896\times10^{-3}\,\text{m/s}^2$

$$V_f = a_E t = (5.896\times10^{-3})(86400) = \boxed{509\,\text{m/s}}$$

Problem 3. Compare product $|\Delta p \Delta x|$ from problems 1 and 2 (assuming each a 1D problem). Discuss.
(on back)

$$\Delta r = |r_f - r_i| = \frac{509}{2}(86{,}400) = \boxed{2.2\times10^{7}\,\text{m}}$$

not small, but force only decreased by factor of 0.9997

2) 

Proton

$$F_{pe} = \frac{k_c q_p q_e}{r^2} = \frac{k_e e^2}{r^2} = m_p a_p \Rightarrow a_p = \frac{k_e e^2}{r^2 m_p}$$

$$a_p = \frac{(9\times10^9)(1.6\times10^{-19})^2}{(0.529\times10^{-10})^2(6.726\times10^{-27}\,kg)} = 1.22\times10^{19}\,m/s^2$$

$$V_f = V_i + a_p t = 0 + (1.22\times10^{19}\,m/s^2)(10^{-18}\,s) = \boxed{12.25\,m/s}$$

$$r_f = r_i + V_{avg}\,t = 0 + \left(\frac{12.25}{2}\right)(10^{-18}\,s) = \boxed{6.1\times10^{-18}\,m}$$

Electron

$$a_e = \frac{k_e e^2}{r^2 m_e} = \frac{a_p m_p}{m_e} = \frac{(1.22\times10^{19})(6.726\times10^{-27})}{(9.11\times10^{-31})} = 9.01\times10^{22}\,m/s^2$$

$$V_f = a_e t = (9.01\times10^{22})(10^{-18}) = \boxed{9.01\times10^4\,m/s}$$

$$\Delta r_f = \frac{V_{avg}}{2}t = \left(\frac{9.01\times10^4}{2}\right)(10^{-18}) = \boxed{4.5\times10^{-14}\,m}$$

3) Sun $\Delta p\Delta x \cong m_s \frac{V_s}{2}\Delta r = (1.989\times10^{30})\left(\frac{1.529\times10^{-3}}{2}\right)(66.1)$

$= 1.0\times10^{29}\ kg\,\frac{m^2}{s^2}$

Earth $\cong m_E \frac{V_E}{2}\Delta r = (5.97\times10^{24})\left(\frac{509}{2}\right)(2.2\times10^7) = 3.3\times10^{34}\ kg\,\frac{m^2}{s^2}$

proton $\cong m_p \frac{V_p}{2}\Delta r = (6.726\times10^{-27})\left(\frac{12.25}{2}\right)(6.1\times10^{-18}) = \boxed{2.5\times10^{-43}\ kg\,\frac{m^2}{s^2}}$

electron $\cong m_e \frac{V_e}{2}\Delta r = (9.11\times10^{-31})\left(\frac{9.01\times10^4}{2}\right)(4.5\times10^{-14}\,m)$

$= \boxed{1.8\times10^{-39}\ kg\,\frac{m^2}{s^2}}$

According to Heisenberg uncertainty principle $\Delta p\Delta x \gtrsim \frac{\hbar}{2}$

or $\Delta p\Delta x \geq 5\times10^{-35}\ kg\,\frac{m^2}{s^2}$ → can't know the product of uncertainties to less than this value