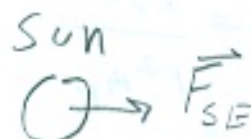


Key



PHYS 1311: In Class Problems
Chapter 3
Feb. 10, 2022



Problem 1. Consider the Sun-Earth system with a center-to-center distance of 1.5×10^{11} m. Suppose that at some instance the Sun's momentum is zero and its location is at the origin. Ignoring all effects but that of the Earth, what will the Sun's velocity, momentum, and position be after 1 day. Compute the same quantities for the Earth ignoring the fact the Earth is in a circular orbit (i.e., assume it to initially be at rest). Compute the change in momentum for the Sun. Compute the center-of-mass for the Sun-Earth system at the initial instance and compare that to the radius of the Sun. Treat this as a 1D problem. (Look up appropriate masses)

a) Sun, $F_{SE} = \frac{GM_S M_E}{r^2} = M_S a_S$

$a_S = \frac{GM_E}{r^2} = \frac{(6.674 \times 10^{-11})(5.97 \times 10^{24} \text{ kg})}{(1.5 \times 10^{11})^2}$

$a_S = 1.77 \times 10^{-8} \text{ m/s}^2$

$t = 1 \text{ day} \times \frac{24 \text{ hr}}{\text{day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ sec}}{1 \text{ min}} = 86,400 \text{ s}$
 $v_f = v_i + a t = 0 + (1.77 \times 10^{-8} \text{ m/s}^2)(86,400 \text{ s}) = 1.529 \times 10^{-3} \text{ m/s}$

Now assume F_{SE} & a_S do not change over this time period

Problem 2. Repeat Problem 1, but replace the Sun by a proton, the Earth by an electron, and a starting proton-electron distance of 0.529×10^{-10} m. Use a time interval of 1.0×10^{-18} s. (Look up the appropriate masses)

(on back)

$r_f = r_i + v_{avg} \Delta t = 0 + \left(\frac{1.529 \times 10^{-3} + 0}{2} \right) 86,400 \text{ s}$
 $= 66.1 \text{ m} \leftarrow \text{small compared to } 1.5 \times 10^{11} \text{ m}$

b) Earth $a_E = \frac{GM_S}{r^2} = \frac{(6.674 \times 10^{-11})(1.67 \times 10^{-27})}{(0.529 \times 10^{-10})^2} = 5.896 \times 10^{-3} \text{ m/s}^2$

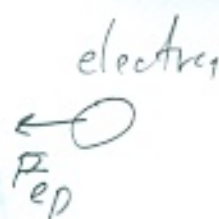
$v_f = a_E t = (5.896 \times 10^{-3})(86,400) = 509 \text{ m/s}$

Problem 3. Compare product $|\Delta p \Delta x|$ from problems 1 and 2 (assuming each a 1D problem). Discuss.

(on back)

$\Delta r = |r_f - r_i| = \frac{509}{2} (86,400) = 2.2 \times 10^7 \text{ m}$ not small, but force only decreased by factor of 0.9997

2)



$$\frac{F_{pe}}{r^2} = \frac{k_e q_p q_e}{r^2} = \frac{k_e e^2}{r^2} = m_p a_p \Rightarrow a_p = \frac{k_e e^2}{r^2 m_p}$$

$$a_p = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(0.529 \times 10^{-10})^2 (6.726 \times 10^{-27} \text{ kg})} = 1.22 \times 10^{19} \text{ m/s}^2$$

$$V_f = V_i + a_p t = 0 + (1.22 \times 10^{19} \text{ m/s}^2)(10^{-18} \text{ s}) = \boxed{12.25 \text{ m/s}}$$

$$r_f = r_i + V_{avg} t = 0 + \left(\frac{12.25}{2}\right)(10^{-18} \text{ s}) = \boxed{6.1 \times 10^{-18} \text{ m}}$$

Electron

$$a_e = \frac{k_e e^2}{r^2 m_e} = \frac{a_p m_p}{m_e} = \frac{(1.22 \times 10^{19})(6.726 \times 10^{-27})}{(9.11 \times 10^{-31})} = 9.01 \times 10^{22} \text{ m/s}^2$$

$$V_f = a_e t = (9.01 \times 10^{22})(10^{-18}) = \boxed{9.01 \times 10^4 \text{ m/s}}$$

$$\Delta r_f = \frac{V_{avg}}{2} t = \left(\frac{9.01 \times 10^4}{2}\right)(10^{-18}) = \boxed{4.5 \times 10^{-14} \text{ m}}$$

$$3) \text{ Sun } \Delta p \Delta x \approx m_s \frac{V_s}{2} \Delta r = (1.989 \times 10^{30}) \left(\frac{1.529 \times 10^3}{2}\right) (66.1) \\ = 1.0 \times 10^{29} \text{ kg } \frac{\text{m}^2}{\text{s}^2}$$

$$\text{Earth } \approx m_E \frac{V_E}{2} \Delta r = (5.97 \times 10^{24}) \left(\frac{509}{2}\right) (2.2 \times 10^7) = 3.3 \times 10^{34} \text{ kg } \frac{\text{m}^2}{\text{s}^2}$$

$$\text{Proton } \approx m_p \frac{V_p}{2} \Delta r = (6.726 \times 10^{-27}) \left(\frac{12.25}{2}\right) (6.1 \times 10^{-18}) = \boxed{2.5 \times 10^{-43} \text{ kg } \frac{\text{m}^2}{\text{s}^2}}$$

$$\text{electron } \approx m_e \frac{V_e}{2} \Delta r = (9.11 \times 10^{-31}) \left(\frac{9.01 \times 10^4}{2}\right) (4.5 \times 10^{-14} \text{ m}) \\ = \boxed{1.8 \times 10^{-39} \text{ kg } \frac{\text{m}^2}{\text{s}^2}}$$

According to Heisenberg uncertainty principle $\Delta p \Delta x \gtrsim \frac{\hbar}{2}$
 or $\Delta p \Delta x \geq 5 \times 10^{-35} \text{ kg } \frac{\text{m}^2}{\text{s}^2} \rightarrow$ can't know the product of uncertainties to less than this value