PHYS 1311: In Class Problems
Chapters 4 and 5
Feb. 16, 2017

Problem 1. Consider the spring-mass system oscillating in the horizontal direction with the mass on a friction-less surface. Take $m = 10$ kg, $k = 100$ N/m, and start the system with an initial velocity of $\vec{v}_i =< 1.00, 0, 0 >$ m/s with an initial position of $\vec{r}_i =< 1.00, 0, 0 >$ m. Determine the amplitude A, angular frequency ω, maximum speed $v_{\max}$, maximum acceleration $a_{\max}$, and a relation for the position x as a function of time t.

$x(t) = A\cos(\omega t + \phi)$, $v(t) = -A\omega\sin(\omega t + \phi)$ $\omega = \sqrt{\frac{k}{m}} = \sqrt{10} = 3.1623 \frac{rad}{s}$

$x(0) = x_0 = 1.00\,m$, $v(0) = v_0 = 1.00\,m/s$

$1 = A\cos\phi$ $1 = -A\omega\sin\phi = -A(3.1623)\sin\phi$

or $1 = \frac{-A\sin\phi\,(3.1623)}{A\cos\phi} = -\tan\phi \cdot 3.1623$

or $\tan\phi = -0.31622$

$\phi = -0.2986\,rad$ $(-0.306\,rad)$

$A = \frac{1}{\cos\phi} = \frac{1}{\cos(-0.2986)}$

$A = 1.054\,m$

$v_{max} = +A\omega = (1.054)(3.1623) = 3.32\,m/s$

$a_{max} = A\omega^2 = (1.054)(3.1623)^2 = 10.500\,m/s^2$

$x(t) = 1.054\cos(3.1623t - 0.2986)$

Problem 2. Given only the distance between the Earth and Moon ($R_{EM} = 3.84 \times 10^8$ m) and that between the Earth and the Sun (1 AU), determine the mass of the Earth and the mass of the Sun. How can we measure R_{EM} or R_{SE}?

$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

$M = \frac{4\pi^2 r^3}{GT^2}$

Earth-Sun

$T_{Earth} = 1\,year = 1 \times 365 \times 24 \times 60 \times 60 = 3.1536 \times 10^7\,s$

$M_{\text{Earth}}{}_{\text{Sun}} = \frac{4\pi^2 r_{ES}^3}{GT^2}$

$r_{ES} = 1\,AU = 1.496 \times 10^{11}\,m$

$M_{Sun} = \frac{4\pi^2(1.496 \times 10^{11})^3}{6.67 \times 10^{-11}(3.1536 \times 10^7)^2} = 1.993 \times 10^{30}\,kg$

Earth-Moon

$T_{moon} = 1\,month = 2.42 \times 10^6\,s$

$M_{Earth} = \frac{4\pi^2(3.84 \times 10^8)^3}{6.67 \times 10^{-11}(2.42 \times 10^6)^2} = 5.72 \times 10^{24}\,kg$