KEY

PHYS 1311 Spring 2022 Test 1
Feb. 3, 2022

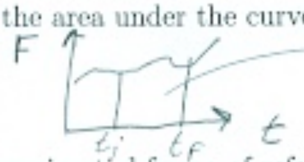
Name ______________________ **Student ID** ______________________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) On a plot of force versus time, the area under the curve from t_i to t_f is the impulse.

True



$F(t)\Delta t = J$

(b) Newton's 2nd law is valid in a noninertial frame of reference.

False   inertial frame

(c) According to Einstein's theory of Special Relativity, the momentum of a particle of mass m cannot exceed mc.

False $p = \gamma m v \xrightarrow[v \to c]{} \infty$

(d) For a position versus time plot, the slope at any point on the curve is the instantaneous velocity.

True

Problem 2. A ball of mass m traveling with velocity $< v_x, 0, 0 >$ hits a wall and rebounds with velocity $< -v_x, 0, 0 >$. (a) What is the change in momentum of the ball? (b) What is the change in the magnitude of the momentum of the ball? (15 points total)

a) $\Delta \vec{p} = \vec{p}_f - \vec{p}_i$ $\qquad \vec{p} = m\vec{v}$

$= m[\langle -v_x, 0, 0\rangle - \langle v_x, 0, 0\rangle] = \boxed{\langle -2mv_x, 0, 0\rangle}$

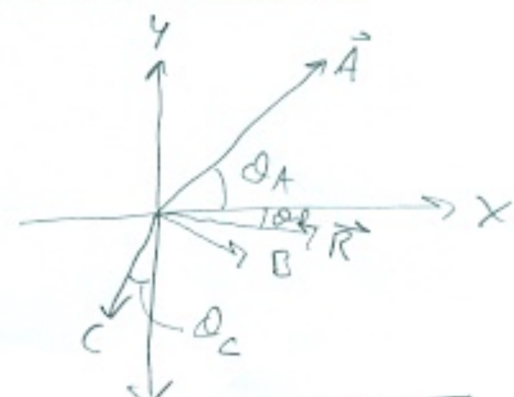
$\Delta|\vec{p}| = |\vec{p}_f| - |\vec{p}_i| = |mv_x| - |mv_x| = \boxed{0}$

Problem 3. Given three vectors in the x-y plane with $\vec{A}$=10 at 45° with respect to the positive x-axis, $\vec{B} = 5\hat{\imath} - 3\hat{\jmath}$, and $\vec{C}$=5 at 30° with respect to the negative y-axis. Determine for the resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ its components, magnitude, and direction with respect to the positive x-axis. (15 points total)

Vector Addition

	X	Y
A	$10\cos 45^\circ = 7.071$	$10\sin 45^\circ = 7.071$
B	5	-3
C	$-5\sin 30^\circ = -2.5$	$-5\cos 30^\circ = -4.330$
R	9.571	-0.259

$\boxed{\vec{R} = \langle 9.571, -0.259, 0\rangle}$



$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$= \sqrt{9.571^2 + (.259)^2 + 0^2}$

$= \boxed{9.575}$

$\theta_R = \tan^{-1}\left(\frac{-0.259}{9.571}\right) = \boxed{-1.550^\circ}$

or $360 - 1.55^\circ = \boxed{358.5^\circ}$

or $\boxed{1.55^\circ \text{ S of E}}$

$\Delta t = 16.2 - 16 = 0.2s$

Problem 4. At $t = \underline{16.0\text{ s}}$ an object with mass 4 kg was observed to have a velocity of $< 9, 29, -10 >$ m/s. At $t = \underline{16.2\text{ s}}$ its velocity was $< 18, 20, 25 >$ m/s. What was the average net force acting on the object? Give in component form and also determine the net average force magnitude. (15 points total) Applied momentum update equation

$$\vec{P}_f = \vec{P}_i + \vec{F}_{avg} \Delta t$$

$$\vec{F}_{avg} = \frac{\vec{P}_f - \vec{P}_i}{\Delta t} = \frac{m(\vec{V}_f - \vec{V}_i)}{\Delta t} = \frac{4kg}{0.2s}\left[\langle 18, 20, 25\rangle - \langle 9, 29, -10\rangle\right]$$

$$= 20\langle 9, -9, 35\rangle = \boxed{\langle 180, -180, 700\rangle N}$$

$$|\vec{F}| = \sqrt{180^2 + 180^2 + 700^2} = \boxed{745 N}$$

Problem 5. The position of a particle as a function of time is given by $\vec{r} = (6.0t\hat{i} + 5.0t^2\hat{j})$ m, where t is in seconds. (a) Find an expression for the particle's velocity $\vec{v}$ as a function of time. (b) What is the velocity vector at $t = 10$ s? (c) What are the units for the constant 6.0? (15 points total)

a) $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\langle 6.0t, 5.0t^2\rangle$$

$$= \langle 6, 10t^2\rangle = \boxed{6\hat{i} + 10t\hat{j}}\ m/s$$

b) $$\vec{v}(t=10s) = 6\hat{i} + 100\hat{j} = \boxed{\langle 6, 100\rangle\ m/s}$$

c) $$x = 6t$$

$$[m] = [Y][s] \Rightarrow [Y] = \left[\frac{m}{s}\right]$$

or $$x = \left(\frac{6m}{s}\right)t$$

Problem 6. (a) Beginning with the following form of Newton's 2nd law,

$$\vec{F}_{net}(t) = \frac{d\vec{p}}{dt}, \tag{1}$$

derive the momentum update relation, but with $\vec{F}_{net}$ a function of time t, using definite integration methods. (b) Consider an electron entering a region in which it experiences a net time-dependent force given by $\vec{F}(t) = < A_0 t, 0, 0 >$ N, with $A_0 = 2.75 \times 10^{-12}$ N/s. If the initial velocity of the electron at $t_i = 0$ is $\vec{v}_i = < 0.01c, 0.01c, 0 >$, what is its final momentum and velocity at $t_f = 1.0 \times 10^{-6}$ s using the equation derived in part (a). You can give both in units of c. (c) Is neglecting relativistic effects justified in this case (i.e., compute γ).? (30 points total)

a) $d\vec{p} = \vec{F}_{net}(t)\,dt \rightarrow \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}_{net}(t)\,dt$

or $\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}_{net}(t)\,dt \Rightarrow \boxed{\vec{p}_f = \vec{p}_i + \int_{t_i}^{t_f} \vec{F}_{net}(t)\,dt}$

b) $\vec{p}_f = \vec{p}_i + A_0 \int_{t_i}^{t_f} \langle t, 0, 0 \rangle\, dt$

$m_e = 9.109 \times 10^{-31}$ kg

$c = 2.998 \times 10^{8}$ m/s

$= m_e \langle 0.01c, 0.01c, 0 \rangle + A_0 \langle \frac{t^2}{2}, 0, 0 \rangle \Big|_0^{1\times10^{-6}}$

$m_e c = 2.73 \times 10^{-24}$ kg m/s

$= \langle 0.01 m_e c, 0.01 m_e c, 0 \rangle + \langle \frac{A_0 t^2}{2}, 0, 0 \rangle$

$= \langle 2.73 \times 10^{-24}, 2.73 \times 10^{-24}, 0 \rangle + \langle 1.375 \times 10^{-24}, 0, 0 \rangle$

$\boxed{\vec{p}_f = \langle 4.105 \times 10^{-24}, 2.73 \times 10^{-24}, 0 \rangle \, \frac{kg\,m}{s}} = \langle 1.368 \times 10^{-32}, 9.1 \times 10^{-33}, 0 \rangle c$

$\vec{v}_f = \frac{\vec{p}_f}{m_e} = \boxed{\langle 4.511 \times 10^{6}, 3 \times 10^{6}, 0 \rangle \, \frac{m}{s}} = \langle 0.015, 0.01, 0 \rangle c$

c) $|\vec{v}_f| = \sqrt{0.015^2 + 0.01^2}\, c = 0.018c$

$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v_f}{c}\right)^2}} = \frac{1}{\sqrt{1 - 0.018^2}} = \boxed{1.00016}$

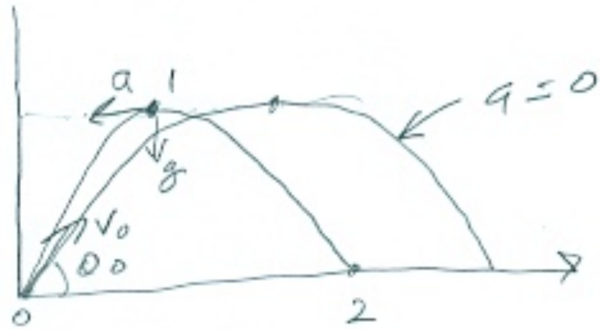
Yes, we can neglect relativistic effects

g

Bonus Problem. A projectile is launched from ground level at an angle θ_0 and speed v_0 into a headwind that causes a constant horizontal acceleration of magnitude a opposite the direction of motion. (a) Derive an expression in terms of a, g, and θ_0 for the launch angle that gives maximum range. (b) What is the angle for maximum range if a is 10% of g?

Time to top of projective motion

$V_{1y} = 0$, $V_{1x} = V_{0x} = V_{2x} = V_0 \cos\theta_0$



y-motion $\qquad V_{0y} = V_0 \sin\theta_0$

$$V_{1y} = V_{0y} - g\,t_1 \Rightarrow t_1 = \frac{V_{1y} - V_{0y}}{-g}$$

$$t_1 = \frac{V_0 \sin\theta_0}{g}, \quad t_2 = \frac{2V_0 \sin\theta_0}{g}$$

x-motion : $X_0 = 0$, $t_0 = 0$

$$X_2 = X_0 + V_{0x} t_2 - \frac{a}{2} t_2^2 = V_0 \cos\theta_0 \left(\frac{2V_0 \sin\theta_0}{g}\right) - \frac{a}{2}\left(\frac{2V_0 \sin\theta_0}{g}\right)^2$$

$$X_2 = \frac{2V_0^2 \sin\theta_0 \cos\theta_0}{g} - \frac{a}{2}\frac{4V_0^2 \sin^2\theta_0}{g^2}$$

use identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$$X_2 = \frac{2V_0^2}{g}\left[\sin\theta_0\cos\theta_0 - \frac{a}{g}\sin^2\theta_0\right]$$

$$X_2 = \frac{2V_0^2}{g}\left[\frac{\sin 2\theta_0}{2} - \frac{a}{g}\frac{1}{2}(1 - \cos 2\theta_0)\right) = \frac{V_0^2}{g}\left[\sin 2\theta_0 - \frac{a}{g} + \frac{a}{g}\cos 2\theta_0\right]$$

$$\frac{dX_2}{d\theta_0} = 0 = \frac{d}{d\theta_0}(\sin 2\theta_0) + \frac{a}{g}\frac{d}{d\theta_0}(\cos 2\theta_0) = 2\cos 2\theta_0 - \frac{2a}{g}\sin 2\theta_0 = 0$$

or $\cos 2\theta_0 = \frac{a}{g}\sin 2\theta_0$

or $\tan 2\theta_0 = \frac{g}{a}$

$$\boxed{\theta_0 = \frac{1}{2}\tan^{-1}\left(\frac{g}{a}\right)} \quad (a)$$

$$\theta_0 = \frac{1}{2}\tan^{-1}\left(\frac{g}{0.1g}\right) = \frac{1}{2}\tan^{-1}(10)$$

$$\boxed{\theta_0 = 42.1^\circ}$$