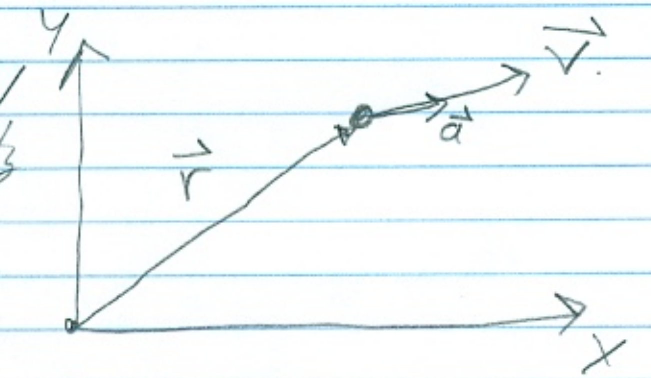
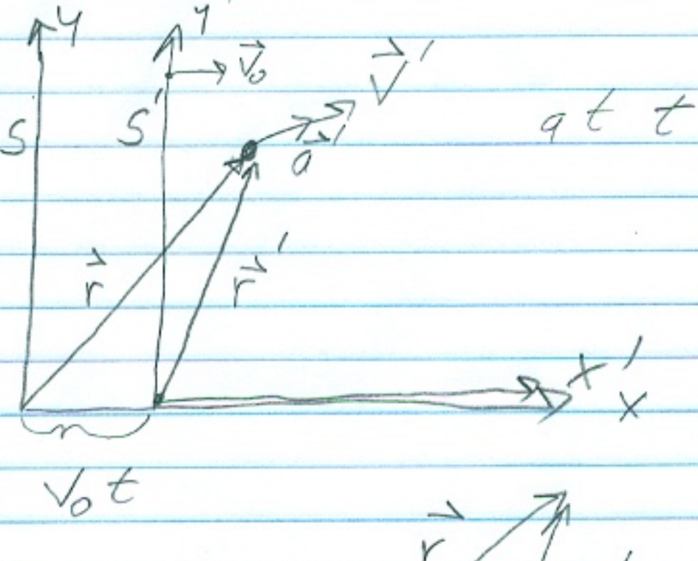
Relative Velocity and the Galilean Transformation (Sec. 4.4)

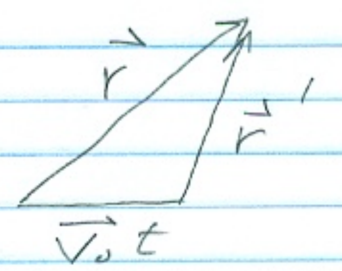
2-22-2021

- Consider our usual x-y cartesian frame (on the board)
- A particle is located at $\vec{r}$ with velocity $\vec{v}$ and acceleration $\vec{a}$



- Call this reference frame S which is at rest in the plane of the paper
- Now consider another reference frame, S', but moving with a constant velocity $\vec{V_0}'$ with respect to frame S,
- Let's let $\vec{V_0} = V_0 \hat{i}$
- Let the frames be coincident at $t = 0$
- The particle is S' is located at $\vec{r}'$ with velocity $\vec{v}'$ and acceleration $\vec{a}'$.





- $\vec{r} = \vec{r}' + \vec{V_0}\, t$

or $\boxed{\vec{r}\,' = \vec{r} - \vec{V}_0 t}$ Galilean transformation for position

- To get velocities, take $\frac{d}{dt}$

$$\Rightarrow \frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \frac{d(\vec{V}_0 t)}{dt} \Rightarrow \vec{V}_0 \frac{dt}{dt} = \vec{V}_0$$

$$\boxed{\vec{V}\,' = \vec{V} - \vec{V}_0}$$

Galilean velocity transformation

Velocity of particle in frame S'

Velocity of particle in frame S X

Velocity of frame S' with respect to S

- How about acceleration

$$\frac{d\vec{v}\,'}{dt} = \frac{d\vec{V}}{dt} - \cancelto{0}{\frac{d\vec{V}_0}{dt}}$$

$$\boxed{\vec{a}\,' = \vec{a}}$$

- Inertial frame — a reference frame moving with constant velocity $\vec{V}_0$
  - not accelerating
  - with respect to another inertial frame
- Noninertial frame — a reference frame that is accelerating

2-23-2021

- Why is this important?

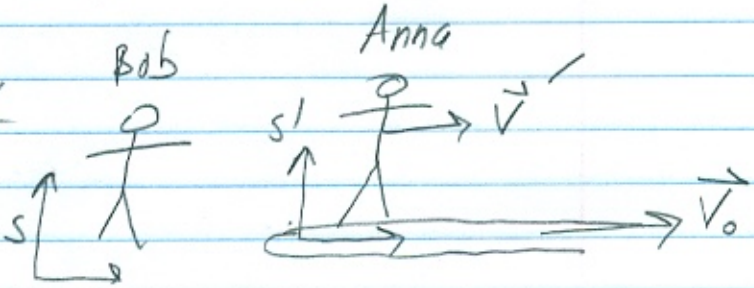
→ Newton's 2nd Law (and 1st law and kinematics) are only valid in an inertial frame

→ These laws break down in a non-inertial frame

Simple example for Velocity

- Bob and Anna are at the airport. Anna gets on a moving walkway. Bob stops before the walkway. The walkway has a velocity of 2 ft/s. Anna walks on the walkway with speed 1 ft/s.

a) what is the velocity of Anna with respect to Bob?



b) If Anna turns and walks toward Bob with speed 1 ft/s, now what is the velocity of Anna with respect to Bob?

Lorentz Transformation Intro 1-26-2016

- what if $|\vec{V}| \geq c$?

- for example our particle problem

$$\vec{V}_0 = \langle 0.8c, 0, 0 \rangle$$

$$\vec{V}' = \langle 0.6c, 0, 0 \rangle$$

$$\vec{V} = \vec{V}' + \vec{V}_0 = \langle 1.4c, 0, 0 \rangle \leftarrow \text{impossible}$$

- Einstein's Principle of Relativity (special)

  1. the laws of physics must be the same in all inertial frames (also from Newton 1st Law)

  2. Velocity of light is c in all inertial frames

Instead of a massive particle, lets use a photon

$$\vec{V}' = \langle c, 0, 0 \rangle$$

$$\vec{V} = \langle 1.8c, 0, 0 \rangle \quad \times$$

- therefore the Galilean transformation fails

$$\vec{r}' = \vec{r} - \vec{V}_0 t \xrightarrow[\text{instead}]{} \begin{aligned} x' &= \gamma(x - V_0 t) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{V}{c^2}x\right) \end{aligned} \qquad \vec{V}_0 = \langle V_0, 0, 0 \rangle$$

$t \neq t'$??

1-26-2022

- This is the Lorentz transformation for position
- To get the Lorentz transformation for velocities, we have to take appropriate derivatives, i.e. $\frac{dx'}{dt'} = V_x'$
- You will do this in PHYS 3700. The result is

$$V_x' = \frac{V_x - V_0}{1 - \frac{V_x V_0}{c^2}} \quad \text{or} \quad V_x = \frac{V_x' + V_0}{1 + \frac{V_x' V_0}{c^2}}$$

- Apply this to our problem, gives

$$V_x = \frac{0.6c + 0.8c}{1 + \frac{(0.6c)(0.8c)}{c^2}} = \frac{1.4c}{1.48} = \boxed{0.946c} < c$$