Example Problem 4.1

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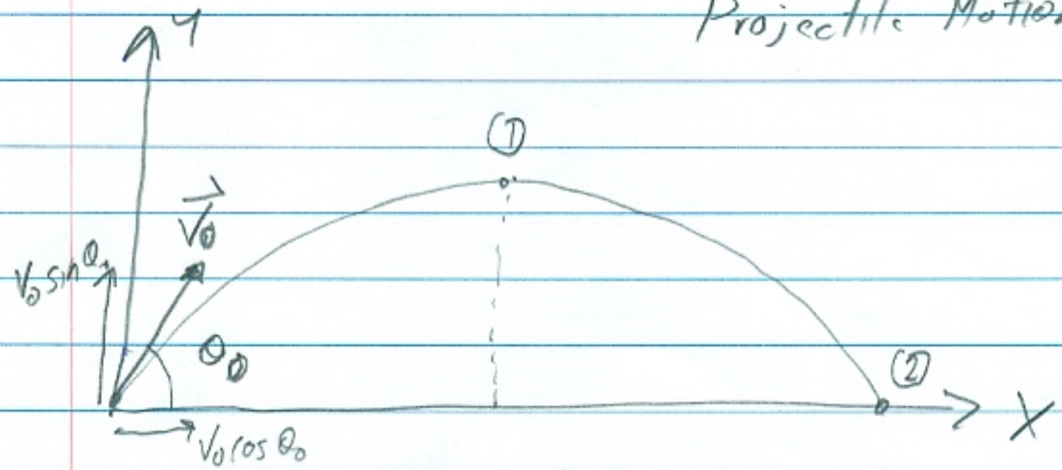
Projectile Motion



- to answer the equation consider motion from ⓪ → ①

- only concerned with motion in y-direction (x motion irrelevant)

- but need velocity components in y-direction

$V_{0y} = V_0 \sin\theta_0$ | also $t_0 = 0$, $y_0 = 0$, $x_0 = 0$

$V_{1y} = 0$ (top of arc)

- Use equation 3 to solve for t_1

→ $V_{1y} = V_{0y} - g(t_1 - t_0)$ solve for t_1

$$t_1 = \frac{V_{1y} - V_{0y}}{-g} = \frac{0 - V_0 \sin\theta_0}{-g}$$

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or $\boxed{t_1 = \frac{V_0 \sin\theta_0}{g}}$

or $t_1 = \frac{(50\, ft/s) \sin 37^\circ}{32.2\, ft/s^2} = \boxed{0.934\, s}$

- Maximum height the soccer ball reaches
- use 1st kinematic eq (could use 2nd or 4th)

$$y_1 = y_{max} = \underbrace{y_0}_{\to 0} + \frac{1}{2}(V_{0y} + \underbrace{V_{1y}}_{\to 0})(t_1 - \underbrace{t_0}_{\to 0})$$

or $y_{max} = \frac{V_{0y} t_1}{2} = \frac{(V_0 \sin\theta_0)}{2}\left(\frac{V_0 \sin\theta_0}{g}\right)$

or

$$\boxed{y_{max} = \frac{V_0^2 \sin^2\theta_0}{2g}}$$

or

$$y_{max} = \frac{(50)^2 \sin^2 37^\circ}{2(32.2)} = \boxed{14.06\, ft}$$

- Now lets look more closely at the motion
- What is the total distance traveled by the ball in the x-direction? $X_2 = ?$
- Use 2nd kinematic equation

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$$X_2 = \underset{0}{X_0} + V_{0x}(t_2 - \underset{0}{t_0}) + \frac{1}{2}\underset{0}{a_x}(t_2 - \underset{0}{t_0})^2$$

or

$$X_2 = V_{0x} t_2 = V_0 \cos\theta_0 \, t_2$$

- but what is t_2 ?

- Since the motion is symmetric about time t_1, $t_2 = 2t_1$

or $t_2 = \frac{2V_0 \sin\theta_0}{g}$

to give

$$X_2 = V_0 \cos\theta_0 \frac{2V_0 \sin\theta_0}{g} = \boxed{\frac{2V_0^2 \cos\theta_0 \sin\theta_0}{g}}$$

or

$$X_2 = \frac{2(50)^2 \cos 37^0 \sin 37^\circ}{32.2} = \boxed{75 \text{ Ft}}$$

$$\boxed{X_2 \equiv \text{Range}}$$

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- How can we determine θ_0 to give the maximum range?

- $\frac{d x_2}{d\theta_0} = 0$

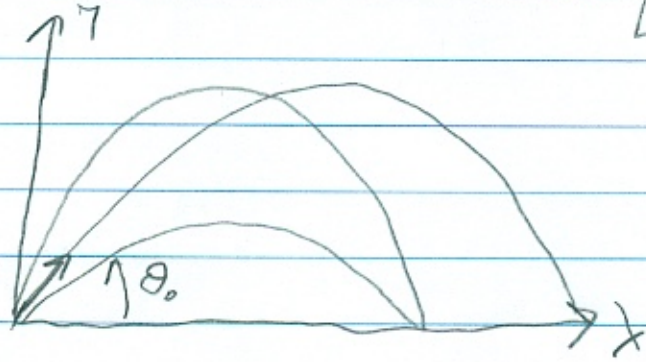
- first simplify x_2 using $\sin 2\theta = 2\sin\theta\cos\theta$

$$x_2 = \frac{V_0^2}{g} 2\cos\theta_0 \sin\theta_0 = \frac{V_0^2}{g}\sin 2\theta_0$$

$$\frac{dx_2}{d\theta_0} = \frac{V_0^2}{g}\cos 2\theta_0 \,(2) = 0$$

or $\cos 2\theta_0 = 0 \Rightarrow 2\theta_0 = 90^0$

or $\boxed{\theta_0 = 45^0}$



- what is velocity of the ball when it hits the ground

- For x-direction $V_{2x} = V_{0x}$

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– For y-direction use third kinematic equation

$$V_{2y} = V_{0y} - g(t_2 - t_0)$$

$$= V_0 \sin\theta_0 - g\frac{2V_0 \sin\theta_0}{g} = -V_0 \sin\theta_0$$

$$= -V_{0y}$$

$$|\vec{V}_2| = \sqrt{V_{2x}^2 + V_{2y}^2} = \sqrt{V_{0x}^2 + (-V_{0y})^2} = V_0$$

$$\theta_2 = \tan^{-1}\left(\frac{V_{2y}}{V_{2x}}\right) = \tan^{-1}\left(\frac{-V_{0y}}{V_{0x}}\right)$$

$$= \tan^{-1}\left(-\frac{V_0 \sin\theta_0}{V_0 \cos\theta_0}\right) = \tan^{-1}(-\tan\theta_0)$$

$$= -\theta_0$$