KEY

PHYS 1311 Spring 2018 Test 3

April 3, 2018

Name ______________________ **Student ID** ______________________ **Score** ________

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) For a non-conservative force, its x-component can be found from the relation $F_x = -dU(x)/dx$.

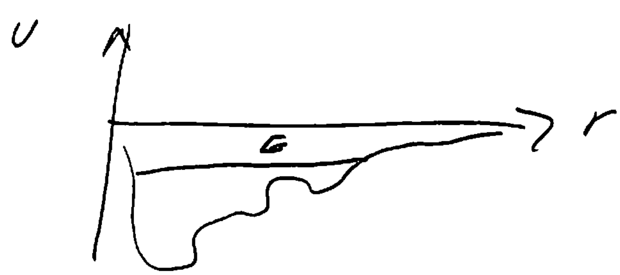
$\boxed{\text{False}}$ A potential energy does not exist for a non-conservative force

(b) For a multidimensional potential energy surface, an equipotential line has a constant potential energy.

$\boxed{\text{True}}$ equipotential $\equiv$ constant potential energy

(c) If the potential energy of a system is defined to be zero when the separation distance goes to infinity, the system is bound if the total system energy is negative.

$\boxed{\text{True}}$ $K+U<0$



Problem 2. Given the vectors $\vec{A} =< 1.00, 2.00, -3.00 >$ and $\vec{B} =< -3.00, 1.00, -2.00 >$, determine (a) $\vec{A} \cdot \vec{B}$, (b) the magnitudes $|\vec{A}|$ and $|\vec{B}|$, and (c) the angle between $\vec{A}$ and $\vec{B}$. (15 points total)

a) $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = (1)(-3) + (2)(1) + (-3)(-2)$

$= -3 + 2 + 6 = \boxed{5}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{1^2 + 2^2 + 3^2} = \boxed{3.742}$

$|\vec{B}| = \sqrt{3^2 + 1^2 + 2^2} = \boxed{3.742}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\phi, \ \phi = \cos^{-1}\left[\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right] = \cos^{-1}\left(\frac{5}{3.742^2}\right)$

$= \boxed{69.1^\circ}$

Problem 3. (a) Starting with the momentum update equation in one dimension, derive the *work-kinetic energy theorem.* Note, you will need to use the chain rule, set-up some integrals, and assume that mass remains constant. (b) If a spring of force constant k_s acts on a mass m, find the work done on the mass as its position moves from x_i to x_f in the horizontal direction (neglect friction and air resistance). (15 points total)

a) $P_f = P_i + F\Delta t$

$\Delta p = F\Delta t$

$\frac{\Delta p}{\Delta t} = F = m\frac{\Delta v}{\Delta t}$

or $\frac{dp}{dt} = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = m\frac{dv}{dx} v$

or $\int_{x_i}^{x_f} F\,dx = \int_{v_i}^{v_f} v m\,dv = m\int_{v_i}^{v_f} v\,dv$

$W = m\left[\frac{v^2}{2}\right]_{v_i}^{v_f} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$

or $\boxed{W = K_f - K_i}$

b) $F_s = -k_s x$

$W = \int_{x_i}^{x_f} F\,dx = -k_s \int_{x_i}^{x_f} x\,dx$

$= -k_s\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$

$W = -\left[\frac{1}{2}k_s x_f^2 - \frac{1}{2}k_s x_i^2\right]$

$\boxed{W = \frac{1}{2}k_s\left[x_i^2 - x_f^2\right]}$

Problem 4. Under certain conditions the interaction between a "polar" molecule such as HCl located at the origin and an ion located along the x-axis can be described by a potential energy $U = -b/x^6$, where b is a constant. Find the force vector on the the HCl due to the ion. (15 points total)

$\vec{F} = -\nabla U$ or $F_x = -\frac{dU}{dx}$, $F_y = -\frac{dU}{dy}$, $F_z = -\frac{dU}{dz}$

$F_x = -\frac{d}{dx}\left(-\frac{b}{x^6}\right) = b(-6)\frac{1}{x^7} = -\frac{6b}{x^7}$

$F_y = F_z = 0$, $\boxed{\vec{F} = \left\langle -\frac{6b}{x^7}, 0, 0 \right\rangle}$

Problem 5. A mass on a spring has an angular oscillation frequency of 2.81 rad/s. The mass has a maximum displacement (when $t = 0$ s) of 0.232 m. The spring constant is 45.9 N/m. a) What is the total energy of the system? b) What is the potential energy stored in the mass-spring system when $t = 1.42$ s? c) What is the velocity of the mass at $t = 1.42$ s? (15 points total)

$\omega = 2.81\,\frac{rad}{s}$, $A = 0.232\,m$, $k = 45.9\,N/m$, $\omega = \sqrt{\frac{k}{m}} \rightarrow m = \frac{k}{\omega^2}$

a) Total, $E = \frac{1}{2}kA^2 = \frac{1}{2}(45.9)(0.232)^2 = \boxed{1.235\,J}$

b) $E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, need x → $K = E - U = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2(E-U)}{m}}$

$x(t) = A\cos(\omega t + \phi)$, $\phi = 0$

$= 0.232\cos\left[\left(2.81\,\frac{rads}{s}\right)(1.42\,s)\right] = -0.1533\,m$

$m = \frac{45.9}{2.81^2} = 5.81\,kg$

$U = \frac{1}{2}kx^2 = \frac{1}{2}(45.9)(0.1533)^2 = \boxed{0.5398\,J}$

c) $v = \sqrt{\frac{2}{m}(E-U)} = \sqrt{\frac{2}{5.81}(1.235 - 0.5398)} = \boxed{0.489\,m/s}$

Problem 6. (a) Beginning with the universal gravitational force relation, $\vec{F} = -(GMm/r^2)\,\hat{r}$, derive the work done by gravity on an object of mass m as it moves from r_i to r_f. (b) If a Tesla Roadster is launched by a giant catapult from the surface of the Earth, what must its initial speed be to escape the Earth? (30 points total)

a) $$W = \int \vec{F}\cdot d\vec{s} = \int_{\vec{r}_i}^{r_f} \vec{F}\cdot d\vec{r} = -GMm \int_{\vec{r}_i}^{\vec{r}_f} \frac{\hat{r}\cdot d\vec{r}}{r^2}$$

$$= -GMm \int_{r_i}^{r_f} \frac{dr}{r^2} \quad \text{since } \hat{r}\cdot d\vec{r} = \underbrace{|d\vec{r}|}_{}\underbrace{|\hat{r}|}_{=1}\underbrace{\cos\phi}_{=0} = dr$$

$$= -GMm \left[\frac{-1}{r}\right]_{r_i}^{r_f} = \boxed{GMm\left[\frac{1}{r_f} - \frac{1}{r_i}\right]} = -\Delta U$$

b) Total energy is $E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$

m = mass of Tesla Roadster

M = mass of Earth

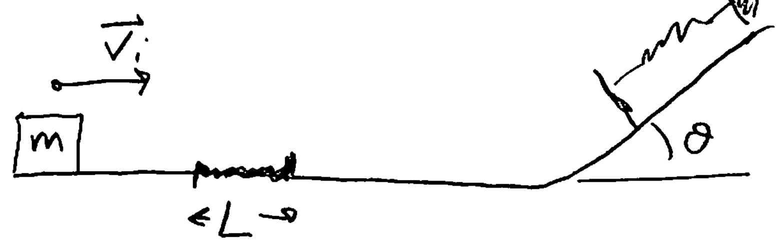
for Tesla to escape Earth $E = 0 = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r}$

$$\boxed{V_{esc} = \sqrt{\frac{2GM}{r}}}\ ,\ r = R_E\ ,\ V_{esc} \neq f(m)$$

$$= \sqrt{\frac{2\,(6.673\times10^{-11})(5.97\times10^{24}\ \text{kg})}{(6380\times10^{3})}} = 11{,}175\ \text{m/s}$$

$$= \boxed{11.2\ \text{km/s}}$$

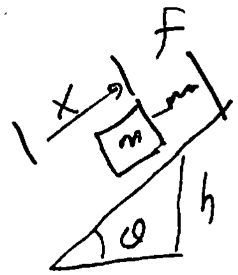
Bonus Problem. A 5.00-kg block has an initial velocity of < 10.0, 0, 0 > m/s. It slides across a flat frictionless surface, but encounters a rough surface with coefficient of kinetic friction $\mu_k = 0.8$. After crossing this surface it encounters again another frictionless surface, but with an incline. At the top of the incline is a spring with force constant of 50 N/m. If the block compresses the spring to a maximum displacement of 1.00 m when the block comes to rest at a height of 2.00 m., what was the length of the rough surface. (5 points total)



$$E_f = E_i + W_{surr}$$

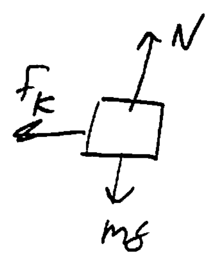
$$E_i = \frac{1}{2} m V_i^2$$

$$E_f = \frac{1}{2} k x^2 + mgh \quad \text{at top}$$



$$W_{surr} = W_{friction} = f_k L = -\mu_k m g L$$

Then total energy equation is



$N = mg$

$f_k = \mu_k N$

$= \mu_k mg$

$$\frac{1}{2} k x^2 + mgh = \frac{1}{2} m V_i^2 - \mu_k m g L$$

solve for L

$$\mu_k m g L = \frac{1}{2} m V_i^2 - \frac{1}{2} k x^2 - mgh$$

$$L = \frac{\frac{1}{2} m V_i^2}{\mu_k m g} - \frac{\frac{1}{2} k x^2}{\mu_k m g} - \frac{mgh}{\mu_k m g}$$

$$= \frac{V_i^2}{2\mu_k g} - \frac{k x^2}{2\mu_k m g} - \frac{h}{\mu_k}$$

$$L = \frac{(10)^2}{2(0.8)(9.8)} - \frac{50(1)^2}{2(0.8)(5)(9.8)} - \frac{2}{0.8}$$

$$L = 6.38 - 0.638 - 2.5$$

$$= \boxed{3.24\ \text{m}}$$