KEY

## PHYS 1311 Spring 2018 Test 2

Feb. 22, 2018

**Name** ______________ **Student ID** ______________ **Score** ______

**Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.**

**Problem 1. Conceptual questions.** State whether the following statements are *True* or *False.* (10 points total, no calculations required)

(a) If the spring force acting on a mass $m$ was given by $\vec{F} = -a^2 x\hat{i}$, the resulting oscillatory motion would not be simple harmonic (where $a$ is a constant).

False $|\vec{F}| = a^2 x = kx$ where $k = a^2$

(b) The orbital speed of Mercury is smaller than that of Jupiter.

False orbital speed $v = \sqrt{\frac{GM}{r}}$ $M$ = mass of sun, $r$ = orbital radius

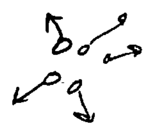
(c) For a firecracker at rest on the street, its total momentum is zero before and after it explodes.

system

True

before: $0$

after immediately



$\vec{P}_i = 0$ $\vec{P}_f = 0$

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**Problem 2.** If a simple pendulum on the Earth is designed to have a period of 1.00 s, what is its length? Would this pendulum's period be longer or shorter on the Moon. (15 points total)

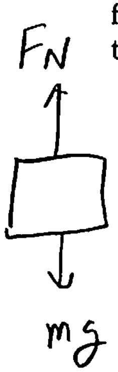
a) $\omega_{sp} = \sqrt{\frac{g}{L}} = \frac{2\pi}{T} \rightarrow T = 2\pi\sqrt{\frac{L}{g}}$ or $T^2 = 4\pi^2 \frac{L}{g}$

or $L = \frac{T^2 g}{4\pi^2} = \frac{(1.0\,s)^2 (9.8\,m/s^2)}{4\pi^2} = \boxed{0.25\,m}$

b) $g_{moon} < g_{earth}$

$T_{moon} > T_{earth}$ $\boxed{\text{longer}}$

**Problem 3.** Off the coast of Alaska during an episode of the *Deadliest Catch*, a lobsterman conducts an experiment by standing on a bathroom scale on his ship. When at the dock with calm waters, the scale reads 830.0 N. When the ship is at sea in a storm, the lobsterman finds a maximum reading from the scale of 1100.0 N and a minimum of 650.0 N. Calculate the maximum and minimum acceleration experienced by the lobsterman. (15 points total)




Weigth $= mg = 830.0\,N$, $F_N \equiv$ what scale reads

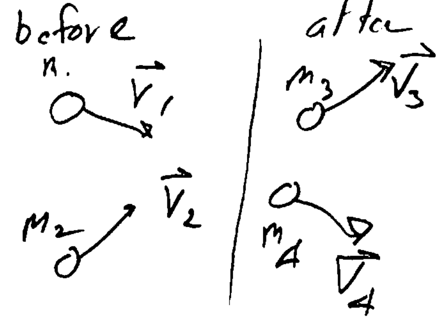
$+\uparrow \sum F_y = F_N - mg = ma_y$

or $a_y = \frac{F_N - mg}{m} = \left(\frac{F_N - mg}{mg}\right) g$

max $a_y = \left(\frac{1100 - 830}{830}\right) 9.8\,m/s^2 = \boxed{3.19\,m/s^2}$

min $a_y = \left(\frac{650 - 830}{830}\right) 9.8\,\frac{m}{s^2} = \boxed{-2.13\,m/s^2}$

**Problem 4.** Two rocks in the asteroid belt collide. Before the collision, one rock had a mass of 10.0 kg and a velocity of $< 4000, -2500, 3000 >$ m/s. The other rock had a mass of 5.00 kg and a velocity of $< -500, 2000, 2500 >$ m/s. During the collision, a 2.00-kg chunk of the first rock breaks off and sticks to the second rock. The remaining 8.00 kg rock has a velocity after the collision of $< 1300, 300, 1800 >$ m/s. What is the velocity of the new 7.00-kg rock after the collision? (15 points total)




Use conservation of momentum

$$\vec{P}_i = \vec{P}_f$$

$$\vec{P}_1 + \vec{P}_2 = \vec{P}_3 + \vec{P}_4$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_3\vec{V}_3 + m_4\vec{V}_4$$

$m_1 = 10$ kg

$m_2 = 5$ kg

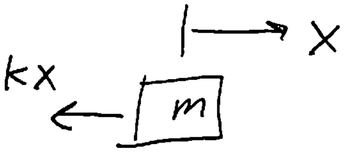
$m_3 = 8$ kg

$m_4 = 7$ kg

$$\vec{V}_4 = \frac{m_1\vec{V}_1 - m_3\vec{V}_3 + m_2\vec{V}_2}{m_4} = \frac{10\langle 4000, -2500, 3000\rangle - 8\langle 1300, 300, 1800\rangle + 5\langle -500, 2000, 2500\rangle}{7}$$

$$\boxed{\vec{V}_4 = \langle 3870, 2490, 4010\rangle \text{ m/s}}$$

**Problem 5.** A spring-mass system oscillates in the horizontal direction without friction. Starting with Newton's second law, derive the period of oscillation of the system in terms of the mass $m$ and force constant $k$. (15 points total)




$$\xrightarrow{+} \Sigma F_x = m a_x$$

$$-kx = m a_x$$

SHM, $x(t) = A\cos(\omega t + \phi_0)$

$$a(t) = -A\omega^2\cos(\omega t + \phi_0)$$

$$\cancel{-}k\left[\cancel{A\cos(\omega t + \phi_0)}\right] = m\left[\cancel{-A}\omega^2\cancel{\cos(\omega t + \phi_0)}\right]$$

$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

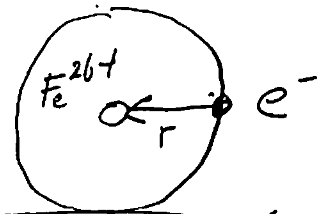
or $T = \dfrac{2\pi}{\omega} = \boxed{2\pi\sqrt{\dfrac{m}{k}}}$

**Problem 6.** Consider an electron in uniform circular motion around a bare iron nucleus (26 protons and 30 neutrons, no electrons) orbiting with a radius of $0.2 \times 10^{-11}$ m. a) Calculate the magnitude of the electric force between the electron and the iron nucleus b) Is the force attractive or repulsive? c) What is the orbital speed of the electron? d) Is this speed relativistic? e) Calculate the magnitude of the electron's momentum. f) Calculate the electron's angular speed, angular acceleration, radial acceleration, and tangential acceleration. (30 points).

$|\vec{r}|$

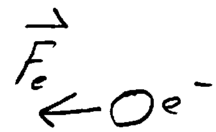
a) $q_1 = 26e, \quad q_2 = -e$

$$|\vec{F_e}| = \frac{k_e q_1 q_2}{|\vec{r}|^2} = \frac{k_e 26 e^2}{r^2}$$




$$= \frac{(8.99 \times 10^{9})(26)(1.602 \times 10^{-19})^2}{(0.2 \times 10^{-11})^2} = \boxed{1.500 \times 10^{-3} N}$$

b) attractive since $q_1 q_2 < 0$




c) $\Sigma F_r = m a_r$ (uniform circular motion)

$$|\vec{F_e}| = \frac{m v^2}{r} \quad \text{or} \quad v = \sqrt{\frac{|\vec{F_e}| r}{m}}$$

$$\text{or } v = \sqrt{\frac{(1.5 \times 10^{-3})(0.2 \times 10^{-11} m)}{9.109 \times 10^{-31}}} = \boxed{5.74 \times 10^{7} m/s = 0.19c}$$

d) $\boxed{\text{Relativistic}}$ barely $\quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{0.191c}{c}\right)^2}} = 1.02$

e) $p = \gamma m v$

$$= (1.02)(9.109 \times 10^{-31} kg)(5.74 \times 10^{7} m/s) = \boxed{5.33 \times 10^{-23} kg \frac{m}{s}}$$

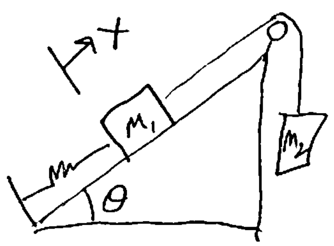
f) UCM → $\omega$ is constant

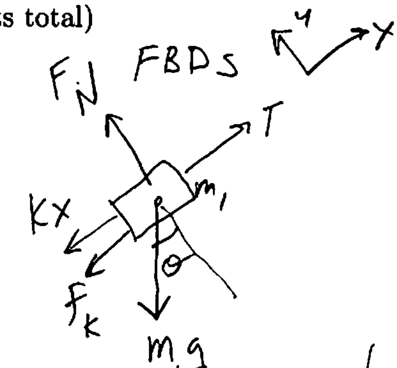
$\boxed{\alpha = 0}$, $a_t = r\alpha = \boxed{0}$

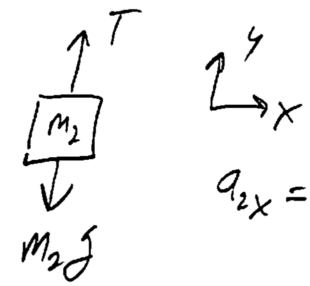
$$\omega = \frac{v_t}{r} = \frac{5.74 \times 10^{7} m/s}{0.2 \times 10^{-11} m} = \boxed{2.87 \times 10^{19} \frac{rad}{s}}$$

$$a_r = \frac{v_t^2}{r} = \boxed{1.65 \times 10^{27} \frac{m}{s^2}}$$

**Bonus Problem.** Consider masses $m_1$ and $m_2$ connected by a rope. $m_1$ is on an incline of angle $\theta$ while $m_2$ is suspended vertically by the rope over a frictionless pulley. $m_1$ slides up the incline and there is friction between $m_1$ and the incline with coefficient of kinetic friction $\mu_k$. On the opposite side of the mass $m_1$ from the rope, a spring is attached to $m_1$ with force constant $k$. If the spring is initially unstretched, derive a relation for the acceleration of $m_1$ as a function of its motion up the incline, $x$, in terms of $m_1$, $m_2$, $g$, $\theta$, $\mu_k$, and $x$. Find the value of $x$ for which $m_1$ comes to rest and give the tension in the rope $T$ at that time. (Figure to be provided). (5 points total)




$a_{2x} = 0$

$+\nearrow \sum F_x = m_1 a_{1x}$

$T - kx - f_k - m_1 g \sin\theta = m_1 a_{1x}$

or

$m_1 a_{1x} = \dfrac{T - kx - \mu_k F_N - m_1 g \sin\theta}{m_1}$

$+\nwarrow \sum F_y = m_1 a_{1y}$

$F_N - m_1 g \cos\theta = 0$

$F_N = m_1 g \cos\theta$

$+\uparrow \sum F_y = m_2 a_{2y}$

$T - m_2 g = m_2 a_{2y}$

$T = m_2 (g + a_{2y})$

$a_{1x} = -a_{2y} = a$

$m_1 a_{1x} = m_2 g + m_2 a_{2y} - kx - \mu_k m_1 g \cos\theta - m_1 g \sin\theta$

$m_1 a = m_2 g - m_2 a - kx - m_1 g (\mu_k \cos\theta - \sin\theta)$

or

$(m_1 + m_2) a = g [m_2 - m_1 (\mu_k \cos\theta - \sin\theta)] - kx$

or

$$\boxed{a = g \frac{[m_2 - m_1 (\mu_k \cos\theta - \sin\theta)]}{m_1 + m_2} - \left(\frac{k}{m_1 + m_2}\right) x}$$

$a \to 0$ $\boxed{x = \frac{g}{k} [m_2 - m_1 (\mu_k \cos\theta - \sin\theta)]}$ $\boxed{T = m_2 g}$