KEY

PHYS 1311 Spring 2018 Test 1
Jan. 30, 2018

Name ____________________ **Student ID** ____________________ **Score** ______

Note: This test consists of one set of conceptual questions, five problems, and a bonus problem. For the problems, you *must show all* of your work, calculations, and reasoning clearly to receive credit. Be sure to include units in your solutions where appropriate. An equation sheet is provided on the last page.

Problem 1. Conceptual questions. State whether the following statements are *True* or *False*. (10 points total, no calculations required)

(a) A particle which has zero mass can travel at a speed greater than that of light.

False

(b) Newton's 3rd Law of Motion applies to the interaction between two objects.

True

(c) For the two vectors $\vec{A}$ and $\vec{B}$, the operation $\vec{A}/\vec{B}$ is valid.

False

(d) Since the mass of the Guardian Galaxy is found to be twice that of the Milky Way, the Universal Gravitational Constant G must be twice its value on Earth.

False

Problem 2. A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration is 3.5 m/s^2 larger than the Honda's 3.0 m/s^2 acceleration, the Honda gets a 1.0 s head start. Who wins? By how many seconds? (15 points total)

$t_f = ?$ $x_i = 0$ $v_i = 0$ $t_i = 0$

Use this kinematic equation

$$x_f = x_i + v_i t_f + \frac{1}{2} a_x t_f^2 = 0 + 0 + \frac{1}{2} a_x t_f^2$$

or $t_f = \sqrt{\frac{2x_f}{a_x}}$

Total Honda time $t_H = \sqrt{\frac{2(400)}{3}} = 16.33\,s$

Porsche time $t_p = \sqrt{\frac{2(400)}{3.5+3}} = 11.09\,s$

Total Porsche time $1 + 11.09\,s = 12.09\,s$ ← $\boxed{\text{Porsche wins}}$

$\Delta t_{PH} = 16.33 - 12.09 = \boxed{4.24\,s}$

Problem 3. (a) If an electron has a speed of 2.5×10^8 m/s, compute the magnitude of its momentum. (b) For a particle of mass m moving at a non-relativistic speed v being acted on by a net external force $\vec{F}_{\rm net}$, derive the momentum update equation and the definition of momentum by starting with Newton's 2nd Law, $\vec{F}_{\rm net} = m\vec{a}$. (15 points total)

a) $|\vec{p}| = \gamma m |\vec{v}|$ $\quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{2.5}{2.998}\right)^2}} = 1.82$

$|\vec{p}| = (1.82)(9.109\times10^{-31}\,kg)(2.5\times10^8\,m/s) = \boxed{4.13\times10^{-22}\,kg\frac{m}{s}}$

b) $\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$

define $\vec{p} = m\vec{v}$ $(\gamma = 1)$ $\Rightarrow \vec{F}_{net} = \frac{d\vec{p}}{dt}$

or $d\vec{p} = \vec{F}_{net}\,dt$

or $\Delta\vec{p} = \vec{F}_{net}\,dt = \vec{p}_f - \vec{p}_i$ $\quad \boxed{\vec{p}_f = \vec{p}_i + \vec{F}_{net}\,\Delta t}$

$\vec{V_f}$

$\vec{F}_{net}$

Problem 4. A small rock in the asteroid belt is acted upon by a constant force of $<10, -50, 20>$ N during a 5 second time interval. At the end of this time, the rock has a velocity of $<100, 50, 100>$ m/s. If the rock has a mass of 5 kg, use the momentum principle to find its initial velocity. (15 points total)

m

Momentum principle $\quad |\vec{v}| << c \rightarrow \vec{p} = m\vec{v}$

$\vec{P}_f = \vec{P}_i + \vec{F}_{net}\Delta t$, divide by m

$\vec{V}_f = \vec{V}_i + \frac{\vec{F}_{net}}{m}\Delta t$

or $\vec{V}_i = \vec{V}_f - \frac{\vec{F}_{net}}{m}\Delta t = \langle 100, 50, 100\rangle - \frac{\langle 10, -50, 20\rangle(5)}{5}$

$= \boxed{\langle 90, 100, 80\rangle \frac{m}{s}}$

Problem 5. If the force acting in Problem 4 is the universal gravitation force,

$$\vec{F}_g = -\frac{Gm_1m_2}{|\vec{r}|^2}\hat{r}, \quad = |\vec{F}_g|\,\hat{F} \tag{1}$$

find the magnitude of the force and $\hat{r}$. If the force is due to an asteroid a distance of 100.0 m from the rock, find $\vec{r}$ and the mass of the asteroid. (15 points total)

$m_2 \qquad r = |\vec{r}|$

$m_1 = 5\text{ kg}$

$|\vec{F}_g| = \sqrt{10^2 + 50^2 + 20^2} = \boxed{54.77 N}$

$\hat{F} = -\hat{r} = \frac{\vec{F}_g}{|\vec{F}_g|} = \frac{\langle 10, -50, 20\rangle N}{54.77 N} = \langle 0.183, -0.913, 0.365\rangle$

or $\boxed{\hat{r} = \langle -0.183, 0.913, -0.365\rangle}$

$m_2 = \frac{|\vec{F}_g||\vec{r}|^2}{G m_1} = \frac{(54.77)(100)^2}{(6.673 \times 10^{-11})5}$

$= \boxed{1.64 \times 10^{15} \text{ kg}}$

$\vec{r} = |\vec{r}|\hat{r} = 100m\langle -0.183, 0.913, -0.365\rangle$

$= \boxed{\langle -18.3, 91.3, -36.5\rangle m}$

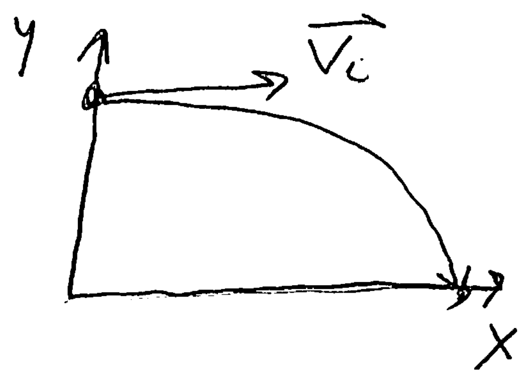
Problem 6. A friend of yours on the UGA baseball team wants to determine her pitching speed. You have her stand on a ledge of height 4.00 m above the ground and throw the ball horizontally. The ball lands a horizontal distance of 25.0 m away. What was her pitching speed? What is the total time the ball is in the air? Give the final velocity of the baseball in component form. Find the magnitude and direction angle of the final velocity. (30 points total)

$\boxed{V_{xi} = ?}$, $V_{xf} = V_{xi}$, $V_{yi} = 0$

$y_i = 4m$, $y_f = 0$, $\boxed{V_{yf} = ?}$

$x_i = 0$, $x_f = 25m$

$t_i = 0$, $\boxed{t_f = ?}$ $\vec{a}_y = -g\hat{j}$



Find time t_f

$$y_f = y_i + V_{yi} t_f - \frac{1}{2} g t_f^2$$

$$0 = y_i + 0 - \frac{1}{2} g t_f^2 \Rightarrow t_f = \sqrt{\frac{2y_i}{g}}$$

$$\text{or } t_f = \sqrt{\frac{2(4m)}{9.8\,m/s^2}} = \boxed{0.9035 s}$$

initial velocity

$$x_f = x_i + \frac{1}{2}(V_{xi} + V_{xf}) t_f = 0 + V_{xi} t_f$$

$$\text{or } V_{xi} = \frac{x_f}{t_f} = \frac{25.0m}{0.9035s} = \boxed{27.67\,m/s}$$

Final velocity

$$V_{yf} = V_{yi} - g t_f = 0 - g t_f = -(9.8\tfrac{m}{s^2})(0.9035s$$

$$= \boxed{-8.854\tfrac{m}{s}} \Rightarrow \boxed{\vec{V}_f = \langle 27.67, -8.854, 0 \rangle \tfrac{m}{s}}$$

$$|\vec{V}_f| = \sqrt{27.67^2 + 8.854^2} = \boxed{29.1\tfrac{m}{s}} \quad \theta_f = \tan^{-1}\left(\frac{-8.854}{27.67}\right) = \boxed{-17.74^\circ}$$

Bonus Problem. The Lorentz transformation for position relating a particle in frame S to a frame S' where S' is moving with constant velocity $\vec{v}_0 =< v_0, 0, 0 >$ is given by

$$x' = \gamma(x - v_0 t), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - \frac{v_0}{c^2}x). \tag{2}$$

Prove by taking derivatives that the transformation for the x' and y' components of the velocity are given by

$$v'_x = \frac{dx'}{dt'} = \frac{v_x - v_0}{(1 - \frac{v_x v_0}{c^2})}, \qquad v'_y = \frac{dy'}{dt'} = \frac{v_y}{\gamma(1 - \frac{v_x v_0}{c^2})}. \tag{3}$$

where $dv_x = dx/dt$, $dv_y = dy/dt$, and $\gamma = (1 - (v_0/c)^2)^{-1/2}$. (5 points total)

$$\frac{dt'}{dt} = \frac{d}{dt}\left(\gamma\left(t - \frac{v_0}{c^2}x\right)\right) \qquad \gamma = \text{constant}$$

$$= \gamma\left(\frac{dt}{dt} - \frac{v_0}{c^2}\frac{dx}{dt}\right) = \gamma\left(1 - \frac{v_0}{c^2}v_x\right)$$

$$\frac{dx'}{dt} = \gamma\left(\frac{dx}{dt} - v_0\frac{dt}{dt}\right) = \gamma(v_x - v_0)$$

$$\frac{dx'}{dt'} = v_x' = \frac{dx'}{dt}\frac{dt}{dt'} = \frac{\gamma(v_x - v_0)}{\gamma(1 - \frac{v_0}{c^2}v_x)} = \boxed{\frac{v_x - v_0}{1 - \frac{v_0}{c^2}v_x}}$$

$$\frac{dy'}{dt} = \frac{dy}{dt} = v_y$$

$$\frac{dy'}{dt'} = v_y' = \frac{dy'}{dt}\frac{dt}{dt'} = \boxed{\frac{v_y}{\gamma(1 - \frac{v_0}{c^2}v_x)}}$$