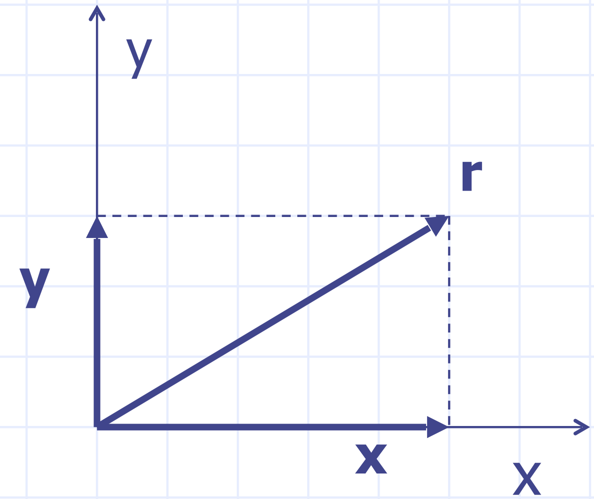
Kinematics in 2D

- Motion in a plane, vertical or horizontal
- But, the motion in the x- and y-directions are independent, except that they are coupled by the time
- Therefore, we can break the problem into x and y ``parts”
- We must use vectors:

position $\mathbf{r} = \mathbf{x} + \mathbf{y}$
velocity $\mathbf{v} = \mathbf{v}_x + \mathbf{v}_y$
acceleration $\mathbf{a} = \mathbf{a}_x + \mathbf{a}_y$



- Usually,

$$\vec{a}_y = -g\,\hat{j}$$

Two Sets of Kinematic Equations

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})(t_f - t_i)$$

$$x_f = x_i + v_{xi}(t_f - t_i) + \frac{1}{2}a_x(t_f - t_i)^2$$

$$v_{xf} = v_{xi} + a_x(t_f - t_i)$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$y_f = y_i + \frac{1}{2}(v_{yi} + v_{yf})(t_f - t_i)$$

$$y_f = y_i + v_{yi}(t_f - t_i) + \frac{1}{2}a_y(t_f - t_i)^2$$

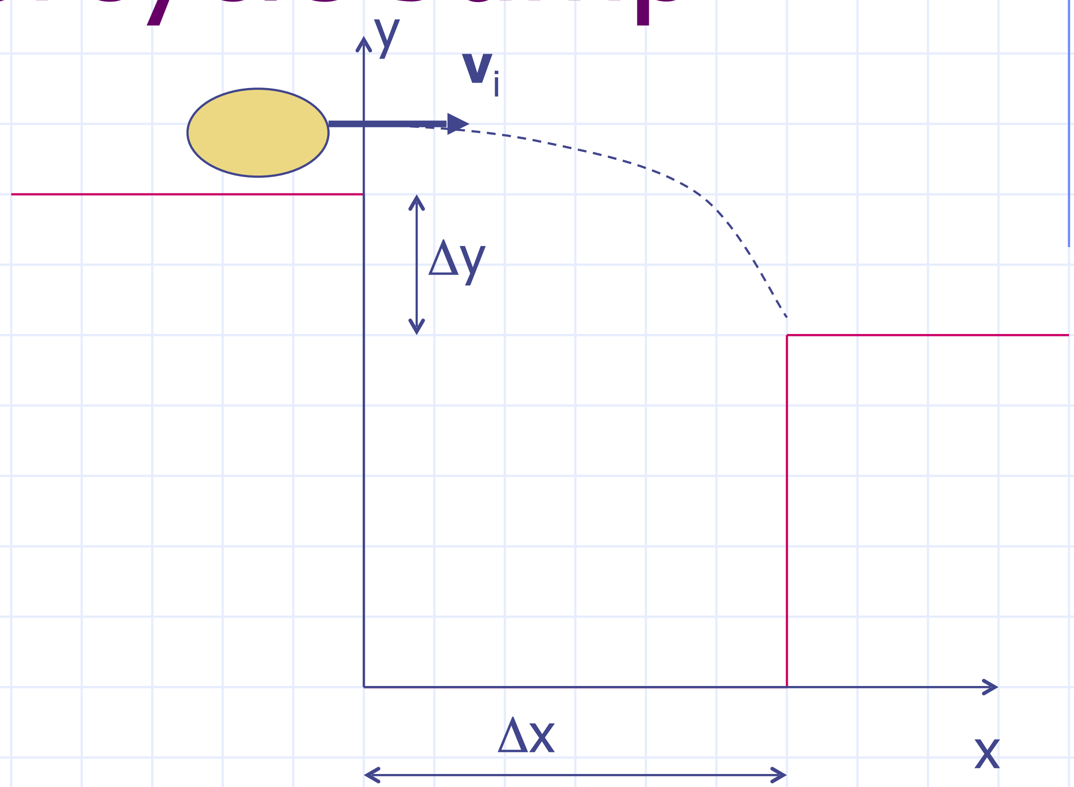
$$v_{yf} = v_{yi} + a_y(t_f - t_i)$$

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

We can solve problems using the same methods as for 1D, but now we need to consider both x and y components simultaneously

Example: Motorcycle Jump

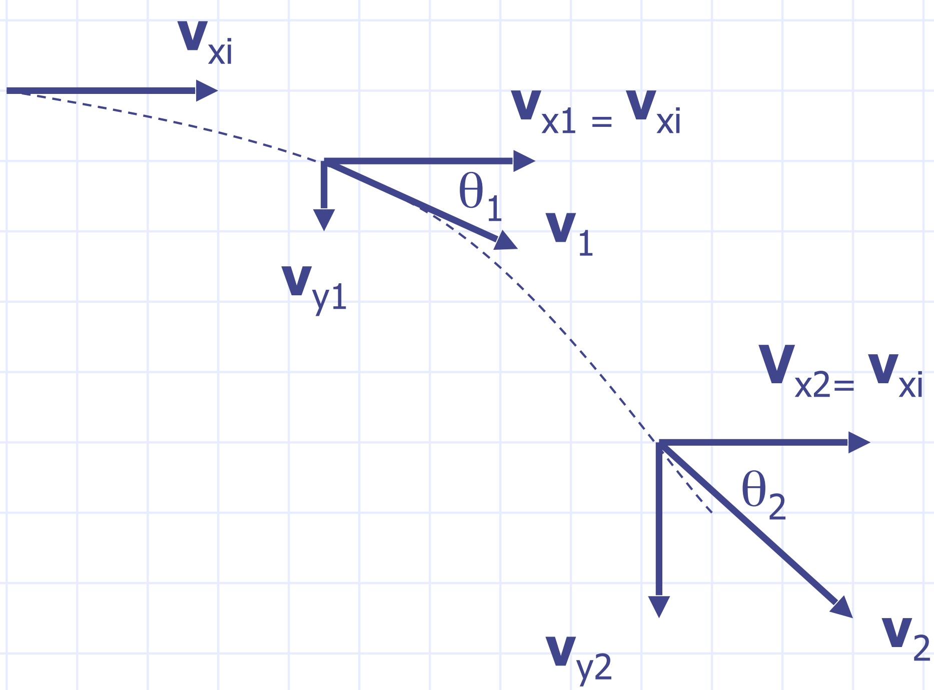
Consider a motorcycle jumping between two buildings separated by a distance Δx. The difference in heights of the buildings is Δy. What initial velocity must the motorcycle have to just make it to the other building? What is the time to cross to the other building? What is the final velocity on impact?



$v_{xi} = ?,\ v_{yi} = 0$

$a_x = 0,\ a_y = -g$

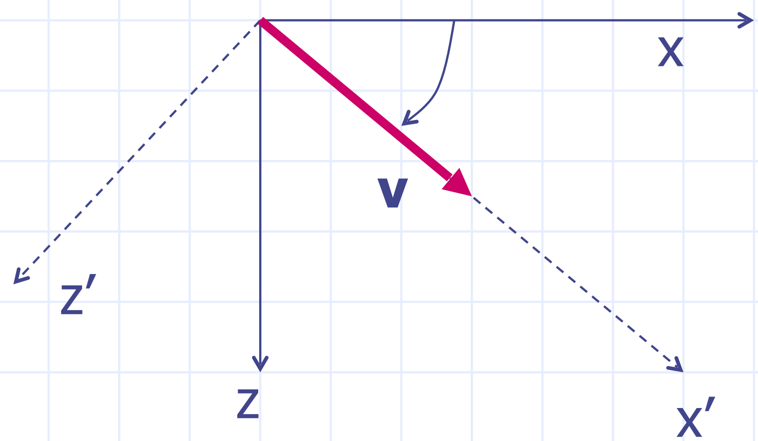
- The x-component v_{xf} of the velocity remains constant throughout the flight time, $v_{xf}=v_{xi}$,
  - we neglect air resistance (for now)
  - $a_x = 0$
  - therefore, nothing to affect x-motion
- Because of gravity, once the motorcycle is in the air, its speed in the y-direction v_{yf} increases from zero, points down, and therefore the height decreases
- The magnitude of the resultant velocity also increases and the angle of the resultant velocity vector (with respect to the x-axis) changes



- x-direction motion is the same as if motion occurred on a flat surface
- y-direction motion is equivalent to dropping the motorcycle

What about 2D motion in the horizontal plane?

- No acceleration due to gravity
- Can rotate coordinate system to reduce problem to 1D
  - if the motion is in a straight line
- If motion has a curvature
  - must be treated as 2D (later)



Return to Motorcycle Problem

x-direction

1. $\Delta x = (v_{xi}+v_{xf})/2\ \Delta t = v_{xi}\ \Delta t$ ➔ $\Delta t = \Delta x/v_{xi}$
2. Same information as first equation
3. $v_{xf}=v_{xi}$ since $a_x=0$
4. Same information as third equation

y-direction

1. $\Delta y = (v_{yf}\ \Delta t)/2$ ➔ $v_{yf} = 2\Delta y/\ \Delta t$
2. $\Delta y = -g\Delta t^2/2$ ➔ $\Delta t = \sqrt{(-2\Delta y/g)}$
3. $v_{yf}= -g\ \Delta t$

- $v^2_{yf}= -2g\ \Delta y$

*three different ways to get v_{yf}

$$\Delta t = \frac{\Delta x}{v_{xi}} = \sqrt{\frac{-2\Delta y}{g}} \quad \rightarrow \quad v_{xi} = \Delta x \sqrt{\frac{-g}{2\Delta y}}$$

What is the final velocity when motorcycle lands on the other roof?

We know from the 3rd x-direction equation that:

$v_{xf} = v_{xi}$

Therefore we need only v_{yf}. From 4th y-direction equation:

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{v_{xi}^2 - 2g\Delta y} = v_{xi}\sqrt{1 - \frac{2g\Delta y}{v_{xi}^2}}$$

If $\Delta y \rightarrow 0$ or $g \rightarrow 0$ or $v_{xi} \rightarrow \infty$, then $v_f \rightarrow v_{xi}$

If $\Delta y \rightarrow -\infty$ or $g \rightarrow \infty$ or $v_{xi} \rightarrow 0$, then $v_f \rightarrow v_{yf}$

Let's add some numbers:

$\Delta x = 50.0$ ft, $\Delta y = -20.0$ ft, $g = 32.2$ ft/s^2

$v_{xi} = \Delta x \sqrt{(-g/(2\,\Delta y))} = (50.0 \text{ ft}) \sqrt{((-32.2 \text{ ft/s}^2)/(2(-20.0 \text{ ft})))}$

$= \boxed{44.9 \text{ ft/s}}$

$\Delta t = \Delta x / v_{xi} = (50.0 \text{ ft})/(44.9 \text{ ft/s}) = \boxed{1.11 \text{ s}}$

Or $\Delta t = \sqrt{(-2\,\Delta y/g)} = \sqrt{(-2(-20.0 \text{ ft}))/(32.2 \text{ ft/s}^2))} = \boxed{1.11 \text{ s}}$

$v_{yf} = -g\,\Delta t = -(32.2 \text{ ft/s}^2)(1.11\text{s}) = \boxed{-35.9 \text{ ft/s}}$

Or $v_{yf} = -\sqrt{(-2g\,\Delta y)} = -\sqrt{(-2(32.2 \text{ ft/s}^2)(-20.0 \text{ ft}))}$

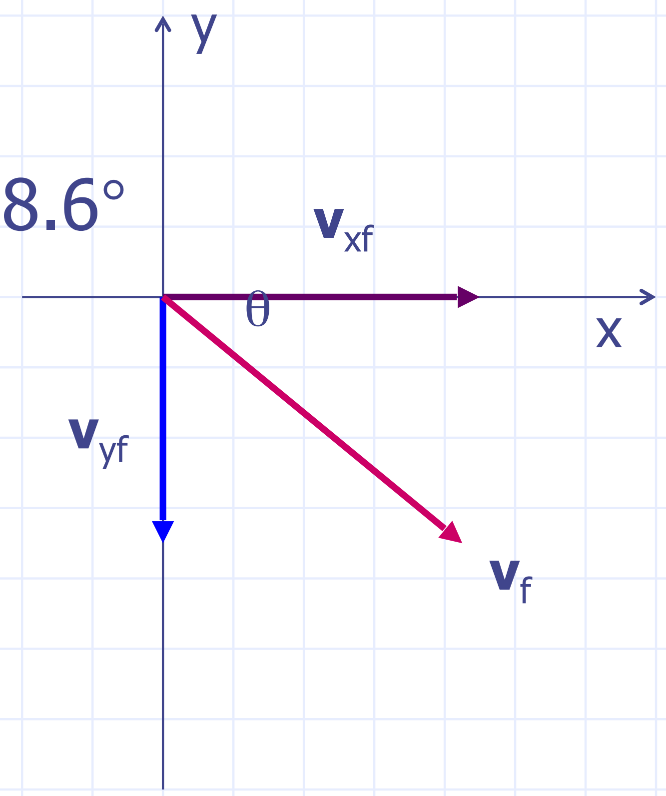
$= \boxed{-35.9 \text{ ft/s}}$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{[(44.9\ ft/s)^2 + (35.9\ ft/s)^2]}$$

$$= 57.5\ ft/s$$

$$\theta = \sin^{-1}(v_{yf}/v_f) = \sin^{-1}(-35.9/57.5) = -38.6°$$

v_f = 57.5 ft/s @ - 38.6°



Example Problem

- A soccer player kicks the soccer ball with an initial speed of 50 ft/s at an angle of 37° with respect to the horizontal. Find the maximum height of the ball's trajectory and the time it is at that point.
- Given: v_0=50 ft/s, θ_0= 37° , v_{y1}=0

  Take: t_0=0, y_0=0

  Find: t_1=? and y_1=h=?

Example Problem

- A skier leaves a ramp of a ski jump with a velocity of 10.0 m/s, 15.0º above the horizontal. He lands on a slope of incline 50.0º. Neglecting air resistance, find (a) the distance from the ramp to where the jumper lands and (b) the velocity components just before landing.